THE METHOD OF STRENGTH CALCULATION IN VOLUME
FRACTURE AND SURFACE DAMAGE

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ABSTRACT
In this paper the development of the methods of strength
calculation are described: an engineering procedure is
developed to perform calculations using the criteria of surface
strength, i.e. wear resistance.

INTRODUCTION
The method of strength calculation has presently reached
perfection. It is impossible to state that similar achievements
have been made in the calculations of friction and wear. In our
view, it is in part explained by the fact they are based on the
mechanics of a discrete contact between bodies with rough
surfaces rather than on the mechanics of a deformable solid. If
the liner wear to the depth corresponding to the height of
projections on a rough surface is accepted zero, then the
interaction between bodies in friction and wear should be
evidently described with allowance for deformation and
damage of nonrough contacting surfaces.

The method of active system (Figure 1) calculation is the
most general [1, 2]. Active systems are such specific
mechanical systems that take up and transmit cyclic loads while
operating under friction (be it sliding, rolling, impact and
others). It is a prime feature of the methods of calculation of
volume and surface strength of components of an active system
is that two effects direct and back are taken into account. Direct
effect: changes of fatigue resistance characteristics of an active
system and/or elements produced by friction and wear
processes. Back effect: changes of friction and wear
characteristics of an active system and/or its elements produced
by alternative stresses (strains) on.

DETERMINATION OF CROSS SECTIONAL
DIMENSION
Determine the diameter of the shaft of the active system
that operates with mechano-sliding fatigue (Figure 1, a). The
condition of strength with the safety factor \( n_{\sigma\tau} \) should be
recorded with the allowance for the direct effect
\[
\sigma \leq [\sigma] = \sigma_{-1T} / n_{\sigma\tau},
\]
where \([\sigma]\) – admissible normal stress. The value \( \sigma_{-1T} \) (the
fatigue limit of the shaft with the allowance for the effect of
friction and wear processes) can be determined either
experimentally [2] or from one of formulas according to [3]
\[
\sigma_{-1T} = \sigma_{-1T} \left( 1 - \frac{\tau_{W}^{2}}{R_{\sigma/\tau}^{2} \tau_{f}^{2}} \right),
\]
where \( \sigma_{-1} \) – fatigue limit \( \tau_{W} = 0 \), \( \tau_{W} = f p_{a} \) – frictional stress, \( f \) –
coefficient of friction, \( p_{a} \) – nominal contact pressure, \( R_{\sigma/\tau} \) –
parameter of the interaction of damages caused by contact \( \tau_{W} \)
and non contact \( \sigma \) stresses, \( \tau_{f} \) – limiting value of frictional stress.

**Figure 1.** Design schemes of typical active systems solid / solid
Assume the shaft is bent with the moment M. The maximum normal stress in it is

\[ \sigma = \frac{M}{W} = \frac{M}{\pi d^2 / 32}. \]  

(3)

Taking into account (2) and (3) in (1), we find the required diameter of the shaft

\[ d_{TF} \geq 3 \sqrt[3]{\frac{32 M n_{0TF}}{\pi \sigma_{-1}}} = \sqrt[3]{\frac{32 M n_{0TF}}{\pi \sigma_{-1} - \frac{1}{R_{\alpha/	au}} - \frac{\tau_f}{\tau_f^*}}} \].  

(4)

Figure 2, a shows a graphic analysis of formula (4) where the ordinate axis is the ratio \( d_{TF} / d_f \) of the diameter of the shaft determined with the tribo-fatigue criterion \( d_{TF} \) or with the mechanical fatigue criterion \( d_f \). A horizontal dotted line in Figure 1 corresponds to the case \( d_{TF} = d_f \). The allowance for friction and wear processes at \( R_{\alpha/	au} = 1 \) is illustrated by a curvilinear dotted line, full lines characterize cases when \( R_{\alpha/	au} > 1 \) or \( R_{\alpha/	au} < 1 \).

![Figure 2. Determination of shaft diameter (a) and selection of material (b)](image)

The general conclusion is the following: the shaft’s diameter determined with the criterion of tribo-fatigue can be substantially smaller or larger than the diameter determined with the criterion of mechanical fatigue provided the bending moment coincides in both cases. It is clear therefore that the traditional method of calculating components of the active system cannot be considered sufficient.

Now determine the radius of the roller for the active system operating with mechno-rolling fatigue (Figure 1, b). In this case the condition of strength with the safety factor \( n_{0FR} \) should allow for the back effect:

\[ p_0 \leq \frac{[p]}{p_{fr}} = \frac{[p]}{n_{0FR}} \],  

(5)

where \([p]\) – admissible contact pressure.

Then the required contact area

\[ A_{FR} \geq \frac{2F_n n_{0FR}}{\pi p_{fr}} = \frac{2F_n n_{0FR}}{\pi p_{fr} \left( \frac{1}{R_{\alpha/p}} - \frac{\sigma^2}{\sigma_{-1}^2} \right)}. \]  

(6)

If the value of the contact area (6) and the shaft’s radius are known, it is easy to calculate (for example, with a corresponding Hertzian solution) the roller’s radius.

**SELECTION OF MATERIAL**

In accordance with the conditions of strength and wear resistance the admissible stress is determined (taking into account direct and back effects): a) for the body (i.e. shaft) and b) for the counter specimen (i.e. sliding bearing or wheel).

The materials for both elements are chosen according to the corresponding admissible stresses.

As an example let us consider the problem of selecting a material for the shaft using condition (1) and (2):

\[ [\sigma]_{TF} = \frac{1}{n_{0TF}} \frac{\tau_f}{\tau_f^*} \geq \sigma. \]  

(7)

From (7) the admissible stress is determined using the tribo-fatigue criterion \([\sigma]_{TF}\), and then the latter serves to determine a specific brand of a material and its condition that maintain the required safety factor.

Figure 2, b shows a graphic analysis of condition (7).

Below we consider a similar problem of selecting a material for the sliding bearing. We have

\[ \tau_f / n_{0FR} = [\tau] \geq \tau_f. \]  

(8)

From (8) formula for determination of \( \tau_f \), yields

\[ [\tau]_{TF} = \frac{1}{n_{0FR}} \frac{\sigma^2}{\sigma_{-1}^2} \geq \tau_f. \]  

(9)

If the admissible stress with the tribo-fatigue criterion \([\tau]_{TF}\) is known, we choose a specific brand of the material and its condition that satisfy the accepted safety factor.

**REQUIREMENTS TO FRICTION COEFFICIENT**

This problem regarding, for example, mechno-sliding fatigue (Figure 1, a) is solved with the help of the condition of strength validity:

\[ \tau_f = f_p \sigma_0 \leq [\tau], \]  

(10)

\[ f_{TF} = \frac{f_p}{\tau_f} \leq \frac{1}{n_{0FR}} \frac{\sigma^2}{\sigma_{-1}^2}. \]  

(11)

It is to emphasize that formulas (11) answer the question what friction coefficient the active system should have to ensure the required reliability in operation. It is established that the active system should have the following friction index [1]:

\[ f_f = \frac{\sigma_a}{\tau_f}. \]  

(12)

In fact, expression (12) is the condition of strength according to the friction parameter, while the value \([f_{TF}]\) should be considered the admissible friction parameter.

The procedure of active system durability and damage intensity assessment is presented in this report.

**REFERENCES**