APPLICATION OF ENERGY PRINCIPLE TO DRY SLIDING FRICTION

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ABSTRACT
A new method named as energy principle method is used to calculate and research the friction force and friction coefficient of two flat elastic surfaces. The studies reveal the relationships between friction coefficient and parameters of a tribo-system such as microstructure of interface, performance of material, operational parameters and geometry parameters. When the height of an asperity increases in the model, the friction coefficient increases rapidly to a critical value, and then hardly increases after that.

Keywords: Energy Principle; Friction force; Friction coefficient; Energy dissipation mechanism

1. INTRODUCTION
To date, friction coefficient cannot be calculated theoretically in engineering design because the mechanism of friction is unclear. In sliding process, friction transforms uniform motion of solid into irregular motion of the atoms, namely mechanical energy into heat. Therefore, kinetic friction is determined by the rate of dissipation of mechanical energy. The mechanism of the energy dissipation, in other words, the origin of friction, is always an important research topic in tribology [1]. In recent years with the development of nano-techniques, such as atomic-force microscope, some progress has been made on understanding the mechanism by investigating the atomic-scale friction models such as IO model, FK model, FKT model and etc [1-3]. However, the models are not suitable for the practical problems and cannot be directly used in engineering.

In the present paper, a new method aimed to provide easy calculation and enough accuracy named as energy principle method is proposed. The energy dissipation mechanism of the model is based on that of the IO model [4]. It is suggested that the oscillation of interfacial atoms in a new equilibrium position after jumping is the key of energy dissipation.

2. ENERGY PRINCIPLE METHOD
In the model, two flat elastic surfaces slide relatively under a load $P$ and a shear force $F$, see Fig.1, where $A$ is the real contact area, and $E_{AB}$ is the elastic modulus of interface.

The friction work is always trying to move the atoms of the interface away from one location of the minimum potential, and the atoms drop back to the next one spontaneously because they must keep balance. The friction work and the friction coefficient can be obtained by calculating the sum of the potential difference of each fluctuation by a numerical method.

The energy dissipation process of the model above includes following four stages. They are (a) start to move; (b) at an equilibrium stage; (c) in the critical equilibrium; and (d) jumping to the next equilibrium position, as shown in Fig.2.

Fig.1 The model of dry sliding friction
Fig.2 Four stages of sliding process
The present model suggests that the vibration energy turning into heat by the interaction of the interfacial atoms plays an important role in energy dissipation mechanism of the IO
model. The changes of the total potential and friction force during sliding are shown in Fig. 3.

![Fig. 3 The change of friction force and total potential](image)

Where, $F$ is the friction force; $a$ is the crystal lattice constant; and $U$ is the total potential.

### 3. DERIVATION OF EQUATIONS

The friction force $F$ and friction coefficient $\mu$ will be calculated by the energy principle method. Here, the situations in Fig.1 are considered: (1) Object $A$ and $B$ have the same geometry parameters and are made of the same material; (2) the crystal faces and directions of the interface are the same during sliding. Therefore, $F$ and $\mu$ can be given as follows.

$$F_{\mu} = \frac{\sum \Delta U}{\Delta a} = \frac{E_{a}A}{4\pi(1+v)} b \frac{E(a-\delta_{a})}{2L}$$  

$$\mu_{\mu} = \frac{F_{\mu}}{P} = \frac{E_{a}A}{4\pi(1+v)} b \frac{E(a-\delta_{a})}{2L}$$

where, $E$ is the elastic modulus; $\delta_{a}$ is the variation of relative distance of the interfacial atoms in the jump; $I$ is inertial moment; $L$ is the height of asperity; and $F_{\mu}(x)$ is the shear force of interface.

To simplify the calculation of Eq.(1) and (2), $F_{\mu}(x)$ in the equations can be regarded as linear without any apparent difference. The results of the simplification are

$$F_{\mu} = \frac{E_{a}A}{4\pi(1+v)} b \frac{E(a-\delta_{a})}{2L}$$  

$$\mu_{\mu} = \frac{F_{\mu}}{P} = \frac{E_{a}A}{4\pi(1+v)} b \frac{E(a-\delta_{a})}{2L}$$

where, $v$ is Poisson ratio; and $b$ is the distance of interface.

In Eq.(1) to (4), $\delta_{a}$ can be calculated as follows.

$$\delta_{a} = \frac{F_{\mu}}{E_{\mu}} - \frac{F_{\mu}}{E_{\mu}} \frac{\nu}{1+\nu} = \frac{E_{\mu}}{E} \frac{\nu}{1+\nu}$$

When the pressure $P$ on object $A$ is not zero, the ratio $a/b$ is changed by the value of $p$:

$$\frac{a+\delta_{a}}{b-\delta_{b}} = \frac{a + \frac{F_{\mu}}{E}}{b - \frac{F_{\mu}}{E}} = \frac{E + p}{E} = \frac{E_{\mu}}{E_{\mu}}$$

During sliding, the crystal directions of the interface are random. Therefore, if object $A$ and $B$ are made of the same material, Eq.(3) and (4) can be revised as

$$F_{\mu} = k_{F}F_{0}$$

$$\mu = k_{\mu} \mu_{0}$$

where $k_{F}$ is a coefficient of random direction.

### 4 DISCUSSION AND CONCLUSIONS

In Eq.(3) and (4), it is shown that $F$ and $\mu$ are determined by both microstructure of the interface and macro-performance of the whole objects. When $L$ increases, $\mu$ increases rapidly to a critical value, and then hardly increases after that, see Fig.4. Therefore, the roughness of optimization can be gained easily by this method.

![Fig. 4 Variation of friction coefficient with height of asperity](image)

In the equation $F_{\mu} = \tau_{A}A$ proposed by Bowden and Tabor [5], $\tau_{A}$ can be calculated by the interaction of interfacial atoms. A more accurate value for $\tau_{A}$ can be gained by considering the energy dissipation process. Eq.(4) is similar to Bowden and Tabor’s equation when $L$ is large enough.

When two flat surfaces of cuboids are elastic, the real contact area increases little when the load on it increases. From Eq.(4) and (6) it can be seen that the friction coefficient is inversely proportional to the pressure approximately. However, if the two objects are elastic spheres, or the value of pressure reaches the yield stress, the real contact area will linearly increase with the load. Hence the friction coefficient keeps unchanged when the load or the apparent contact areas increase. It agrees with Amontons’ law.

In summary, the energy principle method is used to calculate the friction force and friction coefficient of two flat elastic surfaces. The study reveals the relationships between friction coefficient and the parameters of a tribo-system, such as microstructure of interface, performance of material, operational parameters and geometry parameters.

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### REFERENCES


