TERZAGHI'S PIERRE LOTI. Andrew Schofield, Emeritus Professor, University of Cambridge.

1. Vauban's glacis.

Participants in the XV ICSMGE who want to tread in the footsteps of Terzaghi may be drawn to the ‘Teahouse of Pierre Loti’. When Terzaghi returned to Istanbul he wanted first to visit Pierre Loti (Louis Marie Julian Viaud, 1850-1923). Loti's romantic novels introduced French readers to daily life under the last sultans. The Insight Guide (1991) to Istanbul tells us of “Pierre Loti on Eyüp hill overlooking the Golden Horn and the mosque and minarets of old Istanbul”. Visitors can sip Turkish coffee there under shady vines and admire the city. Geotechnical visitors to the place today may also speculate that if Coulomb had come to Istanbul (a century before Loti) he might have different reasons to visit it. He was as much professionally involved in fortress walls and cannon fire as Terzaghi was in consolidation and settlement. He might have wanted to visit the Theodosian walls, ruined by Mehmet II's cannon in the siege in 1453.

The first Emperor Constantine dedicated Constantinople as the new Rome in 330. It faced many threats and was built to command the sea. The Theodosian wall across the peninsula protected it by land. In Roman times siege engines could make breaches in fortress walls, but catapults and battering rams were feeble compared with siege artillery, gun powder, and cannon balls. The city had greatest strength under Emperor Justinian (527-565), the builder of Sancta Sophia, and his Byzantium proved strong enough to survive for 900 years after his death. But five years after Justinian's death, Muhammad was born in Mecca, and a hundred years later the victories of militant Islam brought Saracen Arabs to besiege Constantinople. They brought with them the aged Eyüp Ensari, standard bearer of Muhammad and the last survivor of the Companions of the Prophet. He died before the walls. Islam pressed the assault, but without cannon fire the walls could withstand it. The Byzantine fleet had a new weapon in 673 called “Greek Fire”, a mixture of oils blown by bellows as a flame thrower, which gave them victory at sea.

Byzantium did not fall then, but all nearby lands were eventually lost to Islam; Turks had occupied Gallipoli by 1354. Mehmet II became Sultan in 1451. His plan to conquer Constaninople involved constructing a fort to dominate the Bosphorus and prevent re-supply by sea, called Rumeli Hisari, but he put great confidence in artillery. He got a Hungarian gun-founder called Urban to cast a siege train of seventy cannon. In the siege, cannon fire breached the wall in many places. On 29 May 1453, assault was launched from all sides and took the city. The garrison was overwhelmed. The body of Emperor Constantine XI who led them was never found among the heaps of the slain. The tomb of Eyüp was found and a sacred and beautiful mosque was built over it. Breaches in a fortress wall by close range cannon fire at the base of the wall helped Islam to a momentous victory. Coulomb would have known of it. In the three centuries after Constantiople fell, military science progressed. Marshal Vauban, much concerned with loss of French lives, wrote books on defence and attack of places. To make life more safe in a fort, with less need to manoeuvre in battle in the open and take heavy casualties from horse-drawn artillery, Vauban’s system of fortification was applied to all the forts defending France.

In 1764 when Coulomb was a young French Army engineer officer, the French and the British were at war. He was drafted to Martinique in the West Indies (where a fort was to be constructed to defend the island against British sea power), was put in charge of the work and stayed nine years, became ill and had to return to France. Vauban’s protection system was as follows. A short space away outside a fortress wall a second slightly lower, inward facing new masonry wall was built. Against this new wall a new earth-work with a long outward earth slope called a ‘glacis’ was constructed. This glacis held the attacking force away from the fort and stopped their cannon from shooting at the base of the wall. New fortifications were designed with a star shape in plan. Defensive cannon batteries were placed high on the walls. Both the glacis slope and the ditch between the glacis and the fortress could be swept by cannon fire. To take the fort the attacking troops had to run into this ditch with scaling ladders, supported by attacking grenadiers who carried hand grenades up the glacis. Their leaders lit the fuses. The grenadiers threw their grenades at the defenders on their wall. All attackers, both those with scaling ladders and the grenadiers on the glacis, were exposed to defensive cannon and musket fire at close range. Most geotechnical visitors today will know nothing of the Conquest in 1453.If Coulomb or any veteran of the Duke of Wellington's war in the Iberian Peninsular had walked along the Theodosian walls they would have known all about protection from cannon fire.

2. An alternative to the teaching on friction at the Corps du Génie engineering school.

Vauban saw the importance of training army engineers. He formed the Corps du Génie in 1676 and in 1749 an army engineering school was founded at Mézières. Coulomb was a student who entered that school in 1760. He graduated in November 1761 having studied mathematics and mechanics and a French design handbook, Bélidor (1737) "Architecture Hydraulique". Belidor stated what lateral force acts on a wall that retains a newly placed glacis. He supposed that as the masonry fell, the retained earth would fall in a natural slope at an angle of repose, which he assumed to be 45°. He calculated that the wall had to retain a wedge of earth sliding on this natural slope at the back of the wall. Coulomb (1773) improved this calculation, using calculus to find the location of a worse plane of slip failure within the retained earth, more steep than 45°.

Belidor had a theory that sliding friction depended on the geometry of the irregular surfaces in contact. He assumed that each asperity is a hemisphere, and found that for one hemisphere to pass over the saddle between two other hemispheres the sliding surfaces must separate at a dilation angle of about 30°. This conveniently explained why different materials such as iron and copper, lead and wood, were found to have coefficients of friction of about 1/3. Leslie linked the concept that friction is due to dilation with Coulomb. John Leslie (1766-1832) was a Scottish scientist, elected to be Professor of Physics in the University of Edinburgh because of his work on radiant heat in a book, Leslie (1804), in which he also discusses the generation of heat by friction. Bowden and Tabor (1973), in the 1st edition of their study of sliding friction, quote part of Leslie's book (pp 299-305), where Leslie writes about frictional resistance to sliding as follows.

"If the two surfaces which rub against each other are rough and uneven, there is a necessary waste of force, occasioned by the grinding and abrasion of their prominences. But friction subsists after the contiguous surfaces are worked down as regular and smooth as possible. In fact, the most elaborate polish can operate no other change than to diminish the size of the natural asperities. The surface of a body, being moulded by its internal structure, must evidently be furrowed, or toothed, or serrated. Friction is, therefore, commonly explained on the principle of the inclined plane, from the effort required to make the incumbent weight mount over a succession of eminences. But this explication, however currently repeated, is quite insufficient. The mass which is drawn along is not continually ascending: it must alternately rise and fall: for each superficial prominence will have a corresponding cavity; and since the boundary of contact is supposed to be horizontal, the total elevations will be equaled by their collateral depressions."

Bowden and Tabor show a saw tooth slip plane with slopes up to the tips of successive teeth, and they write that Leslie's criticism of "roughness" theory

"remains unanswered. Of course one can say that work is used in dragging a body up a slope and when it gets to the top it falls with a bang, bending and denting the surface, so that all the work done on it is lost as deformation work during impact. If we adopt this view we have gone a long way from the dragging-up-the-roughness model. We are really talking of a deformation mechanism."

Bowden and Tabor go on to discuss the welding of small areas of contact between metal sliding surfaces, on which they base a theory of sliding friction at a contact between solids.

Coulomb (1781) won a double prize of the French Academy for the paper "Théori des Machines Simples" on friction on bearing surfaces. It included the "friction circle" analysis for plane journal bearings in simple machines and also included slip-ways for launching ship's hulks. He tested surfaces of all materials with all lubricants then available to Naval architects including all types of wood and grease. His paper also speculates that when wood slides on wood, wood fibres brush against each other. As fibres slowly bend they store energy and, when an end of a fibre brushes past a restraint and springs free, part of the energy stored in it is dissipated. Bowden and Tabor show Coulomb's illustration of these brushes of wood fibres. There are transient flows of lubricant among fibres when they move, or when sliding stops or is restarted. Coulomb measured transient variations of resistance to sliding for wooden surfaces that he did not observe for metal surfaces. Coulomb's concept of elastic surfaces in "brushing" contact with each other, and recent work on rolling contact by J. J. Kalker in Delft have similarities.

There is an alternative to thinking of slip planes in soil and sliding friction. H. G. B. Allersma, also in Delft, views aggregates of sand grains under stress with polarised light. Forces are seen to run through columns of grains, aligned like fibres near to the principal stress direction. As stresses change these lines flicker, vanish and re-form. When a column of stressed grains buckles the elastic energy stored in the grains is dissipated, much as in the brush of fibres that Coulomb discusses. This leads to new ways of thinking of the limiting stability of soil. An aggregate of hard soil grains in a slope at an angle of repose can be thought to contain a skeleton of compression fibres. They are not like the skeletons of a crowd of people, closely packed in bodily contact together and shuffling down a slope. In an X-ray image of the flesh and bones of those people the bones would not flicker and re-form. In a yielding granular aggregate grains slip and rotate. The elastic grains will from time to time store and then dissipate elastic energy. As columns form and buckle hard grains will find themselves pinched in columns of highly stressed grains that buckle in compression. In "Critical State Soil Mechanics", Schofield and Wroth (1968) comment that

"At close range we would expect to find many complicated causes of power dissipation and some damage to particles: however we stand back from the small details and loosely describe the whole process of power dissipation as 'friction', neglecting the possibilities of degradation or of orientation of particles. The first equation of the critical states ($q=Mp'$) determines the magnitude of the deviator stress q needed to keep the soil flowing continuously as the product of a frictional constant M with the effective pressure p'."

It is possible that the storage and dissipation of elastic energy in elastic soil grains could be analysed by numerical modelling. A computation would be thought a success if it began with grains of random shape and size and ended with a constant angle for a slope at repose. The invited Special Lecture at the XV ICSMGE "Re-appraisal of Terzaghi's Soil Mechanics", Schofield (2001) has a Fig. 5 showing a slope of repose. Starting as Coulomb did from the observed angle ϕ_d of a slope at repose (not 45°), it relates ϕ_d and the critical state friction coefficient M.

3. Coulomb's adhesion and cohesion tests and his earth pressure calculation.

Coulomb (Heyman 1972) learned that strength of construction materials involves friction and cohesion. He was taught that the internal friction of earth is equal to the angle of repose of earth as a drained natural slope and that cohesion is the resistance that a solid body offers to simple separation into two parts, whether in tension or in shear. He was unsure about such cohesion, and he made and reported his own tests on limestone specimens of two square inch cross section. Tension failure took a force of 430 pounds, and shear failure took 440 pounds. He made the tests several times and found that the force in shear was nearly always larger than the force in tension. What he had learned at school was untrue, but the difference was small. He felt it safe to design fortifications using the value of cohesion measured in a tension test and the angle of friction ϕ_d observed as the angle of repose of a drained slope 'left to itself'. He also stated several times (Schofield 1998) a 'first law of soil mechanics' that 'newly disturbed soil has no cohesion'. It was self evident that newly placed loose earth, dug with picks, shovelled into barrows and tipped behind a new wall, would have no tensile strength. While his earth pressure calculation is still used today, engineers make a different choice of soil strength properties, hoping to find cohesion and friction from appropriate shear box or triaxial tests of samples taken in site investigation.

Coulomb took the wall height to be a and the horizontal distance between the slip plane and the wall to be x. If γ is the unit weight of earth the weight of the sliding wedge is $\gamma ax/2$. If P is the force acting between the wall and the fill, he could resolve to find the force normal to the slip plane

$(\gamma ax^2/2+Pa)/(x^2+a^2)$.

The wedge sliding parallel to the slip plane applies a force that the soil strength must resist

$(\gamma a^2x/2-Px)/(x^2+a^2)$.

Coulomb submitted this work in his Essay in support of a proposal for his election to the French Academy of Science. His Essay also aimed to show his skill in calculus. He solved the general case in which the wall retained ground with undisturbed cohesive strength c, as follows. He knew from experience that if P began at some safe value and was gradually reduced, slip failure would occur on a straight plane. For slip in the general case on a length of the slip plane of $(x^2+a^2)^{-2}$,

$c(x^2+a^2)^{-2} +(\gamma ax^2/2+Pa)/(x^2+a^2)^{-2} \tan\phi = (\gamma a^2x/2-Px)/(x^2+a^2)^{-2}$, hence

$P= \{(\gamma ax/2)(a - x \tan\phi) - c(x^2+a^2) \}/\{a \tan\phi + x)$.

Differentiating to find a minimum when $dP/dx=0$ gave

$0=(c+\{\gamma\, a \tan\phi\}/2)(x^2-a^2-2xa \tan\phi)$.

The location of the worst slip plane was $x= a \tan (45-\phi/2)$. Coulomb had established that this distance x to the worst slip plane would be the same whether or not the ground had cohesion. For this value of x the P had a minimum or Active value P_A.

$P_A= \gamma a^2 \tan^2 (45-\phi/2) - 2ca \tan (45-\phi/2)$.

Coulomb wrote this equation as an expression

$P_A= m\, a^2 - l\, ca$.

He noted a difference between the forces in fluids in which friction and cohesion were both zero, and forces in loose earth. With a fluid his expression reduced to the fluid force.

$P_A= \gamma a^2 /2$.

In fluids there is a unique retaining force. For loose earth there is a range of forces which will retain the ground in equilibrium. They lie between maximum and minimum limits P_P and P_A.

Coulomb calculated a triangular distribution of active pressure on the wall by considering a family of many parallel planes of limiting stress through the loose earth. He tried to consider general curved surfaces of slip by the method of slices, wrongly supposing that the vectors of stress on a length of slip surface and on a vertical plane between two slices would both be at a limiting value. He was working before the equilibrium of stress at a point in a continuum was properly defined. His assumption is true only at a place where the slip surface is inclined to the horizontal at ϕ, not true in general. The concept of the stress tensor was unknown when he died in 1806

4. Rankine's conjugate stresses.

Rankine was born in 1820; his father was a railway engineer, from whom he learned mathematics and mechanics (Cook 1951). At 14 years of age Rankine was given a copy of Newton's Principia (in Latin) which he read and later declared to be the foundation of his knowledge. At 16 he went to the University of Edinburgh. He won a gold medal in his first year, but he had to leave after two years for lack of finance. He worked at first as assistant to his father and then in the construction of railways and harbours, and in water supply. He had become interested in thermodynamics at University and began to publish papers on this and on other topics including shipbuilding. These led to his election to the Royal Society. In 1855 he was appointed to the Regius Chair of Civil Engineering in the University of Glasgow. As was then the custom, he wrote and delivered his inaugural oration in Latin. He was Professor in Glasgow for only 17 years, dying in 1872. He wrote only one geotechnical paper Rankine (1857) 'On the stability of loose earth', and repeated that material in his many text-books.

His first text-book was Rankine (1858) 'A Manual of Applied Mechanics', which is also based on Lamé's (1852) 'Leçons sur la Théorie mathématique de l'Élasticité des Corps solides'. His earth pressure theory arose from his reworking of Coulomb's and Lamé's material. Rankine's teaching of Lamé's ellipsoid of stress involved graphical construction with an ellipse. The introduction of the now familiar stress circle was soon to supercede Rankine's work with the ellipse, and the stress circle will be used below. At a point in space where there is plane stress, assuming that there are major and minor stresses with components (σ_1, σ_2) on planes e and f, the stress components (σ, τ) on a plane g inclined at an angle θ to the plane f are given by equations

$$\sigma = \sigma_1\sin^2\theta + \sigma_2\cos^2\theta = (\sigma_1+\sigma_2)/2 + (\sigma_1-\sigma_2)(\cos2\theta)/2, \text{ and}$$

$$\tau = (\sigma_1-\sigma_2)(\sin\theta\cos\theta) = (\sigma_1-\sigma_2)(\sin 2\theta)/2.$$

The co-ordinates (σ, τ) define a set of points which lie on a circle with centre at $(\sigma_1+\sigma_2)/2$ and radius $(\sigma_1-\sigma_2)/2$. The existence of this circle confirms that there are always two principal stresses at any general point. Let point G with co-ordinates (σ, τ) on the stress circle represent the vector of stress on a plane g. If an inclined line GP is drawn through G parallel to g, it will intersect the circle in a point P called the pole of planes. The components of stress on any other plane are found by drawing through P a line parallel to that other plane. It will intersect the stress circle at a point with co-ordinates that define the components of stress on that other plane. In the stress circle compressive pressure σ and anticlockwise shear τ are positive.

Coulomb had guessed the plane slip surface failure mechanism and considered that in a slope at repose, parallel to the slope, there is a family of planes of limiting stress. Rankine’s concept defined conjugate planes g and k to be such that plane k is parallel to the direction of the vector of pressure on g. The vectors of pressure on each of a pair of conjugate planes g and k have the same obliquity δ. The difference is that on g the shear stress τ is anti clockwise and on k it is clockwise. If the limiting obliquity of stress on one plane is ϕ then on the stress circle there is one point A for clockwise limiting stress and another point B for anticlockwise limiting stress. Rankine considered limiting conditions at a point, where the limiting ratio of major and minor principal stress is $(\sigma_1/\sigma_2) = (1+\sin\phi)/(1-\sin\phi)$. He rejected Coulomb's work in the following words.

“Previous researches on this subject are based (so far as I am acquainted with them) on some mathematical artifice or assumption, such as Coulomb's "Wedge of Least Resistance". Researches so based, although leading to true solutions of many special problems, are both limited in the application of their results, and unsatisfactory in a scientific point of view.”

Rankine wanted research on the frictional stability of a granular mass to be without the aid of any artifice or assumption. He would apply his conjugate stress concept in investigation of stability of 'a mass of earth, or of shingle or gravel, or of any other material consisting of separate grains'. He would base his work on a sole principle that resistance to slip in a loose granular mass is equal to the normal pressure across the slip plane, multiplied by the tangent of the angle of repose ϕ of a general slope of the loose earth. He said that conditions in the conjugate directions are identical, but he did not make experiments. There are experiments that anyone can easily perform. Let two closed cylindrical glass jars (jam jars) half full of sand, one with voids filled with air and one water, be lain on their sides on a table and slowly rolled along, and slowly tilted to stand on their ends. The sand is seen to form slopes at the same angle of repose ϕ_d in both cases. Water is not a lubricant. Rankine had predicted that in a slope at repose there are two families of conjugate slip planes. His calculation about conjugate stresses should have led him to look for two planes. One family is parallel to slope and the other family is vertical. If he had looked and seen that nothing significant happens on vertical planes through the aggregate of soil grains, he might have reached the conclusion that an aggregate of grains needed continuum analysis.

The newly excavated earth that was retained by Coulomb's fortress walls was fully disturbed and re-compacted (rammed). Rankine analysed loose earth. They both developed their theories for granular material at a time when scientists thought that the elasticity of bodies arose from change in the molecular configuration and resulting intermolecular forces. In this Newtonian view of solids the 'molecules' could be reduced to material points which were centres of force. Engineers and scientists at that time would observe phenomena and change their theories. Astronomical data, when reduced to Keppler's Law's of planetary motion, had led to Newton's theory of gravitation. The two views of the nature of light at the time were that it was either a flow of particles or a wave transmitted in the 'luminiferous ether'. Leslie made experiments on radiant heat and wrote about the problem of flow of heat across Space from the Sun. The nineteenth century was to see a series of electrostatic and electromagnetic experiments give data that were reduced to Faraday's Laws. These in turn led to Maxwell's equations. To reach those equations he needed to imagine electrostatic displacement currents in capacitors that were linked with transverse waves in the electromagnetic field. It was a consequence of Maxwell's equations that electromagnetic waves could transmit energy through Space. Reynolds made experiments on the dilation of granular aggregates, showed that a shear distortion produced a volumetric expansion, and imagined that the 'luminiferous ether' must be a continuum made of close packed very small hard elastic grains. He spent his last twenty years of as Professor at Manchester University on a theory that he presented in Cambridge in the Rede Lecture, Reynolds (1902). His association of alternating displacement currents with alternating transverse electromagnetic waves was wrong because dilation of granular aggregates in shear is irreversible plastic behaviour, not reversible elastic behaviour. Relativity, an even more imaginative theory, attributes Maxwell's displacement current to Lorentz's contraction of space rather than to Reynold's dilation of ether.

In the preface to the Lyrical Ballads, Wordsworth wrote that

"Poetry is the spontaneous overflow of powerful feelings: it takes its origin from emotion recollected in tranquillity."

Terzaghi's most imaginative contribution to soil mechanics was made in Istanbul as he worked alone in his laboratory (Goodman 1999) or sat alone in Pierre Loti's teahouse. Old buildings on the Golden Horn, long since demolished, helped him understand slow settlement in terms of primary consolidation of effectively stressed soft ground. He reworked in his memory his experiences of engineering in Europe and in the construction camps of North America. Terzaghi later advocated a shift of the centre of research from the study and the laboratory into the field, and research funding has been devoted to the observational method. But it was to Pierre Loti that Terzaghi returned to recall a time of his own achievement and of tranquillity. Geotechnical visitors to Istanbul may perhaps welcome advice from the Conference organisers as to which teahouse to visit. The Blue Guide (1991) to Istanbul describes a street to the water's edge where another

"pleasant little teahouse overlooking the Golden Horn … is probably the teahouse that Pierre Loti frequently visited during his years in Istanbul, rather than the much publicised one in the cemetery above Eyüp."

That was near to the old buildings where consolidation and settlement were visible. What is the advice of the Conference Organisers on Terzaghi's Pierre Loti? Whatever it is the geotechnical visitor to the Theodosian walls will not now see breaches with the slopes of loose earth as it fell at an angle of repose when the cannons ceased fire. The Janissaries scrambled with difficulty up those slopes. A better approach would have been achieved if the Sultan had had water cannon. The Egyptian army crossed the Suez Canal with jets of water bringing down the steep Canal faces in dense sand, undercutting defensive positions along the crest. The flow of water from fallen sand back into the Suez Canal created the condition of horizontal seepage, in which slopes stand at half the angle of repose. If the Sultan had had water cannon his Janissaries would have had less difficulty when flow stopped, climbing slopes at half the angle of repose.

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