A NEW MODEL OF THERMOELASTOHYDRODYNAMIC LUBRICATION IN
DYNAMICALLY LOADED JOURNAL BEARINGS

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ABSTRACT
A comprehensive method of thermoelastohydrodynamic (TEHD) lubrication analysis for dynamically loaded journal bearings is presented. The Reynolds equation in the film is solved using the Finite Element (FEM) discretization. Thermal distortions as well as the elastic deformation of the bearing surfaces are computed using FEM method. The lubrication film temperature is treated as a time-dependent three-dimensional variable with a parabolic variation with respect to the film thickness. In order to compute the film and solid temperature, a heat flux conservation algorithm, is proposed. Lubrication film temperature is supposed to have a parabolic variation with respect to the film thickness.

INTRODUCTION
Many authors have already investigated the transient Elastohydrodynamic (EHD) behavior of lubricated bearings. Fantino and Frène [1] used substitution methods and plane elasticity relations for the bearing housing. Later the Newton-Raphson method was used, in conjunction with the finite element space discretization and Murty's algorithm (Oh and Goenka [2], and Bonneau et al. [3]). Boedo and Booker's studies [4] represent significant contributions dealing with EHD problems, applied to the connecting rod bearing case. Khonsari and Wang [4] proposed the first transient analyses, but they do not include mass conserving cavitation algorithm. Paranjpe and Han [5] constructed transient THD model with full three-dimensional energy equation including mass-conserving cavitation. Piffeteau and Souchet [6] developed transient TEHD procedure including the influence of the thermal distortion using the two-dimensional energy equation averaged over the bearing length. Kim et al [7] presented a 2D TEHD study for a con-rod bearing. In this study the film temperature is averaged across the lubrication film and the thermal distortion of the bearing surface is estimated using the three-dimensional model of the bearing structure.

However, in order to obtain a good modelization for the dynamically loaded bearing thermal transient behavior, these previous thermal models must use refined mesh beyond the interfaces. This leads to prohibitive computing time that do not allow detailed parametric studies.

TEHD GOVERNING EQUATIONS
Using the usual assumptions of lubrication theory and considering a laminar flow with the inertial effects neglected in the film, a generalized Reynolds equation [8] can be written:

\[ 2B \frac{\partial}{\partial x} \left( G \frac{\partial D}{\partial x} \right) + 2B \frac{\partial}{\partial y} \left( G \frac{\partial D}{\partial y} \right) = 2U \frac{\partial F}{\partial x} + \frac{\partial h}{\partial y} + \frac{\partial h}{\partial t} \]

with \( r \) the effective film thickness, a universal variable \( D \) and a cavitation index \( B \) defined as follows:

- in full (active) zone: \( D = p - p_{\text{ev}}, \quad D \geq 0 \)
  \( B = 1 \)
- in cavitated (non-active) zone: \( D = r - h, \quad D < 0 \)
  \( B = 0 \)

In order to compute the film temperature, a heat flux conservation algorithm, is proposed. Lubrication film temperature is supposed to have a parabolic variation with respect to the film thickness.

Based on the same 2D film mesh discretization used to solve the Reynolds equation, for a \( i \) film element we can write:

\[ \sum_{j=1}^{4} \rho C_p \frac{\partial}{\partial y} \left[ \frac{\partial T_i}{\partial y} \right] + \left[ \int \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right] \, dv \]

\[ - \rho C_p \frac{\partial T_i}{\partial t} - k_i \frac{\partial T_i}{\partial \zeta} \bigg|_{\zeta=0} - k_i \frac{\partial T_i}{\partial \zeta} \bigg|_{\zeta=\zeta_{\text{bc}}} = 0 \]

with: \( \rho, \mu, C_p \) the density, the viscosity and the specific heat of the lubricant, \( j \) one of the \( i \) element boundary, \( k_i \) the lubricant thermal conductivity. Fourier series are used to approximate the temperature variation in the thermal boundary layer.
\[ T(z,t) = T_i + \sum_{i}^{n} \left[ a_i \cos\left( \frac{2\pi}{\Delta T} t - \frac{2\pi}{\Delta T} z \right) + \right. \\
+ b_i \sin\left( \frac{2\pi}{\Delta T} t - \frac{2\pi}{\Delta T} z \right) \bigg] + \phi z \] (3)

where \( z \) is the depthness from the solid surface. \( a_i \) and \( b_i \) are the Fourier coefficients, \( \Delta T \) is the period, \( \phi \) is the mean heat flux, \( \lambda \) is the solid diffusivity and \( T_i \) is the mean solid temperature beyond the thermal boundary layer.

Using the FEM discretization two thermo (\( C_f(i,j) \)) and thermo-elastic (\( C_{ef}(i,j) \)) compliance matrix are precomputed.

For a \( i \) element of the solid surface mesh that is not located on the film/solid interface, the mean temperature \( T_i \) must satisfied:

\[-k\phi = H(T - T_i - T_0)\] (4)

where \( k \) is the solid thermal conductivity, \( \phi \) the mean flux passing through the element surface, \( H \) a heat transfer coefficient and \( T \) the external temperature.

Using the thermal compliance matrix, temperature \( T_i \) can be written in terms of flux \( \phi \) that pass through each element surface:

\[ T(i) = T_0 + \sum_{i=1}^{\text{nsurf}} C(i,k) \phi_k \] (5)

where \( T_0 \) is the unknown mean temperature for the whole solid. The following thermal numerical algorithm will be included in the general numerical algorithm.

**BIG-END CONNECTING ROD BEARING EXAMPLE**

A TEHD study is presented for a typical big-end connecting rod bearing, used in spark ignition engines running at 6500 rev/min. Figure 1 shows the steady thermal fields for the housing and the shaft beyond the thermal boundary layer. The shaft thermal field is steady in a coordinate system that turns with the shaft. It can be observed that the temperature strongly varies across both con rod and shaft bearing, in a range of 130 - 141 °C for the con-rod bearing surface and 83 - 150 °C for the shaft bearing surface. This leads to high modification of the initial shape of the surface. Figure 2 shows a schematic representation of the temperature variation in the solid thermal boundary layer for a point located on the housing surface and for different crank angles. It can be observed that after 1 mm inside the solid, the temperature no longer fluctuates with the time.

**REFERENCES**


