

Measurements of the statistical distribution of the scalar dissipation rate in a turbulent axisymmetric plume

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Abstract – Measurements of mixing in flows created by the injection of a scalar from a finite-size source in a turbulent uniform co-flow with various velocity ratios have been performed with PLIF. The gradients of the normalised scalar (ξ) result in measurements of the scalar dissipation rate χ . The probability density functions of both χ and $\chi|\xi$ deviate slightly from a log-normal distribution close to the source and the corresponding r.m.s. over the mean increase with streamwise distance up to about 2.8 and 1.2 respectively.

Key Words - Turbulent mixing, scalar dissipation, probability density function, small-scale, intermittency.

1. Introduction

In turbulent flows, the destruction of scalar fluctuations is determined by the scalar dissipation rate (χ), defined by

$$\chi \equiv 2D \frac{\partial \xi}{\partial x_i} \frac{\partial \xi}{\partial x_i} \quad (1)$$

where D is the molecular diffusivity and ξ is the scalar concentration. In turbulent reacting flows the scalar dissipation is central to our understanding of the effects of turbulent mixing on the evolution of the chemistry. In addition, the fluctuations of χ can affect practically-important combustion phenomena such as pollutant emission, flame extinction and/or re-ignition and autoignition. A detailed knowledge of the statistical properties of ξ , χ , and χ conditional on a value of ξ ($\chi|\xi$) are required in turbulent reacting flow theories and, specifically, often $P(\chi)$ and $P(\chi|\xi)$ are needed, where P is the probability density function.

The possibility of a lognormal χ was demonstrated experimentally by Gurvich and Yaglom [1], whereas the lognormality of $\chi|\xi$ was assumed by Peters [2] in the context of flamelet modelling of combustion. Since then this statistical distribution has been examined with experiments in non-reacting round jets [3,4], boundary layers [5,6] and both two- [7] and three-dimensional [8] numerical simulations of passive scalar mixing in stationary, homogeneous, isotropic turbulence. These studies have provided evidence to support the approximately lognormal nature of $P(\chi)$, but $P(\chi|\xi)$ is not as systematically examined. Differences between the two pdf's can arise if χ and ξ are not statistically independent, which may happen close to the scalar source. Here, experimental data on $P(\chi|\xi)$ and $P(\chi)$ are provided for a flow that resembles diffusion from a source in approximately homogenous turbulence. The source has finite size and the fluid exits the nozzle at a velocity that can be higher than the co-flow. This geometry is relevant to various combustion engines. The mean (μ) and standard deviation (σ) of χ and $\chi|\xi$ are also quantified.

2. Experimental Methods

Recently, high-resolution, two-dimensional measurements of χ with Planar Laser-Induced Fluorescence (PLIF) were performed in the mixing field formed by the continuous axisymmetric injection of a gaseous fluorescent tracer (acetone) into a turbulent co-flow of air confined in an outer tube of 34mm diameter [9]. Two injector nozzles were used, with 1.1 and 2.2mm diameters. The turbulence in the co-flow was enhanced by a grid 63mm upstream of the nozzle, which resulted in almost homogeneous isotropic turbulence. Particular attention was paid to achieving a spatial resolution at the Kolmogorov lengthscale (η_K), which was estimated from hot-wire velocity measurements. Ref. [9] describes also a method of post processing that ensures quantitatively reliable measurements of χ . As a result, quality checks showed that μ_χ was measured to within $\pm 20\%$. Here, the results from Ref. [9] are used to obtain $P(\chi)$ and $P(\chi|\xi)$ at various locations in the flow and for various conditions.

Each pdf was evaluated at a point in space and over a number of images of ξ and χ , only if at that spatial location the mean ξ and χ (and hence PLIF fluorescence signal) was above 1% of the maximum value for that run. Data for each pdf was compiled by considering the local (inside a window of axial length $2\eta_K$ and radial length $10\eta_K$, amounting to about 500-600 data points) ξ and χ over 200 images, resulting in more than 110,000 data points for that spatial location. The effect of spatial averaging introduced by the finite $2 \times 10\eta_K$ window was investigated by compiling similar pdfs over windows down to $1 \times 1\eta_K$, while accepting the reduced convergence of the pdfs. Conditional quantities were calculated by splitting the values of χ into 30 boxes of ξ spanning the range $\min(\xi)$ to $\max(\xi)$. Pdfs of the resulting conditional $\chi|\xi$ were calculated only if the number of data points within the box was over 2,000 points.

3. Results

Figure 1 indicates that the lognormal description for χ can be reasonable, but that deviations are non-negligible especially at the tails. The data include flows with equal velocity ratio between the nozzle and the co-flow and jet-like flows, where the nozzle velocity was 3-5 times the co-flow. The results of Ref. [9] show that due to the dominance of the co-flow turbulence, large-scale mixing in this flow is adequately approximated as turbulent diffusion from a point source for both equal-velocity and jet-like flows. It is evident that the degree to which $P(\chi)$ is lognormal is independent of position and flow type.

Figure 2 shows that $P(\chi|\xi)$ has almost identical shape to $P(\chi)$, is approximately lognormal but deviations occur at the tails, and that these conclusions are independent of ξ . In more detail, in Figure 3 it can be seen that $P[\ln(\chi|\xi)]$ tended to the Gaussian with increasing downstream distance from the injector nozzle (z) and along the centreline ($r=0$) for the same conditions. Hence, we find that the deviation from lognormality decreased with z . For $P[\ln(\chi)]$, the skewness increased from about -2.5 reaching -0.3 at long z , in agreement with a previous DNS study [8]. The kurtosis decreased from about 11 to 2, similar to the value of 3.2 found in the same reference. Furthermore, it seems that the tails can be described by an exponential distribution, as in Ref. [10].

Finally, the ratio σ_χ/μ_χ was found to be 1.2-2.8, whereas $\sigma_{\chi|\xi}/\mu_{\chi|\xi}$ ranged between 0.8-1.2 and approximately independent of ξ , both increasing gradually with the downstream distance from the source, but not affected by the Reynolds number (Re), the source size (injection diameter) or the global/large-scale nature of the flow ('equal velocity' vs. jets).

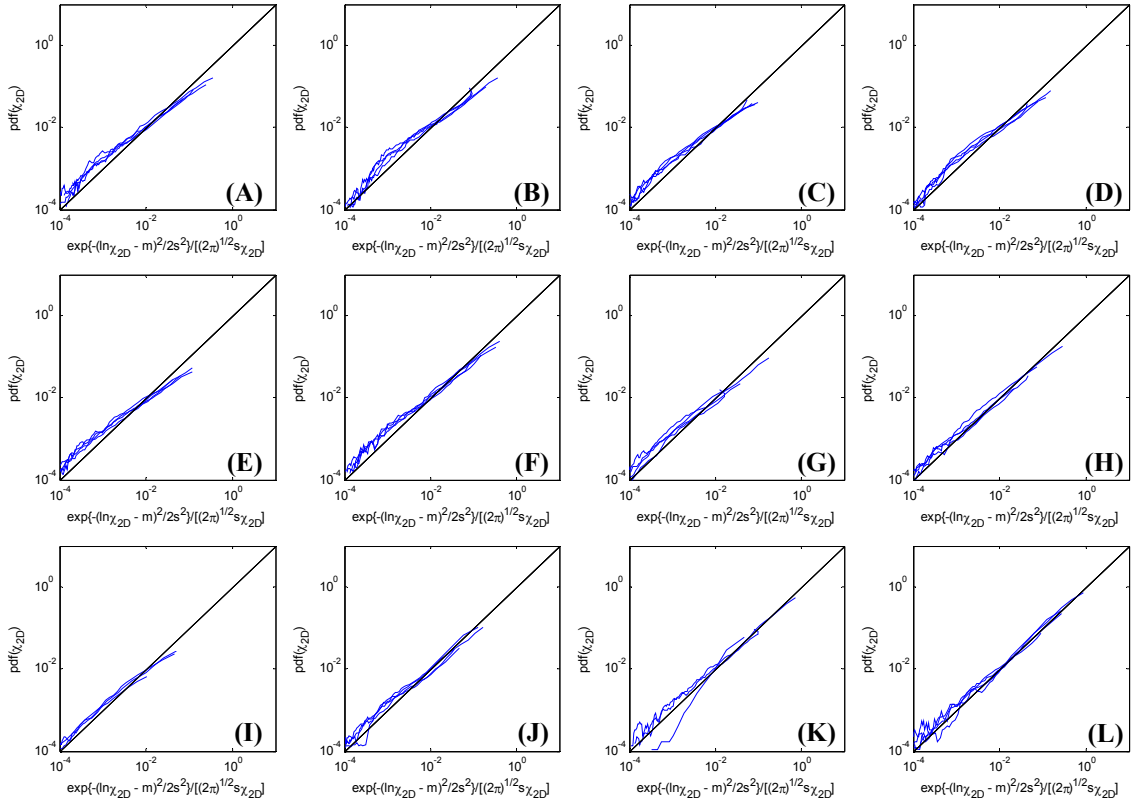


Figure 1. Pdfs of χ at various z for $r=0$. Black lines correspond to a lognormal pdf. (A) – (F) are for ‘equal velocity’ co-flows. (G) – (L) are for jets.

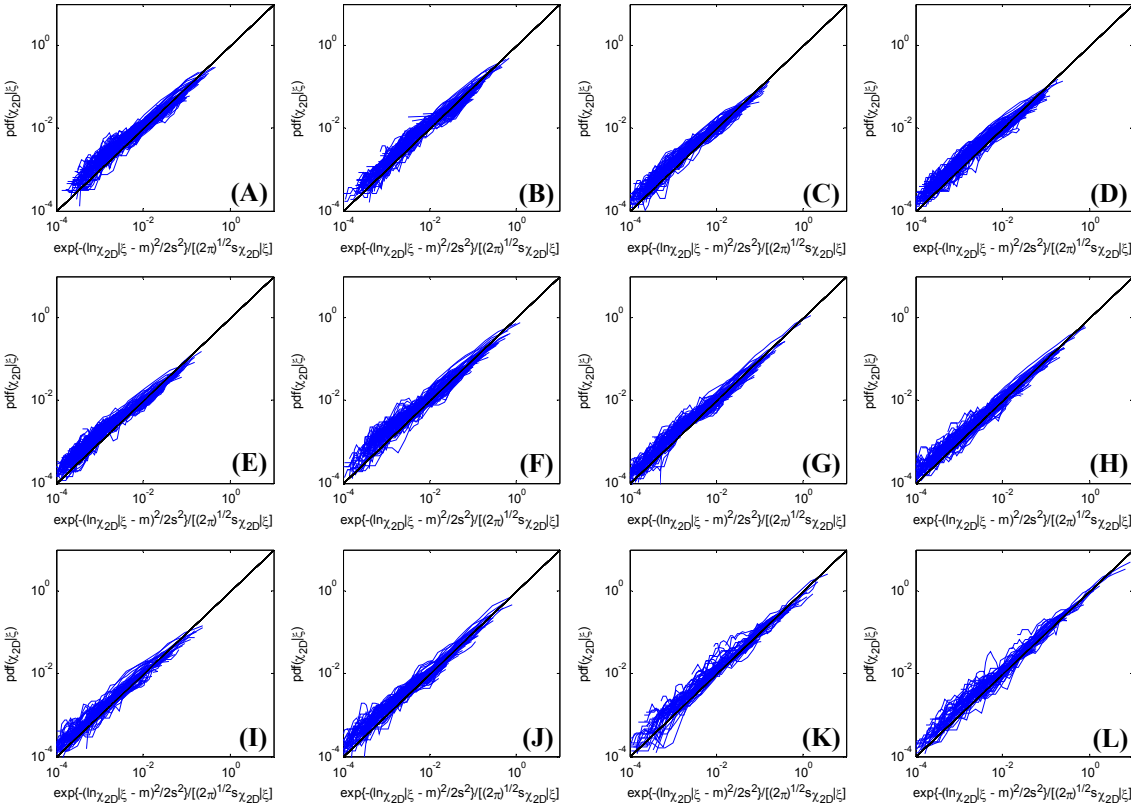


Figure 2. $P(\chi|\xi)$ at various ξ and z for $r=0$. Black lines correspond to a lognormal pdf. The plots correspond to those in Figure 1.

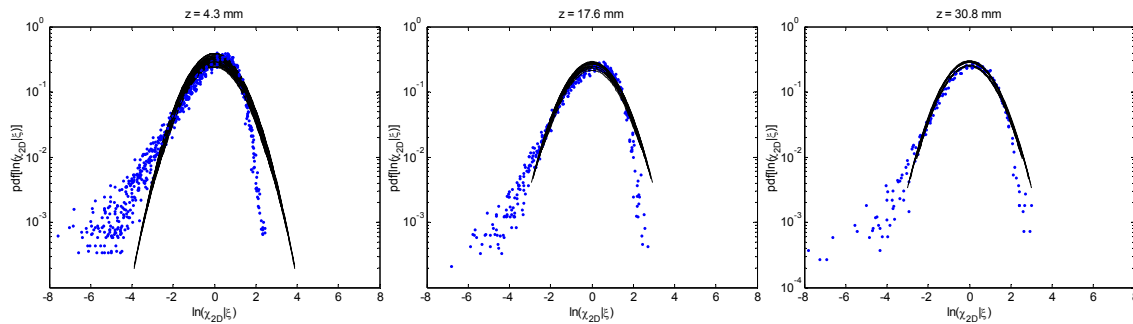


Figure 3. Evolution of $P(\chi|\zeta)$ at a few ζ with z along the centreline. Black lines correspond to a lognormal pdf.

The reported data on the ratio $\sigma_{\chi|\zeta}/\mu_{\chi|\zeta}$ may be considered the main novelty of this experiment. This ratio is very important for turbulent combustion, as extinction events tend to occur at large $\chi|\zeta$, while autoignition tends to occur in low $\chi|\zeta$. The present data can assist the development of models for such phenomena.

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