ABSTRACT

To account for the effects of asperity contacts at various length scales, it is appropriate to characterize an engineering surface as a fractal-regular surface. In spite of significant theoretical advancement, there is a desperate need for experimental verification of the theory of fractal-regular surfaces and a consistent scheme of obtaining the fractal parameters. In the present study, the existence of a fractal region and a regular-shape region in the power spectral density function for fractal-regular surfaces was confirmed experimentally, for the first time, with data obtained from magnetic hard disk and silicon wafer surfaces. A novel scheme involving a variable transformation was developed to extract fractal parameters. This scheme was validated by accurate recovery of fractal parameters from simulated surfaces. The fractal dimension, the fractal roughness parameter and the fractal domain length were found for magnetic hard disk and silicon wafer surfaces.

INTRODUCTION

Because the traditional methods for characterizing surface asperities suffered from strong instrument dependence, fractal characterization of rough surfaces emerged as a topic of importance [1,2,3]. However, an engineering surface cannot adequately be characterized by a pure fractal because it contains a macroscopic shape formed under human interaction. Thus, the concept of a fractal-regular surface was proposed to describe both the macroscopic regular shape formed under human interaction. Thus, the concept of a fractal-regular surface was proposed to describe both the macroscopic regular shape formed under human interaction. Hence, fractal regular surfaces were characterized [4,5]. The fractal dimension, the fractal roughness parameter and the fractal domain length were found for magnetic hard disk and silicon wafer surfaces.

EXPERIMENTAL OBSERVATION OF FRACTAL-REGULAR SURFACES AND A TRANSFORMATION SCHEME FOR EXTRACTING FRACTAL PARAMETERS

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FRACTAL CHARACTERIZATION OF ROUGH SURFACES

A fractal surface, or the fractal regime of a fractal-regular surface, can approximately be represented by the Weierstrass-Mandelbrot (W-M) function as follows [4].

\[ z(x) = L \left( \frac{G}{L} \right)^{D-1} \sum_{n=0}^{\infty} \cos\left(\frac{2\pi n x}{L}\right) \]

where \( z \) is the surface height, \( x \) is the lateral distance, \( L \) is the fractal sample (scan) length, \( G \) is the fractal roughness parameter, \( D \) is the fractal dimension \((1 < D < 2)\), and \( \gamma \) is the (fractal) scaling parameter. To obtain a reasonable spectral line density and phase randomization, \( \gamma \) is usually set equal to 1.5 in the approximation [2]. The mean power spectral density function, \( S_{\text{av}} \), of the W-M function is given by [2]

\[ S_{\text{av}}(\omega) = G^{2(D-1)} \frac{2\ln \gamma}{h} \omega^{-(5-2D)} = h \omega^{-(5-2D)} \]

where \( \omega \) is the spatial frequency, and the spectral coefficient, \( h \), is defined as \( h = G^{2(D-1)/(2\ln \gamma)} \). Equation (2) seems to indicate that a power-law fitting for \( S_{\text{av}} \) versus \( \omega \), represented by a straight line on a double logarithmic scale, would lead to determination of the fractal dimension, \( D \), and the spectral coefficient, \( h \). However, it was found that a power-law fitting directly applied to the power spectral density data of a simulated surface resulted in errors up to 36 percent, with some value of \( D \) residing significantly outside the valid range. This is because the direct power-law fitting is incapable of capturing the trend of the accumulated power, especially under conditions of strong...
fluctuations of the power spectral density value.

**OBSERVATION OF FRACTAL-REGULAR SURFACES**

Fractal-regular surfaces were generated numerically by superimposing the fractal structures based on the W-M function on a parabolic regular shape [6]. The power spectral density data for seven height data sets with different scan lengths for a simulated fractal-regular surface with a fractal dimension, $D = 1.5$, are shown in Fig. 1. Based on different slopes, two distinct regions can be identified. They include a regular-shape region, with the spatial frequency $\omega < \omega_f$, and a fractal region with $\omega \geq \omega_f$, where $\omega_f$ is a fractal transition frequency, equal to the reciprocal of the fractal domain length. The regular-shape region has a steeper slope than the fractal region does. The power spectral density data of a simulated fractal-regular surface for $D = 2$ was also obtained, with a gentler slope in the fractal region.

Experimental surface height data with different scan lengths were obtained from magnetic hard disk and silicon wafer surfaces by using a mechanical stylus surface profiler. From the power spectral density data of both the hard disk (Fig. 2) and silicon wafer surfaces, three regions with different slopes can be observed for each curve. The region with lower frequency values was identified as the regular-shape region due to the approximate relationship, $S \propto \omega^{-4}$ [6]. The “falling tail” of each curve at higher frequency values is a consequence of the quantization of the surface height in analog-to-digital conversion. The intermediate region of each curve is the fractal region. By removing the falling tails, a consistent set of data with both the regular-shape and fractal regions was obtained together with a short, but smooth, transition (Fig. 3). The silicon wafer data indicate a slightly longer transition. These data represent the first experimental evidence for the fractal-regular surfaces based on power spectrum analysis.

**TRANSFORMATION SCHEME**

A new variable, $u = \omega^{(5-2D)^{-1}}$ was introduced to transform Eq. (2) into $S = hu$. A linear fitting based on $S$ versus $u$ is preferred to that based on a nonlinear relationship due to a requirement of the equivalency of the local accumulated power (area under a curve). The search for the coefficient, $h$, in $S = hu$ can be further reduced to the determination of an average value of this coefficient by introducing another variable, the spectral density-frequency ratio, defined as $V = S/u$. Thus, Eq. (2) finally reduces to $V(u) = h = $ constant. The data of $V$ in a non-dimensional form were plotted against $u$, and a linear function fitting was performed with assumed values of the fractal dimension, $D$. The trial value of $D$ that minimizes the magnitude of the slope in the non-dimensional $uV$ plane is the value of $D$ to be found. The quantity, $h$, is found to be the intercept, which is equal to the average value of $V$ for a vanishing slope. When this scheme was applied to simulated fractal-regular surfaces with the original values of the fractal dimension, $D$, being 1.02, 1.5 and 2, the recovered values of the fractal dimension were 1.008, 1.516 and 2.061.

The coefficient, $h$, in Eq. (2) was also recovered. Assuming $\gamma = 1.5$ in the approximation, the recovered values of the fractal roughness parameter, $G$, were $1.05 \times 10^3$ nm and $15.0$ nm, respectively, for the original values of simulated surfaces, $1 \times 10^3$ nm and $10$ nm, respectively. Considering the large range of $G$ spanning orders of magnitude, the recovered values represent a good approximation.

When the transformation scheme was applied to the magnetic hard disk (Fig. 3) and silicon wafer surfaces, the fractal roughness parameter, $G$, and fractal dimension, $D$, were found to be $9.39 \times 10^{-12}$ m and $1.739$ for the silicon wafer surface, and $6.88 \times 10^{-11}$ m and $1.892$ for the magnetic hard disk surface. The fractal domain length, $L_u$, can be found as the reciprocal of the fractal transition frequency, $\omega_f$. For the silicon wafer surface, $L_u = 17.9 \mu$m; for the magnetic hard disk surface, $L_u = 38.8 \mu$m. Although the determination of $\omega_f$ was based on close visual inspection, it has been found that an alternative estimate of $\omega_f$, which is 1.6 times the value shown in Fig. 3, only results in a 1.4 percent change in $D$ and a 35 percent change in $G$ (still a reasonably small change considering the huge range of $G$).

The possibility of releasing the restriction of using a specific value of $\gamma$ in fractal characterization is currently being studied, and results will be published in the near future.

**CONCLUSION**

Experimental evidence for fractal-regular surfaces was provided, for the first time, with the power spectral density data of magnetic hard disk and silicon wafer surfaces. The fractal and regular-shape regions were clearly identified. A transformation scheme was developed to accurately determine the fractal dimension and the fractal roughness parameter from power spectral density data. With this scheme, the values of the fractal dimension of numerically simulated surfaces were recovered within three percent. The fractal dimension, fractal roughness parameter and fractal domain length were obtained for the silicon wafer and magnetic hard disk surfaces.

**REFERENCES**