ON THE FRICTIONAL BEHAVIOR OF AN INCIPIENT-SLIDING CONTACT OF SPHERE-ON-FLAT WITH HIGH NORMAL LOADS

L. Chang
Department of Mechanical and Nuclear Engineering
The Pennsylvania State University
University Park, PA 16802

Abstract – Levinson et al [1] present experimental results that show a steep reduction in the static coefficient of friction of a dry sphere-on-flat contact as the normal load increases. The experiments cover a range of loading conditions from elastic contact to contact with significant plastic deformation. A theoretical analysis is carried out in this study using a contact model of an elastic-plastic sphere on a rigid flat. The shear strength of the contact interface is assumed to be proportional to the contact pressure until it reaches a limiting value that is below the bulk shear strength of the sphere. The theory predicts a friction behavior that is consistent with that from the experimental results in [1] in all key aspects. The analysis reveals that a relatively low limiting shear strength in the contact interface is likely the key factor leading to the dramatic reduction in the friction coefficient and the negligible junction growth obtained in the experiments.

Theoretical Model – The frictional contact between an elastic-perfectly-plastic sphere and a rigid flat at the sliding inception is considered. The contact surfaces are assumed to be perfectly smooth. This sphere-on-flat contact may be viewed as the contact a surface asperity of a spherical tip with a rigid flat. Thus, the model equations developed in Zhang et al [2] for the asperity pressure, area of contact and critical normal approaches are used in this study.

The friction force that can be developed at the sliding inception depends on the shear strength of the contact interface. Experiments [3, 4] have shown that the interfacial shear strength is approximately proportional to the contact pressure for various surface-film materials. Furthermore, Johnson [5] states that, due to contamination or lubrication, the maximum shear stress that the interface can develop is less than that of the bulk solid. In this study, the interfacial shear strength is assumed to be proportional to the asperity pressure until it reaches a limiting value that is below the shear strength of the asperity bulk. It is given by the following equation:

$$\tau = \begin{cases} \alpha p & \tau < \tau_m \\ \tau_m & \tau \geq \tau_m \end{cases}$$

where $\tau$ is the interfacial shear strength of the contact, $p$ is the asperity pressure and $\tau_m$ the limiting value of the interfacial shear strength. Two dimensionless input parameters are used to calculate the friction force with a given applied normal load. One is the shear-strength-pressure proportionality constant, $\alpha$, and the other, the ratio of the limiting interfacial shear strength to the shear strength of the solid bulk, $\overline{\tau}_m$.

Results – The asperity contact equations developed in Zhang et al [2] can be made dimensionless and independent of the size and basic material constants, such as the Young’s modulus, hardness and yield strength, of the asperity. Dimensionless results are obtained and presented in the same format as the experimental results of [1] for easy comparison.

Values for the two dimensionless parameters, $\alpha$ and $\overline{\tau}_m$, described previously need to be chosen for the calculation. That the friction coefficient shown in Fig. 7 of [1] starts around 0.3 at a low normal load suggests that a sensible value for the shear-strength-pressure proportional constant is $\alpha \approx 0.3$. The value for the ratio of the limiting interfacial shear strength to the bulk shear strength is determined by a ‘best-fit’ analysis, guided by the experimental data. This determination yields a value of $\overline{\tau}_m \approx 0.35$.

Figure 1 shows the friction coefficient as a function of the dimensionless normal load, $P/Pc$. The results match reasonably...
well with the experimental data [1] for both the trend and the level. The results are rearranged to show a dimensionless friction force, \( Q/Pc \), against the dimensionless normal load, \( P/Pc \), for small values of \( P/Pc \) up to 14, as is done in Fig. 8 of [1]. Figure 2 shows this result, which also matches reasonably well with the experimental data.

![Figure 2](image-url)

**Figure 2.** Theoretical results corresponding to the experimental data in Fig. 8 of [1]

Levinson et al [1] also measured the areas of contact under various conditions to evaluate the theoretical results of Kogut and Etsion [6] and the theory of friction-induced junction growth of Tabor [7]. In particular, they measured the residual contact area after unloading from a high normal load of \( P/Pc = 220 \), which generated a large amount of contact plastic deformation in the copper specimen. They carried out another area measurement with a new copper specimen; this time, unloading from \( P/Pc = 220 \) with sliding friction. The measured contact areas are essentially the same with or without applying frictional loading, which prompted the authors of [1] to question Tabor’s theory of junction growth.

With the values of \( \alpha = 0.3 \) and \( \tau_m = 0.35 \), which are used to obtain the results of Figs. 1 and 2, the areas of contact are also calculated under various conditions and compared with the experimental results of [1]. At the normal load of \( P/Pc = 220 \), the calculation yields an area of contact normalized by the area of contact at \( P/Pc = 1.0 \) to be \( A/Ac = 84.4 \). For the frictionless contact, the calculated area ratio is 85.9. Physically, the former should not be smaller than the latter. The small percentage difference between the two results is believed to be due to modeling imperfections of Zhang et al [2] as the asperity junction growth is enforced in the model for a given normal approach instead of a given normal load. The two area results calculated above essentially suggest that the model of [2] predicts a negligible junction growth under the given loading-friction condition, consistent with the experimental results of [1].

It may be premature to draw a conclusion that friction generally does not cause a junction growth based on limited experimental data or theoretical results. Figure 3 presents the contact area results calculated under a wide range of loading and friction conditions. The areas obtained with \( \alpha = 0.3 \) and \( \tau_m = 0.35 \) are basically identical to those obtained for the frictionless contact. Furthermore, the results match well with the experimental data shown in Fig. 4 of [1]. However, the theoretical analysis predicts a significant junction growth with a high level of contact friction. At \( P/Pc = 200 \), for example, the friction-induced junction growth is predicted to be about 25% with \( \tau_m = 0.7 \) and 65% with \( \tau_m = 1.0 \).

![Figure 3](image-url)

**Figure 3.** Contact area and friction-induced junction growth with \( \alpha = 0.3 \) (A: frictionless, B: \( \tau_m = 0.35 \), C: \( \tau_m = 0.7 \) and D: \( \tau_m = 1.0 \))

**Conclusion** – A theoretical analysis is carried out in this study using a contact model of an elastic-plastic sphere on a rigid flat. The shear strength of the contact interface is assumed to be proportional to the contact pressure and capped at a limiting value that is below the bulk shear strength of the sphere. The theory predicts a friction behavior that appears to be consistent with that from the experimental results. The analysis reveals that a relatively low limiting shear strength of the contact interface is likely the key factor leading to the steep reduction in the friction coefficient and the negligible junction growth obtained in the experiments.

**References**


