24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

ICSV24
23-27JULY 2017
LONDON CALLING

DEVELOPMENTS IN THE DESIGN OF CENTRIFUGAL PENDULUM VIBRATION ABSORBERS

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For over 60 years, the torsional vibration of high-powered reciprocating aircraft engines has been controlled by centrifugal pendulum vibration absorbers. Loose weights attached to an engine's crankshaft act as tuned-mass absorbers by oscillating at a frequency in proportion to rotational speed. More recently, similar loose masses have been attached to the flywheels of car engines. The need to achieve increased power from fewer cylinders, while reducing weight and improving economy, has exacerbated torsional vibration of the drive train. The dynamics of a wheel carrying many centrifugal pendulums of bifilar design has been the subject of a growing literature, but much less has been written about roller-type pendulums and about overall system performance. When both crankshaft-mounted and wheel-mounted pendulums have been incorporated in the same engine, the system dynamics becomes even more complicated and difficult to predict. The current state of knowledge about practical design limitations will be explained and the need for further research discussed.

Keywords: centrifugal, pendulum, vibration, absorber, design

1. Introduction

The use of a tuned-mass absorber to reduce harmonic vibration of fixed frequency is well-known. In the 1930s, this principle was extended to reducing the torsional vibration of machinery where there is excitation whose frequency increases with speed. Loose masses moving in a curved track or constrained by rollers to move in a curved path serve as tuned-mass absorbers whose natural frequency is proportional to rotational speed, or approximately so. This allows the irregular firing torque of a reciprocating engine, whose frequency also increases with engine speed, to be resisted, at least in theory. There is a good introduction in Den Hartog's classic textbook [2]. The bifilar or Sarazin type of centrifugal pendulum is used widely, while the roller or Salomon type still finds new applications. Examples are shown in figs. 1(a) and (b).

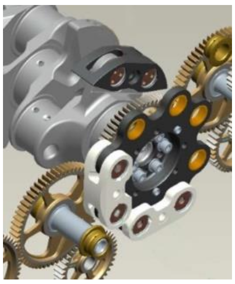


Figure 1: (a) Sarazin or bifilar pendulums

Figure 1: (b) Salomon or roller pendulums

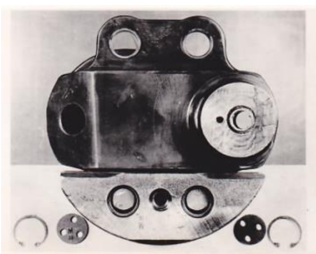


Figure 1: (c) Solid bifilar pendulum design

Figure 1: (d) Laminated bifilar pendulum design

These devices are basically tuned-mass absorbers. They serve to reduce the amplitude of a troublesome resonance by generating negative reaction forces at the pendulums' resonant frequency.

Because centrifugal force acts to hold the pendulum or roller in its equilibrium position, as mentioned, their natural frequency is speed dependent. Each pendulum has a natural frequency that is proportional to engine speed. For example, a fourth-order pendulum, n=4, has a natural frequency of 4 times engine rotational speed. This is important because the excitation harmonics of a reciprocating engine increase in frequency in proportion to engine speed. When the rotational speed is such that the n=4 excitation coincides with a torsional natural frequency of the drive train assembly, large amplitude torsional vibration may occur. Properly working centrifugal pendulum absorbers, tuned to n=4, reduce the n=4 harmonic of excitation at all engine speeds, and therefore they reduce resonant torsional response to this harmonic.

The usual theory for their operation as dynamic absorbers is a constant speed theory. But, in practice, matters are not so simple. At low rotational speed, centrifugal forces are insufficient to hold the moving masses close to their central positions. Instead they rattle within the available clearance. Indeed the name “Rattler” has been registered as a trademark for one particular device of the Salomon type. As engine speed increases, the loose masses are pulled into their central positions. If there is significant excitation, there may still not be enough centrifugal force to generate sufficiently large harmonic reaction forces and some rattling continues. Then, above a critical speed, the centrifugal pendulums overcome the excitation, and pull into synchronism with the torque harmonic to which they are tuned. Above this engine speed, they start to work properly, reducing the amplitude of crankshaft torsional vibration.

So far as the author knows, a comprehensive analytical treatment of the large-amplitude dynamic response of an accelerating engine with centrifugal pendulums has not yet been made. Modelling all aspects of the engine dynamics is extremely complex and most studies have been confined to single pendulums or to several pendulums attached to the same wheel.

2. Constructional details

Commonly used bifilar pendulum designs may have pendulums of solid or laminated construction. Where separate pendulums are attached to each or some of the crankwebs of an engine’s crankshaft, a solid construction may be used, fig 1(c). But for automobile applications, a laminated construction from pressings is usual, fig 1(d).

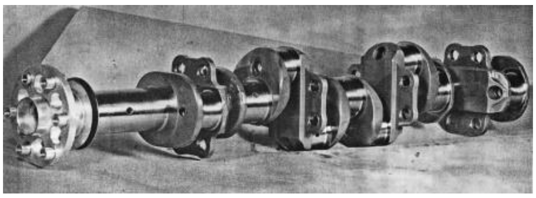


Fig. 2: (a) 8-cylinder aircraft crankshaft with attachments for 8 bifilar pendulums

Fig. 2: (b) Gearbox side of automobile dual-mass flywheel with 4 bifilar pendulums

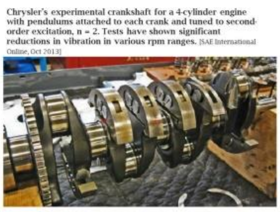


Figure 2: (c) Experimental automobile engine with bifilar pendulums

Fig. 2: (d) TCI “Rattler®” roller-type absorber

In aircraft applications, bifilar pendulums may be attached to the webs of every crank, fig 2(a) but for automobile engines it is more usual to attach pendulums, of either bifilar or roller design, to one of the wheels of a dual-mass flywheel, fig 2(b).

Salomon or roller pendulums are usually wheel mounted, and there may be 6 or more rollers on a single wheel. One commercial off-the-shelf design has 9 rollers, with 6 rollers tuned to one excitation order and 3 to a different order, fig. 2(d).

Most commercial centrifugal pendulum absorbers move in a circular arc with respect to their attachment wheel or crank. As pendulum amplitude becomes large (of the order of 45 degrees) its natural frequency decreases. For this reason, pendulum geometry is usually set so that the pendulum's small-amplitude (linear) natural frequency is higher than its intended value when the pendulum is operating at its design amplitude. For a pendulum designed to absorb *n*th order vibration at rotational speed Ω, the pendulum's natural frequency ω_n is usually set according to $\omega_n^2 = (1+\varepsilon)\, n^2\Omega^2$ where the detuning parameter ε is in the order of 0.1. This is intended to ensure that ω_n is close to $n\Omega$ when the pendulum is working properly.

3. Linear calculations

During design, the expected time-history of torque applied at each crank (of a multi-cylinder engine) must first be computed, including allowance for the reciprocating inertias. This data is then fourier analysed to generate the amplitudes of each order of excitation torque and its variation over the required operating range of the engine. The major orders of excitation, for a 4-cycle (4-stroke) engine are usually the 4^{th} and 2^{nd} orders, in that order.

According to linear theory [2], if a pendulum is tuned precisely to an order of excitation, it will swing to enforce a nodal point for torsional vibration of the system to which it is attached. The centrifugal pendulum acts as an infinite inertia. However, as explained, as the amplitude of pendulum vibration increases, the natural frequency of the pendulum in its centrifugal field reduces slightly. The pendulum then becomes less effective as a vibration absorber. To mitigate this effect, practical pendulum vibration absorbers are "detuned". Typical values range from $\varepsilon = 0.05$ to $\varepsilon = 0.25$. For small amplitudes of vibration (small excitation torque amplitudes), the total effective inertia offered by a pendulum can be shown to be expressed as follows:

$$\textit{Added inertia due to pendulum} = I + m(a+l)^2 + m(a+l)^2/\varepsilon \tag{1}$$

where I = inertia of pendulum (or roller) about its centre-of-mass
m = mass of pendulum (or roller)
a = radial distance from axis of rotation of carrier to pivot point of pendulum (or centre of roller track)
l = length of pendulum (or distance from centre of roller track to centre of roller). For definitions of a and l for a bifilar pendulum, see [6], fig. 2
r = roller radius
ε = detuning

In the case of a roller pendulum, with a roller of radius r, the corresponding expression is

$$\textit{Added inertia due to roller} = I + m(a+l)^2 + m(a + l - I/mr)^2/\,\varepsilon\,(1 + I/mr^2) \tag{2}$$

In both cases, when the detuning ε is zero, the effective inertia is infinite. For a solid, cylindrical roller $I = mr^2/2$, in which case the added inertia due to a roller becomes

$$\textit{Added inertia due to solid roller} = I + m(a+l)^2 + (2/3)m(a + l - r/2)^2/\varepsilon \tag{3}$$

Equations (2) and (3) assume that the rollers do not slip on their tracks, but roll without sliding once they have synchronised with the excitation. Because of the high normal force at the line of contact, this is a good assumption. These results show that, for the same mass, the effectiveness of a roller pendulum is less than that of a corresponding bifilar pendulum, but this disadvantage is compensated by the greater simplicity of the roller system and its ease of manufacture and installation.

These results (1) – (3) are not thought to have been presented in this form before. Their derivation will be published separately.

4. Nonlinear characteristics

A practical design procedure for estimating pendulum amplitude, after first determining the amplitudes of torque excitation, is to make a linear calculation using the effective moment of inertia derived from (1) or (3). It is prudent not to allow this amplitude to exceed about 45 degrees, as will be shown below.

First, an understanding of how the centrifugal pendulum works can be seen from the following quasi-static analysis. Assume that the pendulum swings in a high centrifugal field and the amplitude of motion of its carrier wheel is very small. By making this simplifying assumption, a simple analysis illustrates why centrifugal pendulums becomes ineffective when required to swing through a large amplitude in order to generate sufficient torque.

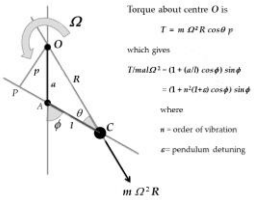


Figure 3: (a) Simplified quasi-static analysis of a centrifugal pendulum. Arm *OA* rotates about the fixed centre *O* at angular velocity Ω. The simple pendulum of point mass *m* and length *l* is hinged at *A*.

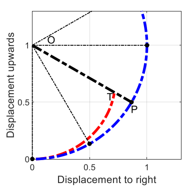


Figure 3: (b) Comparison between paths for constant length pendulum (blue) and constant frequency pendulum (red). Pivot at O, point mass at P (constant length) or T (variable length)

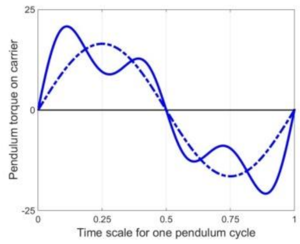


Figure 3: (c) Time history of non-dimensional pendulum torque $T/mal\Omega^2$ during one full period for sinusoidal pendulum motion at an amplitude of 60° with its harmonic fundamental shown by the broken line.

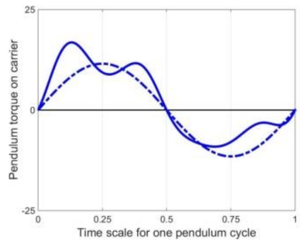


Figure 3: (d) Time history of non-dimensional pendulum torque $T/mal\Omega^2$ during one full period for sinusoidal pendulum motion for a tautochronic pendulum of changing length at an amplitude of 60° with its harmonic fundamental shown by the broken line

Consider the simplified model shown in fig 3(a). Arm *OA* rotates about the fixed centre *O* at angular velocity Ω. A simple pendulum of point mass *m* and length *l* is hinged at *A*. Assume a static analysis, with the only force on the pendulum mass coming from the centrifugal acceleration $\Omega^2 R$ where *R* is the distance from *O* to *C*, the centre of mass. This centrifugal force generates a tension in the pendulum arm, which applies a load at *A* and exerts a torque about *O*. The component of centrifugal force perpendicular to the pendulum causes the pendulum to rotate but does not apply a load at its point of support *A*. The analysis is made slightly more complicated when the additional centrifugal acceleration arising from the pendulum's relative angular velocity $d/dt(\phi)$ is included, but the result is the same in principle. The results of such a calculation are shown in Fig. 3(c). The solid line shows the time-history of non-dimensional pendulum torque $T/mal\Omega^2$ during one full period for sinusoidal pendulum motion at an amplitude of 60°. The fundamental component of this response is shown by the broken line. It can be seen that, during each period of the pendulum's motion, the torque it exerts on its carrier wheel does not change harmonically, but instead follows the irregular curve shown. For this example, the amplitude of motion is taken to be 60°, order n=4 and detuning ε=.25.

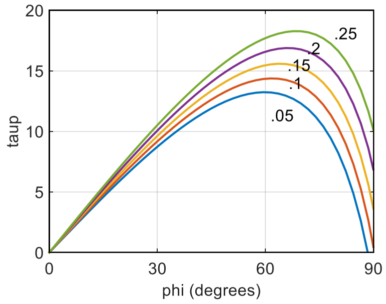
Because the natural period of a centrifugal pendulum increases with amplitude, so that its natural frequency decreases, much has been written about reducing the effective length of the pendulum to achieve a constant frequency pendulum, or so-called tautochronic pendulum [3, 8, 9]. For a simple pendulum attached to a carrier wheel which rotates at constant angular velocity Ω, and for ϕ<1 (rad), the pendulum length must decrease according to the approximate result $l = l_0 (\sin \phi)/\phi$, when the trajectory of the pendulum mass is shown by the red curve in fig 3(b). The calculated pendulum reaction torque for a tautochronic pendulum satisfying this length relationship is shown in fig 3(d). The irregularity of the pendulum torque during a cycle is reduced but so is the amplitude of its fundamental component, and the practical design significance of tautochronic pendulums is unclear. In practice, changing pendulum length has to be achieved by adjusting the profile of the track followed by the rollers that attach each pendulum to its carrier wheel.

5. Steady harmonic calculations

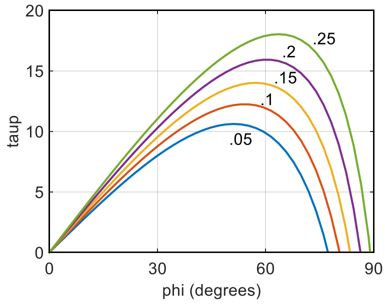
Theoretical calculations of the effectiveness of a particular installation are usually made by the application of some method of harmonic balance in which the motion of the pendulum is approximated by an assumption of harmonic time dependence, responding to the chosen harmonic of engine torque. This approach was adopted by the author in a 1964 paper [6] for which results were computed largely by hand. Applying Matlab® to the same problem allows much quicker results. Typical results are shown below, fig. 5. A bifilar pendulum, in figs 5(a), (c), (e) is compared with a roller pendulum, figs 5(b), (d), (f). In (a) and (b), the amplitude of (non-dimensional) pendulum torque is plotted against pendulum amplitude, in (c) and (d), pendulum torque versus carrier amplitude and, in (e) and (f), pendulum amplitude versus carrier amplitude. The assumption in all these results is that the excitation is purely harmonic and that the corresponding harmonic of pendulum amplitude and carrier amplitude is identified by the "harmonic balance" that is carried out. For the graphs below, a version of the Ritz Minimising Method has been used, but other methods produce similar results

The essential result is that, as pendulum amplitude increases, the carrier reaction torque increases until it reaches a maximum, but thereafter decreases. The corresponding carrier amplitude at first increases with increasing torque amplitude, but then reduces to zero at an "optimum" condition, before changing phase and approaching a jump instability. There is little to choose between the effectiveness of a bifilar pendulum and a roller pendulum of the same mass, less so than for the linear calculations. However, as already pointed out, where the bifilar construction fits within the crankcase of an engine, it is usually more effective because its pendulum can be heavier than that of a wheel-mounted roller pendulum.

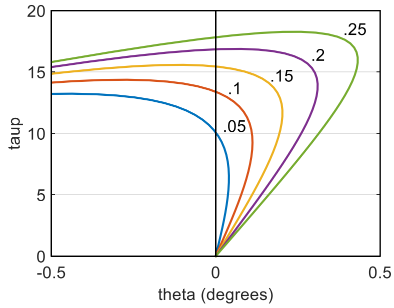
In the literature a lot of attention has been given to the interaction that may occur when there are multiple pendulums on the same wheel, whether bifilar pendulums or roller pendulums. The upshot seems to be that, provided the detuning ε is not zero, there is unlikely to be interference between multiple pendulums, and this seems to be borne out by practical experience. As explained above, as an engine speeds up, at first its loose pendulum weights rattle, when there is insufficient centrifugal force to pull them into synchronism. Sometimes they can be heard audibly falling into synchronism, when there is a change in the noise emitted and engine smoothness improves. This behaviour is known to operators of aircraft with reciprocating engines. A 1976 Operator's Manual, approved by the FAA, for aircraft engines with centrifugal pendulum vibration absorbers, carried this cautionary warning: "*These engines are equipped with a dynamic counterweight system and must be operated accordingly, ... Use a smooth, steady movement of the throttle (avoid rapid opening and closing). If this warning is not heeded, there could be severe damage to the counterweights, roller and bushings*."



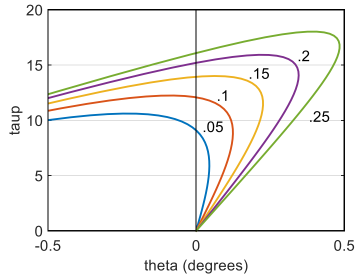
(a) Non-dimensional bifilar pendulum torque versus pendulum amplitude



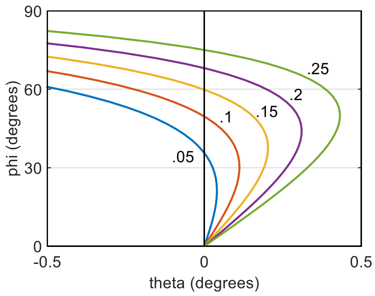
(b) As (a) except for roller pendulum



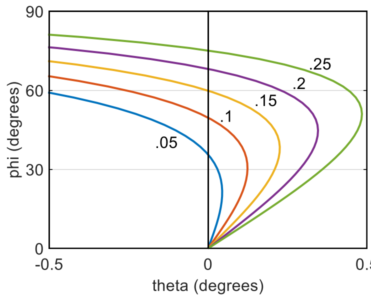
(c) Non-dimensional bifilar pendulum torque versus carrier amplitude



(d) As (c) except for roller pendulum



(e) Bifilar pendulum amplitude versus carrier amplitude



(f) As (e) except for roller pendulum

Figure 5: Approximate harmonic response amplitudes for a bifilar and a roller pendulum ($\gamma=I/mr^2=0.5$, $\rho=r/l=4$) that have synchronised with the $n=4$ order of engine excitation for different values of detuning ε

6. System calculations

Relatively little attention has been given to the analysis of multi-pendulum configurations, when crankshaft-mounted centrifugal pendulums are combined with wheel-mounted pendulums on the engine's flywheel. Establishing an accurate model for the torsional vibration response of a system without absorbers, and then introducing multiple centrifugal pendulums, which are essentially nonlinear devices, brings formidable computational problems. But interesting results are found. There is space for just one example. Figure 6 shows results for a simple two degree-of-freedom torsional system, with one pendulum on the wheel that carries 4th order excitation. The horizontal axis in figure 6(a) represents frequency, expressed as the ratio of the speed of rotation to the speed at which the 4th order of Ω equals the torsional natural frequency of the system without its pendulum Ω_c. The top graph shows loci of carrier amplitude plotted against frequency; the middle graph, pendulum amplitude against frequency, the bottom graph pendulum reaction torque amplitude against frequency. Loci are plotted for four different excitation torque amplitudes (which are constant): 5 (red), 10 (green), 13.2 (black) 13.6 (blue) and 25 (magenta). The system parameters are chosen so that the second wheel acts as a vibration absorber for the first wheel when $\Omega = 0.5\ \Omega_c$. For this example $\varepsilon = 0.06$. When the torque amplitude is 13.6, a nonlinear instability occurs when Ω/Ω_c approaches 1, the system's critical speed. For torque of 13.2, this instability is just avoided. An interesting feature arises for speeds close to $\Omega/\Omega_c = 0.5$. Additional instabilities occur. A much magnified detail of the top graph in figure 6(a) is shown in 6(b). This looping of the loci is a curious complication, with one side of each loop describing an unstable solution from the harmonic balance calculation. The unstable solutions do not occur in practice. The inset view shows the same behaviour found in an earlier analysis [7].

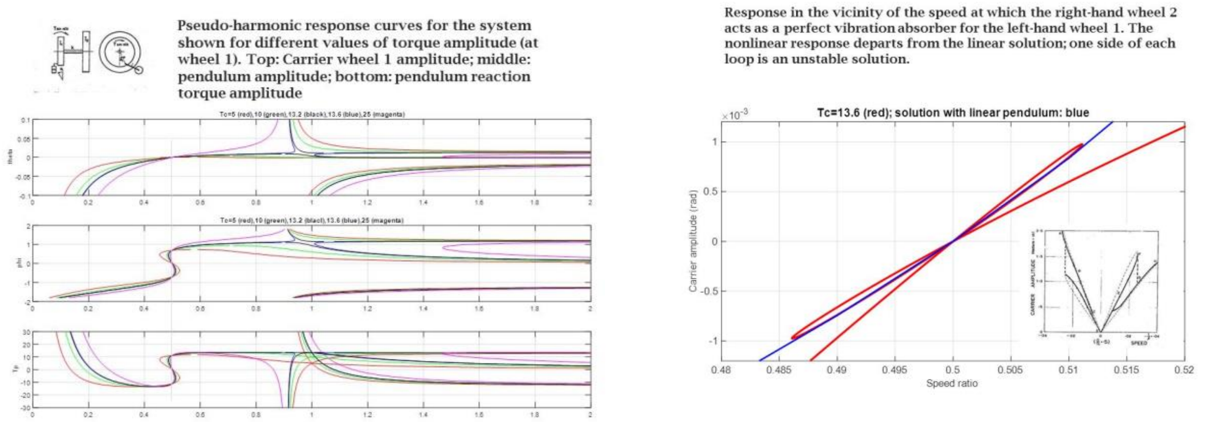


Figure 6: (a) Left view: Approximate harmonic response of a torsional system with two wheels, with excitation on the wheel carrying a pendulum (see inset, top left). (b) Right view: enlarged view of carrier amplitude against speed around the speed at which the second wheel acts as a vibration absorber for the first wheel.

Now that centrifugal pendulum absorbers are increasingly used in automobile engines, and as designs combining crankshaft-mounted with flywheel-mounted pendulums become more common, research to study operation under transient conditions is needed to explore the disruptive effect of engine acceleration, and to widen the speed range over which satisfactory vibration absorption can be achieved.

7. Measurements and practical design

The concept of a tautochronic bifilar pendulum was introduced by Denman in 1992 [3]. Denman showed that the optimal pendulum centre-of-mass curve was an epicycloid and that, for harmonic terms at the excitation frequency, such tautochronic pendulums were found to be somewhat more effective in eliminating harmonic excitation than traditional pendulums. As mentioned, in putting this theory into a practical design, there is a problem that the required tautochronic track of the rollers that support a bifilar suspension only differs by a small amount from a circular track.

The results of measurements on a typical automobile centrifugal vibration absorber show that the trajectory of the centre-of-mass follows is circular to close accuracy. To convert this to a tautochronic path, the tautochronic path would deviate by less than 1mm from a circular path over the operating range of the pendulum (about 60° amplitude). These measurements were made by a camera fitted with a multi-exposure shutter focussed on the sharp corner of a pendulum, fig. 4(b). By moving only the pendulum while shooting multiple images, fig 4(c), its trajectory could be identified accurately and compared with an exactly circular trajectory, fig 4(d). The divergence from a perfect circle was extremely small and appeared to be within the limits of manufacturing accuracy. This complication in machining a non-circular track to the accuracy required for a tautochronic path mitigates against the adoption of a tautochronic design.

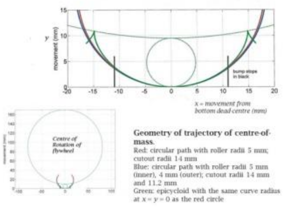


Figure 4: (a) Theoretical track for the roller of a bifilar tautochronic pendulum (green) compared with tracks for constant length pendulums (with two different rollers)

Figure 4: (b) Experiment to record the pendulum trajectory for a typical automobile bifilar centrifugal pendulum

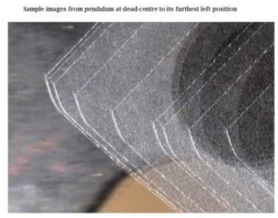


Figure 4: (c) Multi-exposure recording the trajectory of the bifilar pendulum in fig 4(b)

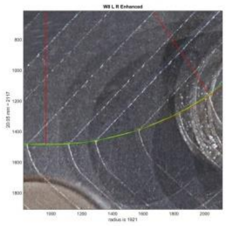


Figure 4: (d) Best-fit trajectory (green) compared with circular trajectory (red) for the bifilar pendulum in fig 4(b)

There are numerous practical considerations that have not been mentioned. Strength is one. Very high g forces are generated on the pendulums, which may be as high as 1000g, and creep of the support structure under these loads may occur during long service. Lubrication is obviously important under the highly-stressed rolling-contact conditions. For crankshaft-mounted bifilar pendulums, this may not be a problem because of the crankshaft lubrication system, but roller pendulums may need a low-viscosity high-pressure lubricant to be sealed within each roller's housing; otherwise surface pitting may occur due to surface fatigue under high contact stresses. The roller surface may have to be circumferentially grooved to provide a lubrication pathway. To reduce wear, further complications may include the introduction of shrink-fit liners, as shown in fig. 1(b). And to increase the magnitude of the reaction torque generated by a pendulum, tungsten (ρ=19.5) may replace stainless steel (ρ=8). There is considerable skill in designing a satisfactory absorber that will provide enough vibration absorption and will last the working life of an engine. Both the design and the analysis of these devices still bring formidable challenges.

8. References

The author is grateful for information from or discussions with Engineered Propulsion Systems Inc. www.eps.aero; Lycoming Division of the Avco Corporation www.lycoming.textron.com; Schaeffler Technologies GmbH & Co. KG www.schaeffler.de; TCI Automotive www.tciauto.com; Vibration Free Ltd www.vibrationfree.co.uk .

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