THE VALIDITY OF THE REYNOLDS EQUATION IN MODELING HYDROSTATIC EFFECTS IN GAS LUBRICATED TEXTURED PARALLEL SURFACES

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ABSTRACT
The pressure distribution and load carrying capacity for a single 3D dimple, representing laser surface texturing (LST) of gas-lubricated tribological components with parallel surfaces, were obtained via two different methods of analysis: 1) a numerical solution of the exact full Navier-Stokes equations; 2) an approximate solution of the much simpler Reynolds equation. Comparison between the two solutions illustrated that the differences in load carrying capacity were negligible for clearances that are 3% or less of the dimple diameter. At larger realistic clearances the error in the load carrying capacity may reach a maximum of 10%.

INTRODUCTION
LST has gained an increasing interest in recent years as a means for enhancing tribological performance [1]. Several theoretical models based on solving the Reynolds equation, e.g. [2, 3] that were developed so far showed good agreement with experimental results (see [3]). However, it was argued on several occasions that the Reynolds equation may not be valid for the particular LST parameters in use and that the full Navier-Stokes (NS) equations should be employed. Thus, a discussion of the validity of the Reynolds equation for solving LST problems seems appropriate.

Several analyses [4-6] were published in recent years that evaluated the validity of the Reynolds equation in cases where surface roughness is included. Stokes, NS, and the Reynolds equations were solved to compare the pressure distributions and load carrying capacities. This was done for incompressible and compressible flows between two surfaces having roughness of various forms. In the incompressible cases the potential cavitation effect on load capacity was treated with a simplistic approach (Sommerfeld condition) or neglected completely and hence, the conclusions regarding the validity of the Reynolds equation for incompressible lubricants may be inaccurate. In the compressible cases it was found that in spite of local differences in the pressure distribution, the different solutions for the load carrying capacity are in good agreement.

The purpose of the present work is to clarify this issue for the case of a hydrostatic compressible gas flow, applicable, for example, in high pressure LST gas seals.

ANALYTICAL MODEL
Fig. 1(a) shows a segment of an infinitely long stationary strip of micro-dimples. The dimples have a spherical geometry with a base radius \( r_p \). Each dimple is located within an imaginary rectangular cell of dimensions \( 2r_1 \times 2r_1 \). A cross section through one of the dimples is presented in Fig. 1(b) showing the dimple depth \( h_p \) and its location on a top surface that is separated by a nominal clearance, \( c \), from another bottom stationary flat and smooth surface. The origin of a coordinate system \((x,y,z)\) is located at the bottom surface just below the center of the dimple as shown in Fig. 1(b). The \( x \) axis points in the direction of a pressure drop.

Figure 1. The Geometrical model: (a) a segment of infinitely long strip of dimples; (b) a cross-section at the middle of one imaginary cell.

The full 3D Navier-Stokes equations for steady state Newtonian ideal gas in a laminar flow, neglecting external
forces, are:
\[ \rho \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{2}{3} \frac{\mu}{\partial x_i} \left( \rho \frac{\partial u_i}{\partial x_j} + \rho \frac{\partial u_j}{\partial x_i} \right) \]
where \( i \) and \( j \) are dummy and free indices, respectively.

The steady state continuity equation for compressible gas flow and the isothermal flow ideal gas state equation are given by:
\[ \frac{\partial}{\partial x_i} (\rho u_i) = 0 \quad \text{and} \quad p/\rho = \text{const} \]

Equations (1) and (2) represent the steady state five scalar conditions for velocity and pressure.

Because of symmetry consideration the equations were solved for one half of the domain with appropriate boundary conditions for velocity and pressure.

RESULTS AND DISCUSSION

A basic assumption for the validity of the Reynolds equation states that no recirculation regions should appear in the flow field. Fig. 2(a) shows such a case while in Fig. 2(b) the violation of this assumption is presented.

![Figure 2](image)

Figure 2. Streamlines of the gas flow at the mid cross-section of a dimple: (a) without flow recirculation, \( c=3\mu\), \( h_b=15\mu\); (b) with flow recirculation at the top of the dimple, \( c=5\mu\), \( h_b=20\mu\).

Fig. 3 portrays the differences in the dimensionless load carrying capacity, \( W \), that were obtained from the two different solutions. The solid lines in Fig. 3 present the results obtained from the NS equations vs. the aspect ratio \( \varepsilon = c/2r_0 \), for 3 values of the dimensionless clearance \( \delta = \varepsilon/2r_0 \). The single dashed line shows the results obtained from the Reynolds equation for the entire range of \( \varepsilon \) and \( \delta \). The load capacity, \( W \), obtained from the NS equations is larger than that obtained from the Reynolds equation, and the difference increases with increasing dimensionless clearance \( \delta \). The maximum difference between the results obtained from the two solutions is about 2% at \( \delta = 0.01 \) for \( \varepsilon < 0.35 \), 7% at \( \delta = 0.03 \) and no more than 11% at the largest clearance of \( \delta = 0.05 \).

![Figure 3](image)

Figure 3. Comparison of load carrying capacity, \( W \), obtained from the NS (solid lines) and the Reynolds (dashed line) equations.

CONCLUSION

The Reynolds equation solution for a hydrostatic compressible flow over a single dimple was compared with a solution of the Navier-Stokes equations for the same problem. Significant pressure variations across the film thickness where found near the leading and trailing edges of the dimple where the film thickness gradient is discontinuous. However, these local differences have little effect on the load carrying capacity. For clearances, \( c \), that are 3% or less of the dimple diameter the Reynolds equation is valid over the entire range of practical dimple depth values. At clearances as large as 5% of the dimple diameter the error in the load carrying capacity may reach 10%. Thus, the use of the Reynolds equation yields reasonable predictions for a wide range of realistic clearances. For higher clearances, employment of the Reynolds equation may still provide a rough estimate of the load carrying capacity but caution must be exercised.

REFERENCES