MODELLING THE WEAR PROCESS OF INHOMOGENEOUS BODIES

I. G. GORYACHEVA.
Russian Academy of Sciences, Institute for Problems in Mechanics
pr. Vernadskogo, 101, bld.1, Moscow 119526, Russia, email: goryache@ipmnet.ru

ABSTRACT
The approaches of contact mechanics are used to evaluate the evolution of the contact characteristics in wear process of inhomogeneous bodies (coated bodies, two-phase composition, bodies with inclusions, etc.). The mathematical model is formulated and used to study the kinetics of the wear process depending on the parameters of inhomogeneity such as size and density of inclusions, waviness at the coating-substrate interface, local hardening parameters, etc.

INTRODUCTION
Wear of surfaces leads to the continuous irreversible changes of the surface macroshape in time. If two bodies are in contact the shape variation simultaneously influences the contact pressure. All contact characteristics (pressure distribution, shape variation, size and position of contact region, approach of bodies) are unknown functions of time in this case. Calculation of the wear process for different junctions is a necessary condition for design of long-life machines.

The approaches of contact mechanics are used to evaluate the evolution of the contact characteristics in wear process of inhomogeneous bodies (coated bodies, two-phase composition, bodies with inclusions, etc.). The mathematical model is formulated and used to study the kinetics of the wear process depending on the parameters of inhomogeneity such as size and density of inclusions, waviness at the coating-substrate interface, local hardening parameters, etc.

MATHEMATICAL FORMULATION OF WEAR CONTACT PROBLEMS
The mathematical formulation of the wear contact problems includes the wear equation which describes the dependence of the linear wear \( w(x,y) \) (irreversible change of the linear dimension of the body in the direction perpendicular to the rubbing surface \( xoy \)) on the contact pressure \( p \) and on the sliding velocity \( V \).

Analysis of a number of wear equations obtained theoretically and experimentally shows that in many cases this dependence can be presented in the form

\[
\frac{\partial w}{\partial t} = K_w p^\alpha V^\beta
\]  

where \( \frac{\partial w(x,y,t)}{\partial t} \) is the wear rate, \( K_w(x,y) \) is the wear coefficient, and \( \alpha \) and \( \beta \) are parameters which depend on material properties, friction conditions, temperature, etc.

The system of equations used to analyze the wear contact problem also includes the relation between the surface displacements \( u_z(x,y,t) \) due to deformation and the contact pressure \( p(x,y,t) \)

\[
u_z(x,y,t) = A[p(x,y,t)], \quad \text{contact condition}
\]

\[
u_z(x,y,t) + w(x,y,t) = \Phi(x,y,t), \quad \text{and equilibrium equation}
\]

\[
\iint_{\Omega(t)} p(x,y,t)\phi(x,y)dxdy = P(t)
\]

where \( A \) is the operator which is known for each specific problem, the function \( \Phi(x,y,t) \) depends on the geometry of the contacting bodies and the velocity of their relative motion, \( P(t) \) is the normal load applied to the contact, the function \( \phi(x,y) \) is determined by the surface macroshapes.

The solution of the system of equations (1)–(4) makes it possible to determine the contact pressure \( p(x,y,t) \), surface displacements due to deformation \( u_z(x,y,t) \), the shape of the worn surface \( w(x,y,t) \), and approach \( D(t) \) of bodies due to their deformation and wear.

STEADY-STATE SOLUTION
Under some conditions the system of equations (1)–(4) has a steady-state solution which determines the contact pressure \( p_s(x,y) \) in the steady state wear process

\[
p_s(x,y) = \lim_{t \to \infty} p(x,y,t) = \frac{D_s}{K_s(x,y)V^\beta(x,y)}
\]

The necessary conditions of the existence of the steady-state regime is the independence on time of the operator \( A \), and stabilization in time the external parameters
such as the approach rate ($\lim_{t \to +\infty} D(t) = D_\infty$) or the load ($\lim_{t \to +\infty} P(t) = P_\infty$), relative sliding velocity ($\lim_{t \to +\infty} V(x, y, t) = V_\infty(x, y)$), contact region ($\lim_{t \to +\infty} \Omega(t) = \Omega_\infty$), etc.

A necessary condition for the asymptotic stability of the steady-state wear is discussed in [2].

The substitution of (5) in Eqs. (1)–(4) makes it possible to calculate the asymptotic values of the functions $u_\infty$, $w$, $P$, i.e. to find the pressure and internal stress distribution, and the shape of the worn surface after the running in stage of the wear process.

**WEAR OF NONUNIFORMLY HARDENED SURFACES AND BODIES WITH INCLUSIONS**

Different technical methods used for hardening of surfaces change their properties and essentially influence the character of the surface wear during the friction process. The mathematical model of the wear contact problem with variable wear coefficient is used to study the wear kinetics and shape changes for hardened surfaces and two-phase materials.

As an example, the solution of the system of equations (1)-(4) for the case of 2-D periodic contact problem for the elastic body hardened within the strips ($nl + a \leq x < (n+1)l$, $-\infty < y < +\infty$) is presented ($l$ is a period). The wear coefficient is a step function:

$$K_w = \begin{cases} K_{w1}, & x \in [nl, a+nl], \\ K_{w2}, & x \notin [nl, a+nl], \end{cases} \quad (K_{w1} > K_{w2}).$$

The operator $A$ has the following form

$$A[p(x, t)] = -\frac{2}{\pi E} \int_0^\infty p(x', t) \ln 2 \left| \frac{\pi(x-x')}{l} \right| dx'.$$

The results of calculations of the steady-state shape of the worn surface for various values of the dimensionless parameters $\tilde{a} = a/l$, $m = 1 - \left( K_{w2} / K_{w1} \right)^{1/a}$ are presented in Fig.1.

So during wear of a surface hardened inside strips, there arises an operational waviness, the parameters of which depend on the ration of the wear coefficients of the hardened and unhardened zones and their characteristic dimensions. The similar results are obtained for the two-phase composition, and for the bodies with inclusions.

**WEAR OF COATED BODIES**

The kinetics of changes of all the contact characteristics (pressure distribution, shape variation, size of the contact zone, etc.) and the coating thickness during the wear process is analyzed based on the method of the wear contact problem solution described in [2]. The coatings bonded to the flat base surface or to the surface having the regular microgeometry are under consideration. Figure 2 illustrates the contact of the punch, which wears the elastic coating bonded to the elastic substrate. The surface of the base has a wavy shape and is described by the even periodic function. The dependence of the pressure distribution, the shape of the worn surface on time for different geometry characteristics and relative mechanical properties of coating and substrate is analyzed.

**CONCLUSIONS**

The mathematical formulations and solutions of the wear contact problems for inhomogeneous bodies are presented, which provide the analysis of the influence of the mechanical and geometrical properties of the contacted bodies on the evolution of the contact and internal stresses, on surface shape variation, and on the approach of the bodies in the wear process. The results obtained can be used to predict the lifetime of the components and to optimise the wear process.

**ACKNOWLEDGEMENTS**

This research was carried out under the financial support of the Russian Foundation for Basic Research (grant No 04-01-00766) and INTAS (grant No 03-51-6046).

**REFERENCES**