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THE EFFECT OF SPHERICAL PROJECTILE SPEED IN RICOCHET OFF WATER AND SAND

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Summary—Experimental results are reported for the ricochet of steel and duralumin spheres (3/8 in. and 1 in. dia.), from shallow depths of water and dry sand. The critical angle for ricochet off water is shown to *increase* with speed to approach the theoretical limit of $18/\sqrt{\sigma}$ deg. For ricochet off sand the critical angle *decreases* with speed but a cut-off angle exists, $82/\sqrt{\sigma}$ deg, for which no ricochet occurs at any speed.

NOTATION

- a radius of spherical projectile
- d diameter of spherical projectile
- g gravitational constant
- v linear speed of projectile
- v_1 exit speed after ricochet
- v_0 initial speed of projectile
- v_{oc} initial projectile speed at which ricochet occurs for a given impact angle
- $\bar{v}$ mean speed during impact
- x, y horizontal and vertical co-ordinates from an origin at the centre of the sphere as it touches the water surface
- $\bar{F}$ mean Froude number $= \bar{v}^2/ag$
- H depth of water
- L vertical lift force
- β angle between forward motion vector and the outward drawn normal at an element on the surface of the sphere
- θ_c critical impact angle calculated from basic Birkhoff theory
- θ_g critical impact angle including the effect of projectile weight
- θ_o initial angle of impact
- θ_{oc} critical angle of impact at speed v_{oc}
- θ_1 angle of exit
- ρ, ρ', ρ_s density of liquid, projectile and sand, respectively
- σ specific gravity of projectile material
- σ_s specific gravity of sand

INTRODUCTION

A SURVEY of empirical knowledge and some history of its applications together with an account of early experimental testing concerning the phenomena of ricochet will be found in ref. (1). The earliest reference to its use has however recently been identified in the second edition of 1643 of the book, "The Art of Shooting in Great Ordnaunce" by the one-time master-gunner to Queen Elizabeth I of England, William Bourne (died 1583) of Gravesend. Bourne's book includes sketches which incontrovertibly represent ricochet of shot from the sea and he wrote, about "... shot that doth graze or trondle (or glance) either upon the land or the water ... by what proportion the shot doth strike or hit the ground or water, by that proportion the shot shall rise againe ...". Another reference,[2] not mentioned in ref. (1) makes statements about the necessity for a spherical projectile to spin in order to cause successful ricochet from water many times.

A theory of ricochet was proposed by Birkhoff *et al.*[3] and detailed predictions from it are given in ref. (1). The latter theory is unable to account for some of the phenomena associated with ricochet including the effect of spin and of the linear speed of the projectile on that impact angle (i.e., the angle between the launch direction and the water surface) which is the largest for which a single ricochet will occur; this is known as the critical angle. Hutchings[4] has recently proposed however that Rayleigh's expression[5] for the lifting pressure on a wide flat lamina skimming a water surface in steady motion, the hydrodynamic flow everywhere being plane, is more appropriate than the model of Birkhoff *et al.*, in particular, since it can be made to take account of spin.

In this note preliminary experiments[6] are reported concerning the effect of speed on the critical angle of ricochet. To the best of the authors' knowledge, the only previous results on this topic are given in ref. (7). May reported an increase of about 20% in θ_c as the initial speed v_0 increased from 200 to 2000 ft/sec.

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The consistency in our findings regarding ricochet from water is clear and our results concerning sand highlights an hitherto unidentified phenomenon.

AN OVER-ESTIMATE OF THE EFFECT OF FROUDE NUMBER ON CRITICAL ANGLE OF IMPACT

In ref. (1) two aspects of ricochet were examined using the theory proposed by Birkhoff *et al.* First, the critical angle of impact, θ_c, was calculated above which ricochet cannot occur. Following this a more detailed analysis of the trajectory of the projectile was performed by solving the system of governing algebraic–differential equations numerically. The weight of the projectile was neglected in both of these analyses in comparison with the hydrodynamic forces on the sphere. The omission of projectile weight is reasonable for high impact speeds, however in some of the experiments described below relatively low speeds were used (down to approximately 30 ft/sec) and so we briefly consider here the effect of including projectile weight in the calculations.

The theory of Birkhoff *et al.*,[1,3] assumes that the pressure on an element of a spherical projectile in process of ricochet is $(1/2)\rho(v\cos\beta)^2$ where $v\cos\beta$ is the speed of the water along the normal to the element; all effects of splash are neglected so that the total vertical lift is assumed to be due to vertically resolved elemental pressure on the projectile surface below the water-line. This is given by

$$L = \frac{1}{2}\rho v^2 \cdot \frac{\sin^4\phi_o}{4} \cdot \frac{\pi a^2}{2}, \tag{1}$$

see Fig. 1. The equation of motion[3] for a solid sphere which has a flat trajectory and including its weight is

$$\frac{4}{3}\pi a^3\rho' v^2 \frac{d^2y}{dx^2} = \frac{1}{2}\rho v^2 \frac{(a^2-y^2)^2}{4a^4}\frac{\pi a^2}{2} - \frac{4}{3}\pi a^3\rho' g. \tag{2}$$

Rearranging (2) and integrating once leads to

$$\int_{\theta_g}^{0} d\left(\left(\frac{dy}{dx}\right)^2\right) = \frac{3}{32a^5}\frac{\rho}{\rho'}\int_a^{-a}(a^2-y^2)^2\,dy - \frac{2g}{\bar{v}^2}\int_a^{-a}dy, \tag{3}$$

where $\bar{v}$ is an average speed of passage of the projectile through the water. θ_g is the angle of impact for which total immersion of the sphere occurs at the instant that horizontal motion ensues. Thus from (3) we have

$$\theta_g^2 = \frac{1}{10}\frac{\rho}{\rho'} - \frac{4ag}{\bar{v}^2} \equiv \theta_c^2 - \frac{4}{\bar{F}}, \tag{4}$$

where $\bar{F}$ is the mean Froude number and θ_c is the critical angle of impact calculated previously[1,3] by neglecting projectile weight. Using the entry speed in equation (4) for θ_g might be expected to predict an over-estimate for an experimentally determined critical angle of impact, θ_{oc}, corresponding to an entry speed of v_{oc} for two reasons. Obviously since $v_{oc} > \bar{v}$ this will produce an increased value for θ_g. Secondly the critical condition for ricochet to occur is not as assumed in equation (3). Because of the weight of the projectile there will be a net downward force at the instant of total immersion and therefore it will not emerge from the water. The limiting depth should have been fixed at that value for which the lift force is just equal to the projectile weight. Both of these objections can be overcome by performing a numerical integration of the governing equations similar to that mentioned above.[6] All the theoretical results referred to below in fact were obtained in this way. However, at least qualitatively, equation (4) describes the effect of entry speed on the critical impact angle. At low speeds it would be expected to be much reduced whereas it would be expected that $\theta_g \to \theta_c$ as v_{oc} increases.

EXPERIMENTAL

Water

Tests were carried out using a cartridge gun mounted on a cast iron platform to fire projectiles into a Perspex tank (6 ft long, 1 ft wide and 1 ft deep), filled with water. Different diameter gun barrels (to accommodate different diameter spheres) were supported by a carrier mounted on a massive cast-iron platform, so that a particular angular position in the vertical plane could be selected. The end of the tank was covered by a layer of Plasticine for safety and a wooden plate resting on supports of variable height made a false bottom on the tank.

Variations in projectile speed were obtained by using different cartridge strengths and by the initial setting of the projectile in relation to the barrel entry. Projectile

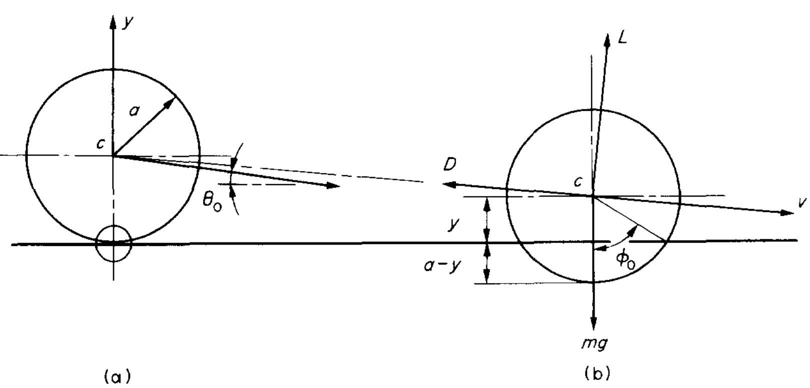

FIG. 1. Entry of sphere at angle, θ_0. (a) At point of initial contact; (b) partially immersed.

speed was measured inside and outside the barrel; in the one case two sets of thin insulated leads a known distance apart were connected by the projectile to give a start signal on a timer followed by a cut-off signal on reaching the second pair of leads. Speed outside the barrel was measured using a photocell system.

The setting of the projectile entry angle was correct to about 1/4°. Projectile exit speed, v_1, and exit angle, θ_1, were measured using a 35 mm camera with a stroboscope flashing at a known rate. Tests were made, by firing projectiles through cardboard screens set perpendicular to the gun barrel axis, to examine projectile deviation from this line. No deviation was observed at more than 50 ft/sec and only 1/4° in a 4 ft range at less than 50 ft/sec. No projectile speed in these tests exceeded 300 ft/sec and speed measurement was thought to be accurate to better than ± 15 ft/sec.

Sand

Experiments similar to those performed using water were also performed using dry sand of specific gravity 2.7 with angular grains about 1 mm dia. A wooden tank 8 in. deep and 4 ft long contained the sand.

Measurement of θ_0, v_0 and θ_1, v_1 were made using the same techniques as for the water experiments. The length between make and break of contract, l, maximum depth of penetration, z, and breadth, b, of the plough were measured directly from the plough left in the sand by the sphere.

RESULTS

Summary of results for ricochet off water

(1) A solid steel sphere of 1 in. dia. was fired over a range of speeds when the impact angle, θ_0 as determined by the initial setting of the gun, had been fixed. The

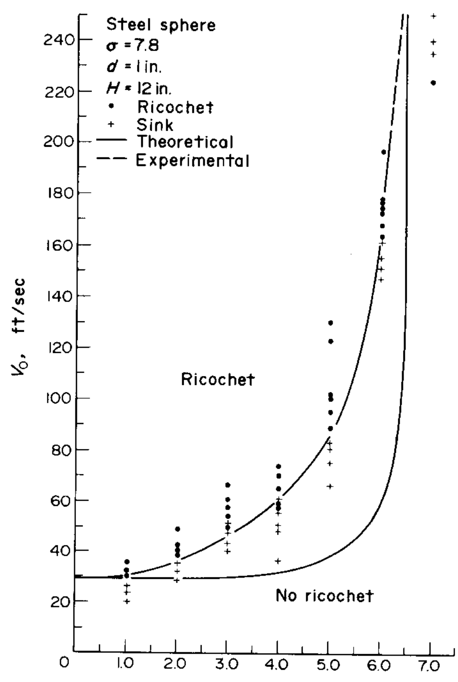

FIG. 2.

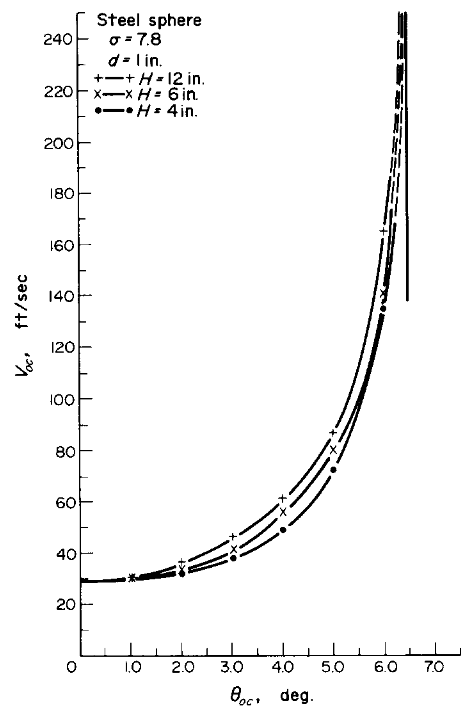

FIG. 3.

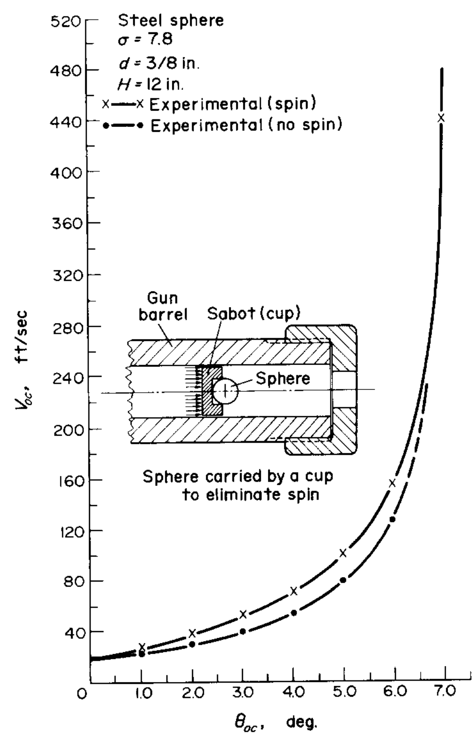

FIG. 4.

speed at which ricochet then occurred as the speed was increased by small amounts could thus be carefully identified. The variation of ricochet speed with impact angle as determined from a large number of tests appears as in Fig. 2.

(2) Tests similar to those above were performed for different depths of water, see Fig. 3. As is apparent and would be expected, the shallower the water the smaller the speed required for ricochet for a given θ_{oc}.

(3) Fig. 4 shows the results of tests aimed at determining the effect of (uncontrolled) spin. By using a sabot (see the inset diagram) to carry a 3/8 in. dia. steel sphere, it was ensured that no initial spin was imparted. Projecting spheres along from the gun barrel gave them top or forward spin, the consistent results from which, show that for a given speed, a lower initial impact angle is required for achieving a ricochet. The effect of the spin seems to be, at most, to require a 1° difference.

(4) The variation of exit speed v_1 after ricochet, with impact angle, for a constant impact speed by a steel sphere is compared in Fig. 5 against that theoretically predicted using the Birkhoff *et al.* premises.

(5) It was found that the exit angle θ_1 is about 0.81 times the entry or impact angle θ_0 see Fig. 6. Always $\theta_1 < \theta_0$ and theoretically $\theta_1 \simeq 0.9\theta_0$. Similar results prevail for both aluminium and steel.

(6) In Fig. 7 the length of plough of steel shot fired at a given speed when ricocheting off water, for chosen values of entry angle is compared with that predicted theoretically for a small, heavy particle. The form of the experimentally obtained relationship well verifies the theoretical one including the existence of a minimum length, but there is a smaller disparity between absolute values for the two than would have been expected.

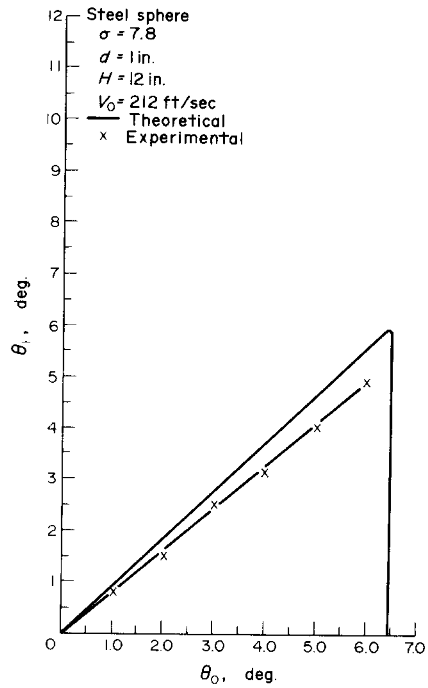

FIG. 6.

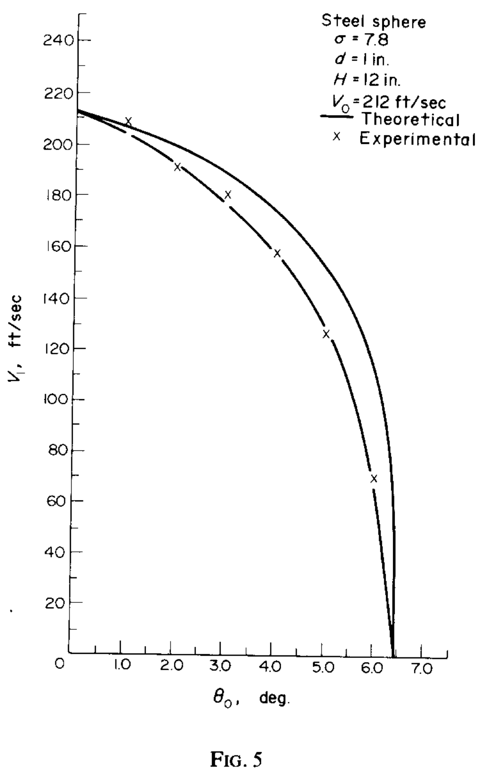

FIG. 5

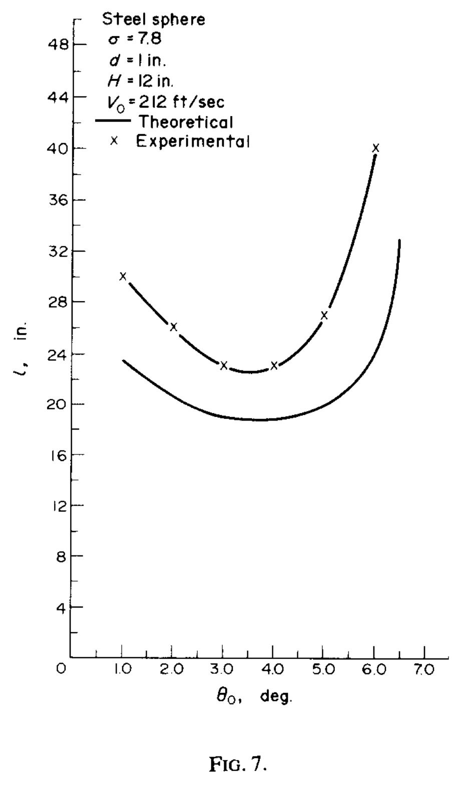

FIG. 7.

Summary of results for ricochet off dry sand

(1) The most surprising of the experimental results reported in this Note, concern the existence of a cut-off angle for the ricochet of a solid sphere from dry sand of specific gravity 2.7. In Fig. 8 a curve is indicated showing for steel ($\sigma = 7.8$) the projectile speed necessary to secure ricochet at a given angle of impact; clearly θ_c *increases* with decrease in speed. The ricochet angle (i.e., see refs. (1) and (3)) $18°\sqrt{\rho_s/\rho'} = 11°$, appears to be approached at high speeds. The sudden cut-off at 29° at a speed of 380 ft/sec is however remarkable; the corresponding angle for duralum ($\sigma = 2.7$) was found to be 51° and for lead ($\sigma = 11.2$) is 26°. These results for an initial cut-off angle can be summarized by $82°\sqrt{\rho_s/\rho'}$ and predicted[1] using $1.38\rho(v \cos\beta)^2$ rather than $0.5\rho(v \cos\beta)^2$. For all three materials, the cut-off speed was about 380 ft/sec. It may be noted that in tests requiring a bullet to be fired through dry sand, Allen *et al.*,[8,9] noted a change in velocity-resistance relationship at about 340 ft/sec and associated it with the speed at which shock waves could be propagated through the medium.

(2) The curve of plough length vs impact angle at a given speed, see Fig. 9 shows a minimum, as did Fig. 7.

(3) As in Fig. 6 for water, so for richochet off sand, the entry angle is 0.81 times the exit angle, see Fig. 10.

(4) A typical Plaster of Paris cast of a plough in sand is shown in Fig. 11. The maximum depth of the cavity is closer to the entry than the exit, and the projectile evidently climbs out of it along a nearly straight path. Also shown in Fig. 12 is a lead sphere, which has been heavily scored during an impact-ricochet.

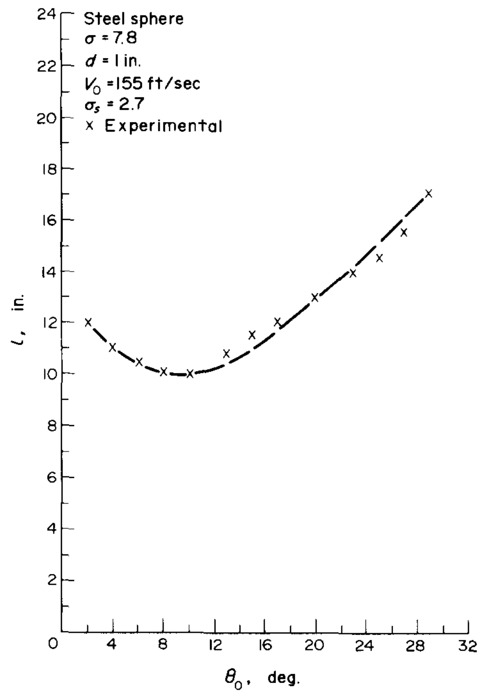

FIG. 9.

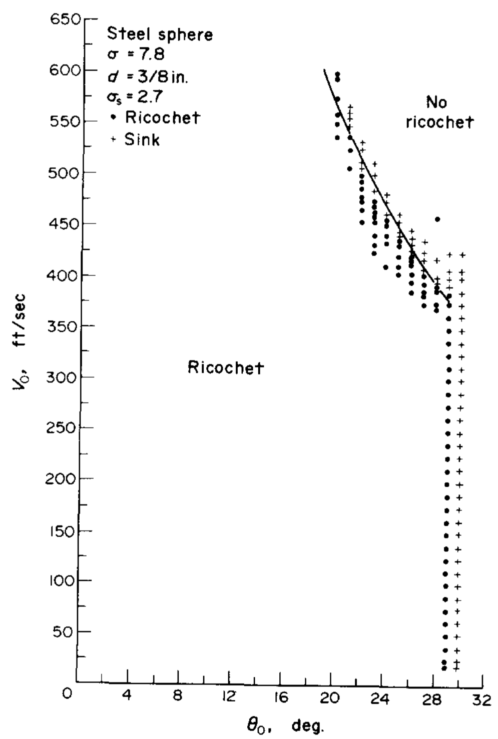

FIG. 8.

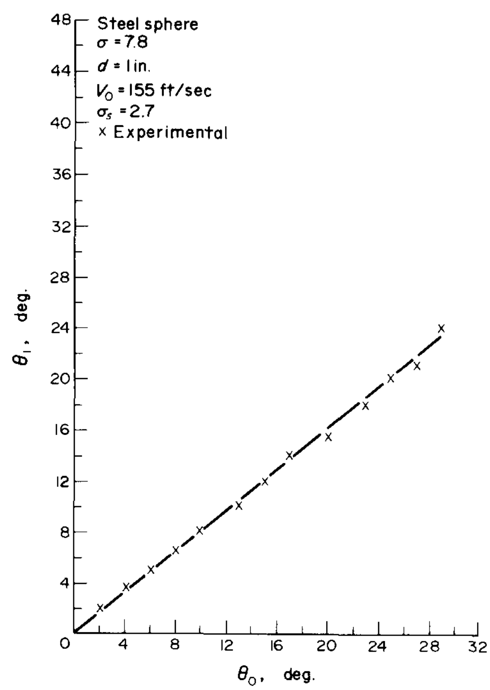

FIG. 10.

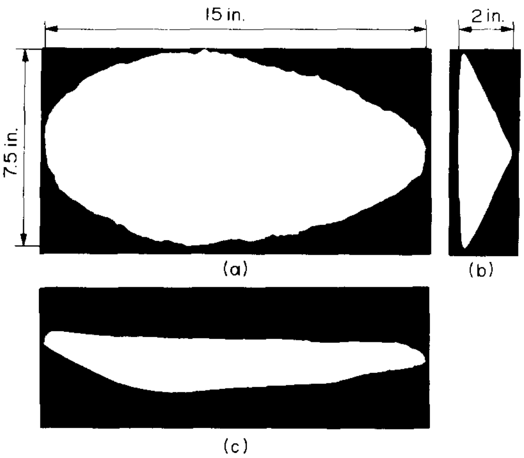

FIG. 11.

CONCLUSIONS

The results of the experiments described above, whilst confirming the well-known characteristics of ricochet behaviour reported elsewhere, indicated that there are certain features of the problem which have not been reported in depth before. In particular the dependence of the critical impact angle on impact speed, spin and depth of water all merit further study. Furthermore, the preliminary results on ricochet from sand lead one to suspect that the nature of the forces responsible for ricochet varies significantly with impact speed.

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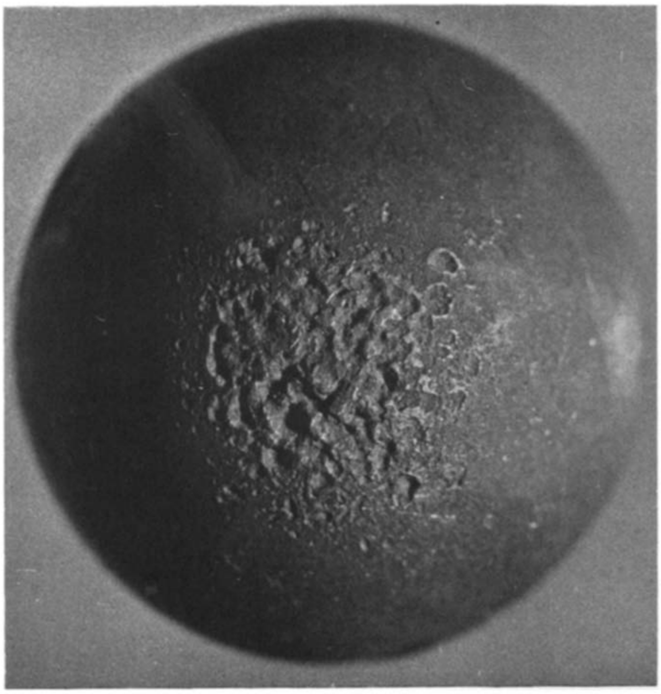

FIG 12.