THE INSTANTANEOUS FRICTION FORCE BETWEEN TWO SLIDING BODIES WHEN OBSERVED FROM A THIRD MOVING BODY

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ABSTRACT

This paper pertains to classical relativity, or using basic science to derive theoretical equations from a frame of reference located on different moving bodies.

PROBLEM DEFINITION

Derive the equations required to predict the forces on the free-body diagrams of two sliding bodies when observed from a third moving body, using the principle of virtual work.

Consider the case of three bodies moving with constant linear velocity with respect to one another, where two of the bodies, “f” and “s”, have a sliding contact between them.

Fig. 2 Velocities relative to body “g”.

\[
\begin{align*}
V_g &= \frac{\infty - a}{\infty + a} \cdot V_s \\
V_g &= \frac{\infty + a}{\infty - a} \cdot V_c \\
V_g &= \frac{V_s}{V_f} V_c
\end{align*}
\]

Determine the sliding velocity on the slower body “s” relative to body “g”. The known sliding velocity of the faster body, \( V_f \), can be used to determine the sliding velocity on the slower body, \( V_s \), utilizing the instant center for relative motion.

Calculate the work done on the faster body “f” for an observer considered at rest on the origin, \( O_g \), of body “g”.

\[
W_{\text{fr}} + W_{\text{out}} + W_{\text{friction}} = 0
\]

\[
F_f X_{fg} - F_0 X_{fg} - (X_{fg} - X_{sg}) W_n = 0
\]

\[
F_f X_{fg} - F_0 X_{fg} - (1 - \eta s) X_{fg} W_n = 0
\]

\[
\frac{X_{sg}}{X_{fg}} = \frac{V_s}{V_f} t; \quad F_f - F_0 - (1 - \eta s) W_n = 0
\]

Calculate the work done on the slower body “s” for an observer considered at rest on the origin, \( O_g \), of body “g”.

\[
F_f X_{sg} - F_0 X_{sg} + (X_{fg} - X_{sg}) (1 - \eta s) W_n = 0
\]

\[
F_f X_{sg} - F_0 X_{sg} + (1 - \eta s) W_n = 0
\]

Calculate the work done on the body “f” for an observer considered at rest on the origin, \( O_s \), of body “s”.

\[
F_f X_{fs} - F_0 X_{fs} - X_{fs} W_n = 0; \quad F_f - F_0 - W_n = 0
\]

Calculate the work done on the body “s” for an observer considered at rest on the origin, \( O_f \), of body “f”.

\[
-F_s X_{sf} + F_0 X_{sf} - X_{sf} W_n = 0; \quad F_s - F_0 + W_n = 0
\]

It can be seen that for the observer on the third moving body, the sliding displacement is not equal to the displacement of the sliding body. When the sliding displacement of the sliding friction force is not the same as the displacement of the sliding body, then the theoretical instantaneous sliding friction force equation must express the percentage of instantaneous sliding motion taking place at the point of contact. This is necessary so that all the forces on the free-body diagram of the sliding body can act over the same virtual displacement that is required to obtain equilibrium on the isolated body. For the observer on one of the sliding bodies, the sliding displacement is equal to the displacement of the sliding body. The existing friction force equation for the observer on one of the sliding bodies will not require any expression for the percent sliding.
GENERAL EQUATIONS

The instantaneous equations for two sliding bodies having relative motion in the same plane when observed from a third moving frame of reference:

\[ V_{sf} = \left( 1 - \frac{V_s}{V_f} \right) V_f \]  
Sliding velocity on faster body \hspace{1cm} (1)

\[ V_{ss} = \left( 1 - \frac{V_s}{V_f} \right) V_s \]  
Sliding velocity on slower body \hspace{1cm} (2)

\[ F_\mu = (1 - \frac{V_s}{V_f}) \mu W_n \]  
Sliding friction force \hspace{1cm} (3)

\[ P_\mu = (1 - \frac{V_s}{V_f})^2 V_f \mu W_n \]  
Sliding power loss \hspace{1cm} (4)

\[ \% = \left( 1 - \frac{V_s}{V_f} \right) \]  
Percent sliding \hspace{1cm} (5)

It can be seen that the intrinsic equation for the percent sliding is included in each of the general equations as a result of deriving the equations from a third moving body.

The correct sign must be applied to the velocities for all the general equations. If both velocities are in the same direction, the percent sliding equation will be less than 100%, but if the velocities are in the opposite direction, the percent sliding equation will be greater than 100%.

Einstein’s special theory of relativity postulates that the laws of physics should be so stated that they apply relative to any frame of reference. The general equations apply relative to the observer on the third moving body. When applying the laws of classical physics, the velocity of the observer is equal to zero when the observer is considered at rest in an inertial reference frame. When zero is substituted for the slower velocity, it allows the general equations to also apply relative to the observer on one of the sliding bodies.

COEFFICIENT OF FRICTION

Figure 3 shows the coefficient of friction using the derived friction force (Eq. 3) to interpret Crook’s test data, [Ref. 1, Fig. 21].

![Figure 3 Coefficient when observed from a third moving body](image)

CONCLUDING REMARKS

Having an equation for the instantaneous sliding friction force that is more general in nature should allow development of a more accurate empirical equation for the instantaneous coefficient of sliding friction, especially at low sliding velocities where the sliding displacement can be extremely small compared to the displacement of the sliding body.

This paper will shed some light on the reason why elastohydrodynamic (EHD) tractive predictions have historically overestimated sliding friction at low sliding velocities, and researchers have not offered any explanation as to why this occurs. The EHD tractive predictions have replaced the existing friction force equation that is only valid if the sliding displacement is equal to the displacement of the sliding body. This condition could be achieved by holding one roller stationary on a roller test machine; but with both rollers rotating, if we consider the first data point in Fig. 3, we find the displacement of the sliding body to be more than seven hundred times greater than the sliding displacement. The EHD tractive predictions for gears, roller test machines, or any two sliding bodies must be multiplied by the percent sliding (Eq. 5) that is required to obtain equilibrium when observed from a third moving body. This will provide a beneficial refinement when making tractive predictions or developing the instantaneous flow properties of the lubricant, especially under high pressure with transient shear stress at low sliding velocity.

The differential equation that relates viscosity, film thickness, pressure, and sliding velocity was first developed by Osborne Reynolds and is named for him. The Reynolds equation was derived from a coordinate system located on one of the sliding bodies. If the Reynolds equation was derived from a third moving coordinate system, it would provide the first step towards deriving an equation for predicting the lubricant film thickness from a third moving body. Based on visual observations on ground gears, there seems to be a strong influence of the sum velocity \( \sum V = V_f + V_s \), which may not be adequately expressed in Professor Dowson’s [Ref. 2], EHD film thickness calculation. Hopefully, when the EHD film thickness equation is derived from the third moving body, the influence of the sum velocity will be adequately expressed.

When applying the equations in this paper to measured data, the observer must first determine if he is at rest on one of the sliding bodies or on the third moving body based on how the experimental data was collected. We must observe events from a third moving body when interpreting experimental data from gears and roller machines when both bodies are rotating and the measured data is obtained relative to ground. We have been using an equation for the sliding friction force that is only valid for the observer on one of the sliding bodies and is invalid for the observer on the third moving body.

REFERENCES
