RESOLVING THE CONTRADICTION OF ASPERITIES PLASTIC TO ELASTIC MODE TRANSITION IN CURRENT CONTACT MODELS OF FRACTAL ROUGH SURFACES

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ABSTRACT

The elastic-plastic contact model of fractal rough surfaces offered by Majumdar and Bushan in 1991 (the MB model) is revisited. According to the MB model, the contact mode of a single fractal asperity transfers from plastic to elastic when the load is increased, and the asperity's contact area grows and becomes larger than a critical area, which is scale independent. This surprising result of the MB model is in contrast with classical contact mechanics where an increase of contact area due to increased load, is associated with a transition from elastic to plastic contact. The present study describes a revised elastic-plastic contact model of a single fractal asperity showing that the critical area is scale dependent, contrary to the MB model prediction. The new model also shows that a fractal asperity behaves as would be expected from classical contact mechanics namely, as the load and contact area increase a transition from elastic to plastic contact takes place in this order.

INTRODUCTION

The pioneering fractal contact model of MB [1], as well as other recent contact models, relay on a fractal description of rough surfaces, generated by the "Weirestrass- Mandelbrot" (W-M) function presented in [2-3] as:

$$z(x)=G^{D-1} \sum_{n=0}^{\infty} \cos(2\pi^n x)/y^{D-Dw}$$

However, the MB model has some drawbacks, mainly due to the assumption that a contact spot, formed at the contact plane, is the base of a single asperity (i.e. \(l=l\) in Fig. 2). Accordingly, by definition, the asperity is fully deformed by that plane, meaning that the interference \(\omega\) is identical to the asperity height \(\delta\).

In order to identify the contact mode, the MB model wrongly examines the asperity total height: \(\delta=G^{D-1} d^{(2-D)/2}\), rather than its real interference, with respect to the critical interference adapted from GW [4], and expressed in fractal terms as: \(\delta=(H/2E)^{3/4}\). As a result, a scale independent critical area, \(a_c\), of the contact spot is obtained in the form \(a_c=G^2/(H/2E)^{3(2-D)}\) and a wrong conclusion is made that when \(a<a_c\), the contact mode is plastic, and it becomes elastic when \(a>a_c\). This scenario, in which an increasing load and contact area lead to plastic to elastic transition, seems to be non-physical.

THE FRACTAL CONTACT MODEL

Figure 1 describes schematically the geometry of the contacting rough surfaces. Contrary to the GW model [4], which assumes that all asperities have spherical summits with a constant radius of curvature, in the present model the radius of curvature is scale dependent.

![Fig. 1- Contacting rough surfaces](image)

When a normal load is applied, the two surfaces, of which one is a rigid flat, approach each other to a separation distance \(d\). The rigid flat (contact plane in Figs. 1 and 2) "truncates" the original rough surface, forming a "truncation plane". The intersection spots having different diameters \(l_n\), (see Fig. 2), are all located at that plane.

In the following, we will concentrate on the contact problem of a single fractal asperity of scale \(l\) (represented by the \(n^{th}\) term of the W-M function). Fig. 2 shows schematically the basic parameters of such an asperity. Naturally, the
deformation of any contacting asperity can be elastic, elas-
tropic or fully plastic [5].

Fig. 2- The main parameters of a typical asperity, shown in
the case where \( \omega > \omega_c \), i.e. during plastic de-
formation

The asperity profile, height, and its tip radius are given
in [1] by the expressions:
\[
\varepsilon(x) = G^{D-1/2} \cos(\pi x / L), \quad \delta = G^{D-1/2},
\]
and \( R = \rho / \pi G^{D/2} \), respectively.

In our model, the interference \( \omega \) is considered as an
independent parameter. In the elastic regime the true area
of contact is \( a = \pi R \omega \).

By substituting the expression for \( R \) we have:
\[
a(\omega) = \frac{\pi}{\pi G^{D-1}}
\]
\[
\omega = o(\omega) = \frac{4}{3} E R \omega^{3/2}
\]  [4].

Substituting \( R \) and \( a(\omega) \) from (2), we get:
\[
P_e = \frac{4E}{3\pi} \left( \frac{l^0}{G^{D-1}} \right)^{3/2} \omega^{3/2} = \frac{4E \sqrt{\pi}}{3} \frac{G^{D-1}}{l^0} \omega^{3/2}
\]

Hence, the relation between the contact area \( a \), and the load \( P \) is of the form:
\[
a(P) = \frac{P^{3/2}}{l^{3/2}}
\]

As shown in [5], the critical deformation \( \omega_c \), for the
inception of plastic contact of a hemisphere as:
\[
\omega_c = \left( \frac{\pi KH}{2E} \right)^{1/2} R
\]
where \( H \) is the hardness of the softer asperity, and \( K \) is a hardness
parameter [5]. Substituting \( R \) we get:
\[
\omega_c = \left( \frac{KH}{2E} \right)^{1/2} l^{3/2} / G^{D-1}
\]

In addition, the critical area of contact would be:
\[
a_c = \frac{\pi}{\pi} \frac{R}{\omega_c} = \frac{1}{\pi} \left( \frac{KH}{2E} \right)^{1/2} \left( \frac{l^0}{G^{D-1}} \right)^{3/2}
\]

Hence, for \( \omega < \omega_c \) (\( \omega < a_c \)), the deformation is elastic, and
for \( \omega > \omega_c \) (\( \omega > a_c \)), the deformation is plastic, contrary the finding
of MB described before.

Substituting (4) in (3), we get the critical load in the form:
\[
P_c = \frac{4KH}{6\pi E^2} \left( \frac{l^0}{G^{D-1}} \right)^{3/2}
\]

Based on the asperity profile, it is easy to see that the
relation between the deformation \( \omega \) and the truncation diameter
\( l \), is in the form:
\[
\omega = \delta - \varepsilon(l/2) = G^{D-1/2} \left[ 1 - \cos\left( \pi l / 2l \right) \right]
\]

Therefore, the relation between the deformation and the
asperity height is in the form:
\[
\frac{\omega}{\delta} = \cos\left( \pi l / 2l \right)
\]  (see Fig.3).

Comparing (7) with (4), we find the condition for elastic
contact (i.e. \( \omega < \omega_c \)) as:
\[
1 - \cos\left( \frac{\pi l}{2l} \right) < \left[ \frac{KH}{2E} \right]^{1/2} \left( \frac{l^0}{G^{D-1}} \right)^{3/2}
\]

The right hand side of the inequality is always positive,
while the left hand side can vary from zero to one. Hence, we
conclude that, for any given asperity of scale, \( l \) there is always a
range of the ratio \( l / l \), which can fulfill the elastic contact
condition. If the asperity base diameter \( l_b \), is small enough, then
a transition from elastic to plastic contact can occur. When
the load is increased, the diameter \( l_b \), of the truncation area increases
to a point where the elastic contact condition is violated. The
possibility of asperity transition from elastic to plastic contact
mode as its loading increases is in agreement with classical
contact mechanics, contrary to the opposite unrealistic finding
of the MB 1991 model.

The relation between the critical deformation and the
asperity height takes the form:
\[
\frac{\omega}{\delta} = \left[ \frac{KH}{2E} \left( \frac{l^0}{G^{D-1}} \right)^{3/2} \right]^{1/2}
\]

Hence, like the critical area, it is scale dependent. Given the
possible range of parameters: \( 1 < D < 2, 10^5 \leq l / G \leq 10^6 \), and
\( 10^5 < KH / 2E < 10^7 \), we find that \( \omega / \delta \) can have a wide range of
values, from \( \omega / \delta \ll 1 \), where the deformation turns plastic
almost immediately, and up to \( \omega / \delta \gg 1 \), where the deformation
is elastic even at the case of full deformation. Fig. 3 shows the
relations \( \omega / \delta \) and \( \omega / \delta \), for a certain set of values in the
range \( 0 \leq l / l \leq 1 \). Since \( \omega < \delta \), an elastic-plastic transition occurs
before a full deformation of the asperity.

Fig. 3- The interference ratio \( \omega / \delta \) as a function of the
truncation diameter ratio \( l_b / l \)

CONCLUSIONS

It was shown that the critical interference, the critical
contact area and the critical load, of a fractal asperity are all
scale dependent, so there is an individual critical area \( a_c \) for
each asperity, depending on its own geometry. The model
shows that the deformation process in each asperity starts
always from an elastic regime. Increasing the load causes a
corresponding increase in the interference and the contact area.
Depending on the specific surface and material parameters,
elastic to plastic transition eventually occurs.

REFERENCES

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