STABILITY AND DYNAMICS OF NON-LINEAR SLIDER BEHAVIOUR DUE TO DISK WAVINESS IN THE PRESENCE OF INTERMOLECULAR AND ELECTROSTATIC FORCES

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ABSTRACT
In this paper we present a theoretical investigation of the stability and the dynamics of the non-linear behavior of a slider at very low head media spacing. A single DOF head disk interface (HDI) model, with constant air bearing stiffness and damping has been used to study the effect of disk waviness on the nonlinear slider dynamics in the presence of intermolecular and electrostatic forces. A variational approach based on the principle of least action was used to derive the equations of motion of the slider. Further, a stability criteria was derived that helped to better understand the instabilities that appear in slider when the slider is flying in close proximity to the disk surface. Due to extremely nonlinear nature of the interaction between the slider and the disk, we observed some strange features of the motion of the slider. In particular the effects of the nonlinear interaction force, air bearing stiffness and damping on the instabilities of the periodic motions of the slider are discussed in detail. We found that the branch associated to the disk waviness frequencies larger than the resonance frequency is always stable and the branch associated to the disk waviness frequencies smaller than the resonance frequency exhibits two stable domains and one unstable domain. This analysis was further extended to include the nonlinear nature of air bearing stiffness and damping as well as contact at the HDI.

1. INTRODUCTION
As the spacing between the slider and the disk decreases in hard disk drives, the linear bit spacing in the magnetic recording can decrease. Thus, in order to increase the areal density to 1Tbit/in², the mechanical spacing between the slider and the disk has to be reduced to less than 5nm. On the other hand, due to this reduction in spacing between the slider and the disk, new forces between the slider and the disk come into play such as intermolecular and electrostatic forces. A study of the effect of intermolecular and electrostatic forces was presented in previous papers [1,2,3]. Besides Wu and Bogy [4] have also shown that there is a reduction in fly height due to intermolecular forces (IMF) for sub 5nm flying sliders. Thornton and Bogy [5] also predicted instability due to these forces at the HDI. Here we extend our analysis and focus on the effect of disk waviness on the stability and dynamics of slider in the presence of intermolecular and electrostatic forces.

2. HEAD-DISK INTERFACE MODEL
The differential equation that describes the motion \( z(t) \) of the slider is given by

\[
m \frac{d^2 z(t)}{dt^2} + c \frac{dz(t)}{dt} + k z(t) = F_{\text{disk}} \cos(\omega t) - \nabla V[z(t)]
\]

where \( m \), \( c \), and \( k \) are respectively the slider mass, air bearing damping and air bearing stiffness respectively and \( \omega_0 \) is the resonance frequency. \( F_{\text{disk}} \cos(\omega t) \) is the force due to disk waviness where \( F_{\text{disk}} = k_n a_{\text{disk}} \). Here \( \omega \) is the disk waviness frequency and \( a_{\text{disk}} \) is the amplitude of disk waviness. \( V[z(t)] \) is the interaction potential between the slider and the disk and it can be expressed as [6]:

\[
V[z(t)] = -\frac{H dxdy}{12\pi(D + z)^2} - \frac{\varepsilon_0 k_x v^2 dxdy}{2(D + z)}
\]

where \( H \), \( \varepsilon_0 \), \( k_x \), \( v \), \( dxdy \) and \( D \) are the Hamaker constant, permittivity constant \((8.85 \times 10^{-12} \, \text{farad/m})\), dielectric constant of the medium \((1 \text{ for air})\), potential difference between the slider and the disk, area of slider in close proximity with the disk and the distance between the disk and the equilibrium position of the slider, respectively. A variational method based on the principal of least action is used in this analysis [7,8,9]. The action \( S[z(t)] \) is a functional of the path \( z(t) \) and is extremal between two fixed instants \( t_a \) and \( t_b \).

\[
S[z(t)] = \int_{t_a}^{t_b} L(z, &t) dt
\]

Here \( L \) is the Lagrangian of the system. The main aim of the use of the variational principal is to employ a harmonic trial function of the form \( z(t) = A(t) \cos(\omega t + \varphi(t)) \) that allows us to perform a non-perturbative analytical treatment in which the dissipation is included. Amplitude \( A(t) \) and phase \( \varphi(t) \) are assumed to be slowly varying functions with time compared to the period \( T = 2\pi/\omega \). Since the slider responds with a delay to the excitation, the sign of the phase chosen means that \( \varphi \) varies in the domain \([-180^\circ, 0^\circ]\). To get the equations of motion in amplitude and phase, we assume a long duration \( \Delta t = t_b - t_a \) such that \( \Delta t >> T \) and calculate the action as a sum of small pieces of duration \( T \). The effective Lagrangian is then calculated as follows:

\[
L_c(A, \dot{A}, \varphi, \dot{\varphi}) = \frac{1}{T} \int_{nT}^{(n+1)T} L(z, &t) dt
\]
Here \( L_e \) is the mean Lagrangian during one period and appears as an effective Lagrangian for a large time scale compared to the period. Since the period \( T \) is small regardless to \( \Delta t = t_b - t_s \) during which the total action is evaluated, the continuous expression of the action is:

\[
S = \int_{t_s}^{t_b} L_e(A, \dot{A}, \varphi, \dot{\varphi}) \, dt
\]

where the measure \( dt \) is such that \( T << \Delta t << \Delta t \). Applying the principle of least action \( \delta S = 0 \) to the functional \( L_{se} \), we obtain the Euler-Lagrange Equation for the effective Lagrangian. The equations of motion of the stationary solutions \( A \) and \( \varphi \) are then obtained by setting first and second derivatives of \( A \) and \( \varphi \) to be zero. Thus, we obtain two coupled equations in \( A \) and \( \varphi \), expressed as:

\[
\cos(\varphi) = \frac{A}{F_{\text{disk}}} \left( k - m \omega^2 \right) - \frac{HDA}{2\pi F_{\text{disk}}(D^2 - A^2)^{3/2}} - \frac{Ae_1 k_e v^2}{F_{\text{disk}}(d^2 - a^2)^{3/2}}
\]

\[
\sin(\varphi) = -\frac{c \omega A}{F_{\text{disk}}}
\]

3. RESULTS

The curve in fig. 2 exhibits a hysteretic cycle (ABCD) due to the nonlinear force that characterizes bifurcations from a mono-stable to a bi-stable state. We observe from fig. 2 that there exists a domain of distance \( D \) where the slider may show unstable behavior. In the domain ABCD three values of the amplitude can be reached for a given value of the slider-disk equilibrium separation. When the slider approaches the disk, the amplitude slowly increases up to point D and then jumps to point A. At this point a further decrease in the distance between the slider and the disk, results in the decrease of the amplitude. When the slider moves away from the disk then first the amplitude increase up to the point B and then jumps to point C. A further increase in the distance between the slider and the disk will result in decrease in the amplitude.

![Fig. 1 Evolution of resonance peak computed for four different values of \( D \): (a) 10nm, (b) 5nm, (c) 3.5nm and (d) 3.2nm. Red lines correspond to the value of the phase ranging from \( 0^\circ \) to \(-90^\circ \) as \( \cos(\varphi) > 0 \), and blue lines corresponds to the value of the phase ranging \(-90^\circ \) to \(-180^\circ \) as \( \cos(\varphi) < 0 \). This is in agreement with the sign convention of the phase of the assumed harmonic solution of the form \( \varphi(t) = A(t) \cos(\omega t + \varphi(t)) \). The two branches define the distortion of the resonance peaks as a function of \( D \), where red and blue lines gives the evolution of resonance peak for frequency values below and above the resonance, respectively. It was also found that blue lines correspond to always stable solution and red lines correspond to conditionally stable solution. Details about the stability analysis will be presented at the conference.

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