Part IA Paper 3 : Electrical and Information Engineering

LINEAR CIRCUITS AND DEVICES
EXAMPLES PAPER 4

Operational Amplifiers

This Examples Paper contains questions based on material in Lectures 17-18. Where applicable, the lecture material on which the question is based is shown e.g. [L17]. Questions of Tripos standard are marked * .

Figure 1 shows the two main configurations employed in op-amp circuits and is referred to in several questions.

Revision Question (answers on p5)

(a) In the circuit of Fig. 1(a), if the input voltage v_i is 3 mV RMS from a microphone, R_1 = 3 kΩ, R_2 = 500 kΩ and the op-amp used is ideal, calculate the output voltage v_o.

(b) In the circuit of Fig 1(b), if the input voltage v_i is 5 mV RMS from a vibration sensor, R_3 = 99 kΩ, R_4 = 1 kΩ and the op-amp is ideal, calculate the output voltage v_o.

(c) Which amplifier circuit inverts as well as giving gain?

(d) Which circuit has a very high input resistance and so is used if the source of v_i has high internal resistance?

Which circuit has a possibly low, but *well-defined* input resistance and so can be used for power matching to its input source?

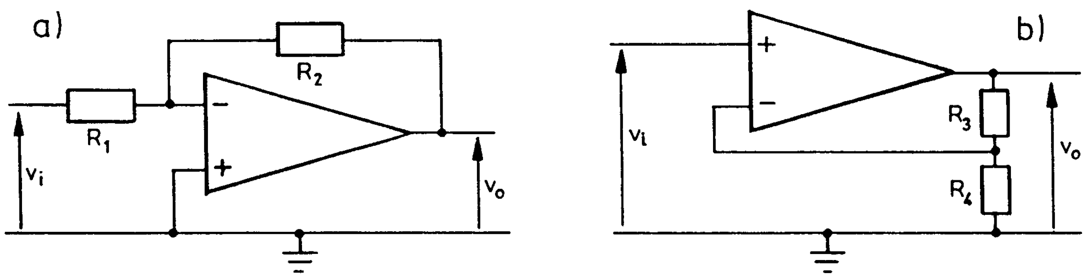

Figure 1

1. Assuming that an ideal op-amp is available, state which of the circuits in Fig. 1(a) and Fig. 1(b) would be chosen for the following applications, and determine suitable resistor values for each case:

(a) to amplify signals of 20 mV amplitude to yield an output of 1 V amplitude <u>without</u> inversion. Assume that the resistor network loading the amplifier output should present a load of about 10 kΩ;

(b) to amplify 2 mV signals from a source which performs best when driving an effective load of 600 Ω and to yield to an output of 0.5 V (an inversion is permissible);

(c) to amplify 2 mV signals from a source from which the minimum possible current should be drawn and yield an output of 0.5 V. It does not matter whether or not the output is inverted. If necessary, assume that, as in part (a), the resistor network loading the op-amp output should present a load of about 10 kΩ [L17]

2. (a) In the operational amplifier circuits of Fig. 1, assume the op-amp devices used have finite values of gain A and input resistance R_i, but have negligibly small output resistance. Derive algebraic expressions for the gain v_o/v_i of each circuit. Show that these equations reduce to those given in lectures for ideal op-amps:

Fig. 1a: $v_o/v_i = -R_2/R_1$ Fig. 1b: $v_o/v_i = (R_3 + R_4)/R_4$

* (b) Derive expressions for the input resistance of each circuit in Fig. 1 when the non-ideal op-amps of part (a) above are used. To simplify the algebra, use the <u>ideal</u> equations given above in terms of R_1 and R_2, R_3 and R_4, to express the relationship between v_o and v_i .

(c) If $A = 10^4$, $R_i = 1$ MΩ, $R_1 = 10$ kΩ, $R_2 = 100$ kΩ, $R_3 = 90$ kΩ, $R_4 = 10$ kΩ,

(i) compare the actual gain of each circuit to the gain which would be obtained if A and R_i tend to infinity;

(ii) evaluate the input resistance of each circuit. [L17]

* 3. Figure 2 shows the circuit for a summing amplifier. Assuming that the op-amp is ideal, show that v_o is given by:-

$$v_o = -\left(\frac{R_f}{R_1} \cdot v_1 + \frac{R_f}{R_2} \cdot v_2 \right)$$

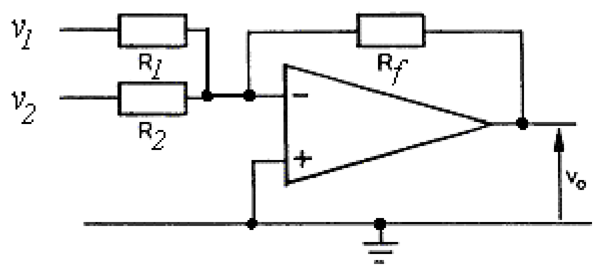

Fig. 2

The sinusoidal a.c. signal sources, v_1 and v_2 , each have an output impedance of 200 Ω.

(a) The summing circuit is required to produce an output $-(200\, v_1 + 40\, v_2)$. If R_f = 100 kΩ, what values of resistance are required for R_1 and R_2?

(b) Show how an additional stage using a second op-amp can be used to produce an output $200\, v_1 - 40\, v_2$

The op-amps are to be provided with electrical power from a supply that delivers +V and –V relative to earth. If v_1 and v_2 have amplitudes 10 mV and 50 mV respectively, determine the minimum power supply voltage V required for your circuit to work correctly. [L17]

* 4. The gain of an *internally compensated* operational amplifier varies with frequency according to the expression: $A = A_0/(1 + \mathrm{j}\, f/f_c)$. Such an op-amp with $A_o = 10^4$ and f_c = 10 Hz is used in the circuit of Fig. 1(b) with R_3 = 99 kΩ and R_4 = 1 kΩ. If R_i is very large, show that the expression for gain, v_o/v_i , reduces to $(R_3 + R_4)/(R_4 + (R_3 + R_4)/A)$. Hence estimate the magnitude of the gain of the amplifier circuit at 1 Hz, 100 Hz and 10 kHz. [L18]

Simulated Inductor

* 5.In the operational amplifier circuit of Fig. 3, $R_1 >> R_2$. Derive an algebraic expression for the complex input impedance (v_1/i_1) of the circuit in terms of R_1, R_2, C and angular frequency ω, assuming that the op-amp device is ideal. By considering the form of this expression, show that at low values of ω the impedance has the same form as that of a series-connected resistance and inductance, but that at high frequencies, the input impedance approaches R_1. [L18]

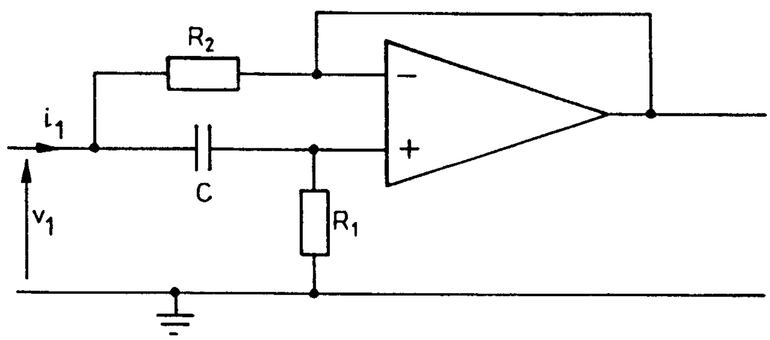

Fig. 3

Op-amp applications

6. The op-amp circuit of Fig. 4 is called a *transconductance amplifier*. It generates a current i which is proportional to the input voltage from a source, v_S. In this example, the source has high internal resistance R_S, and cannot itself supply significant current. The current is passed through a standard resistance R and drives a calibrated moving coil current meter of resistance R_L, to allow voltage v_S to be read out.
Write down an expression for the *transconductance* (i/v_S) of this circuit if the op-amp is ideal (A, $R_i \to \infty$, $R_o \to 0$). [L17]

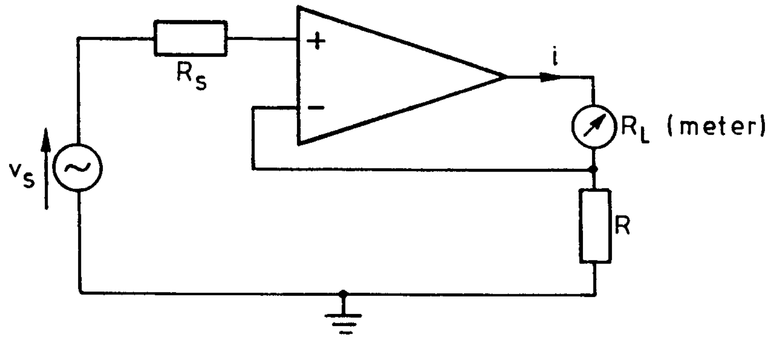

Fig. 4

* 7. The amplifier circuit shown in Fig. 5 is called a *transimpedance amplifier*. It is commonly used to amplify very small alternating currents and produce a proportional output voltage. In this example the signal current originates in a capacitive source and is of order 10^{-11} A. The op-amp used has a forward voltage gain of 10^4, but infinite input resistance and negligibly small output resistance.

Obtain an expression for the output voltage per unit current input (the *transimpedance*). Estimate the half-power bandwidth of the circuit for C = 60 pF and R = 10^{10} Ω.

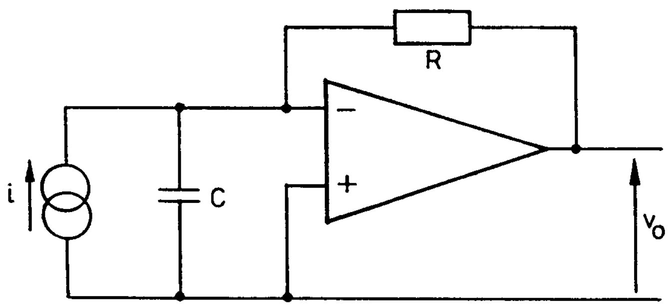

Fig. 5

Unfortunately, the high value of the resistance R cannot be obtained without introducing some stray capacitance into the circuit, with the effect of connecting a capacitor C_S *in parallel with* R. If C_S = 2 pF estimate the upper –3dB point for the circuit. [L18]

ANSWERS

Revision Question

(a) 500 mV RMS (b) 500 mV RMS (c) Fig. 1(a) inverts

(d) Fig. 1(b); Fig. 1(a).

1. (a) Fig. 1(b), $R_4 = 200\ \Omega$, $R_3 = 9800\ \Omega$

   (b) Fig. 1(a), $R_1 = 600\ \Omega$, $R_2 = 150\ k\Omega$

   (c) Fig. 1(b), $R_4 = 40\ \Omega$, $R_3 = 9960\ \Omega$

2. (a) $-AR_2R_i/(AR_1R_i + R_2R_i + R_1R_i + R_1R_2)$; $\rightarrow -R_2/R_1$ when A large

   $A(R_iR_3 + R_iR_4)/(AR_iR_4 + R_iR_3 + R_iR_4 + R_3R_4)$; $\rightarrow (R_3 + R_4)/R_4$ when A large

   (b) $R_1/(1 - R_2/AR_1)$; $AR_iR_4/(R_3 + R_4)$.

   (i) 0.11% low; 0.1% low (ii) 10.01 kΩ; 1000 MΩ.

3. $R_1 = 300\ \Omega$; $R_2 = 2300\ \Omega$. At least $\pm$ 4 V.

4. 99; 98.5; 9.95.

5. $R_2(1 + \omega^2C^2R_1R_2)/(1 + \omega^2C^2R_2^2) + j\omega CR_2(R_1 - R_2)/(1 + \omega^2C^2R_2^2)$

6. 1/R

7. $-AR/\{(A + 1) + j\omega CR\}$; 2.7 kHz; 8Hz.

Tripos Questions

2000 Part IA, Paper 3 Q2
2001 Part IA, Paper 3 Q2
2002 Part IA, Paper 3 Q4
2003 Part IA, Paper 3 Q1
2004 Part IA, Paper 3 Q1
2005 Part IA, Paper 3 Q2

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