GRANULAR FLOW LUBRICATION:  
A LATTICE-BASED CELLULAR AUTOMATA MODELING APPROACH

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ABSTRACT

Liquid lubricants break down at extreme temperatures and promote stiction in micro/nano-scale environments. Consequently, using flows of solid granular particles as a “dry” lubrication mechanism in sliding contacts was proposed, because of their ability to carry loads and accommodate surface velocities. Granular flows are highly complex flows that in many ways act similar to fluids, yet are difficult to predict because they are not well understood. Granular flows are composed of discrete particles which display fluid and solid lubricant behavior with time. This work describes the usefulness of employing lattice-based cellular automata (CA) as a tool for modeling granular flows in tribological contacts. The granular kinetic lubrication (GKL) continuum modeling approach has been successful at predicting trends gleaned from experiments conducted with granules in a couette shear cell. These results are used as a benchmark for determining the effectiveness of the CA modeling results. While the CA model was constructed entirely from rule-based mathematics, velocity and solid fraction results from the simulations were in good agreement with those from the GKL model benchmark results.

INTRODUCTION

Future turbine engine technologies are predicting operating temperatures up to 800°C. However, at these temperatures, conventional liquid lubricants fail. Additionally, liquid lubricants promote stiction which is detrimental to the successful operation of micro/nano-scale systems. Consequently, researchers have proposed solid/particulate lubrication which can lower friction and prevent wear in sliding contact interfaces [1, 2]. One type of particulate lubricant is granular flows, which have demonstrated the ability to act as a hydrodynamic fluid by carrying loads and accommodating relative surface velocities [3, 4].

Modified Navier-Stokes equations and particle dynamic (PD) simulations have been used to model granular flows with limitations of complexity and computational cost. Lattice-based cellular automata (CA) [5] present a simple, flexible and computationally inexpensive approach for this problem. CA employs rule-based mathematics to describe physical processes which are subsequently converted into computer simulations that display emergent system behavior. Continuum behavior of granular flow is the result of discrete granules being energized by particle-wall and particle-particle collisions. Hence, CA simulations which are governed by the local collision rules are an attractive approach that has proven successful in predicting other properties of granular flows [6, 7]. The current work employs CA modeling approach to the problem of granular flows in a couette interface.

CONTINUUM MODEL

The physical problem being modeled in this work is a couette-type of granular flow, where the top wall is stationary, while the bottom wall moves with a velocity \( u = U \) in the positive x direction. Because the flow is steady (\( \partial / \partial t = 0 \)) and fully developed (\( \partial / \partial x = 0 \)), the flow only varies across the film for the infinitely wide bearing assumption. The governing equation for velocity \( u(y) \) and solid fraction \( V(y) \) are defined in Eq. (1) and (2), respectively.

\[
\frac{d^2 u}{dy^2} + B_1 \left( \frac{du}{dy} \right)^2 = 0
\]

\[
\frac{dv}{dy} + B_2 \left( \frac{dv}{dy} \right)^2 + B_3 \frac{du}{dy} + B_4 \left( \frac{dv}{dy} \right)^2 = 0
\]

The details of the B coefficients can be found in [2].

LATTICE-BASED CA SIMULATION

The simulation of the couette flow problem is done on a rectangular grid which corresponds to the granular film. The grid is filled with two types of particles: boundary particles and object particles representing wall and granules respectively. Each grid location can be filled with only one particle capturing the volume occupying property of the granules.

The movement of the particle is obtained by updating its coordinates at each time step. Since the grid is rectangular, the particle can have one of eight discrete directions. The speed of the particle is defined by “time factors”, which represents the number of steps before the particle can advance one grid step. The lower wall is moving and energizing the system.
Rules of evolution for this simulation are the movement of the granules and granular collisions. Granular collisions can be particle-particle or particle-wall collisions. There will be a separate set of rules for boundary interactions and particle-particle interactions (e.g., Figure 1). The Lower boundary induces a change in direction because it is moving.

<table>
<thead>
<tr>
<th>Upper Boundary</th>
<th>Lower Boundary</th>
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<tbody>
<tr>
<td>InPut</td>
<td>Output</td>
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<td><img src="image2.png" alt="Image" /></td>
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Figure 1: Sample rules for Boundary interaction

When the particle with speed $u_p,i$ collides with the boundary, the post collision speed $u_p,f$ of the particle depends on the roughness $R$ and coefficient of restitution of the wall $e_w$ as in Eq. (3) and (4). Note, $U$ is velocity of the lower wall.

$$u_p,f = u_p,i + R (U - u_p,i) \quad (3)$$

$$u_p,f = U - e_w (U - u_p,i) \quad (4)$$

When there is a collision between two particles with velocities $u_p1,i$ and $u_p2,i$, the post-collision speed $u_p1,f$ and $u_p2,f$ of both the particles depend on the coefficient of restitution of the particles $e_p$ in Eq. (5) and (6).

$$u_p1,f = \frac{(1- e_p) u_p1,i + (1+ e_p) u_p2,i}{2} \quad (5)$$

$$u_p2,f = \frac{(1- e_p) u_p1,i + (1+ e_p) u_p2,i}{2} \quad (6)$$

The system evolves in discrete time steps, where the location, direction, and speed of the particles are updated at each step. The simulation is run until it reaches a steady-state and the position and velocity data at steady-state are tabulated.

RESULTS AND DISCUSSION:

The GKL continuum modeling approach has been successful at predicting trends gleaned from experiments conducted with granules in a couette shear cell. Figure 2 shows the velocity for the GKL continuum model across the film. There is slip at the upper and lower boundaries, as is evident by the deviations from lower normalized wall velocities ($u/U$) which are 0 and 1, respectively. The CA simulation also captures this as shown in Fig. 3. The linear trend of the curve looks more Newtonian. It did not capture the plug flow center region seen in Fig. 2.

Figure 4 shows the solid fraction across the channel for the GKL continuum model. The solid fraction is minimum at the walls and maximum in the center, which is a common characteristic of granular flows in couette cells. In Fig. 5, the CA simulation also captures this. However, the profile is asymmetrical in contrast to the continuum solid fraction. The actual range of solid fraction profiles is different because continuum model can have maximum solid fraction of only 0.65 because it has spherical particles. While CA model can have maximum solid fraction of one as it has rectangular particles. Both profiles show that solid fraction is close to their maximum possible values at the center region. The deviations in the CA model are attributed to the fact that rules we enforced were based on our simplest understanding of colliding spheres. Thus, disagreement between the continuum and lattice-based CA results does not indicate absolute failure.

CONCLUSION

The trends of CA simulation are satisfactory considering the following:

- The computational cost of employing CA model.
- The simplicity of rules describing local granular collisions which yield the correct global behavior.
- CA results for velocity capture the well-known slip behavior of granular flows near boundaries.
- Results of solid fraction capture the well-known characteristic of the high concentration of granules in the center region.
- CA provides an alternative rule-based mathematics approach to elaborate theoretical math models.

REFERENCES


