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A COMPARISON OF PREDICTION MODELS FOR VIBRATIONS FROM UNDERGROUND RAILWAY TRAFFIC

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Abstract

Two prediction models for calculating the vibration from underground railways were developed: an insertion gain model and a coupled periodic finite element-boundary element (FE-BE) model. The insertion gain model is a semi-analytical three-dimensional model that accounts for the dynamic interaction between the track, tunnel and soil. The coupled periodic FE-BE model is based on a subdomain formulation, where a boundary element method is used for the soil and a finite element method for the tunnel. The invariance or periodicity of the tunnel and the soil is exploited in both approaches to formulate the track-tunnel-soil interaction problem in the frequency-wavenumber domain and to compute the wave field radiated into the soil. Results obtained with the two models for a concrete tunnel, embedded in a homogeneous fullspace are compared. The advantages and limitations of both models are highlighted. Both models enable to investigate the inherent physics of underground railway vibration and to control the vibrations propagating from the tunnel.

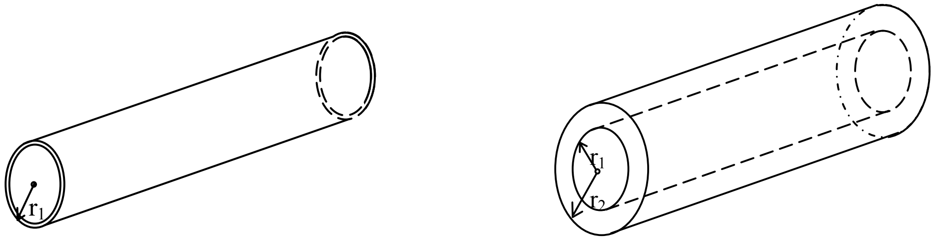
INTRODUCTION

Ground-borne vibrations induced by underground railways are transmitted to adjacent structures and are a major environmental concern in urban areas. Within the frame of the EC-Growth project CONVURT [1], two approaches for modelling the propagation of vibrations caused by underground railway traffic were developed: an insertion gain model and a coupled periodic finite element-boundary element (FE-BE) model. In the present work, a comparative study of both models is carried out.

The coupled periodic FE-BE approach is an efficient prediction tool built using the finite element method and the boundary element method. It accounts for the complex periodic geometry of the tunnel and the layering of a semi-infinite soil medium. However, it cannot easily be used as a design tool on account of the long running time involved. The insertion gain model, which is a three-dimensional semi-analytical model, can be used to reduce computational time and can be very useful in understanding the generation and propagation mechanisms. After a short overview of both models, results obtained with the two models are compared for the case of a concrete tunnel, embedded in a homogeneous fullspace.

THE INSERTION GAIN MODEL

Forrest [5], Hussein and Hunt [7] and Hussein [6] have developed a semi-analytical insertion gain model, that accounts for the essential three-dimensional dynamic interaction between the track, the tunnel and the soil. The system of the tunnel and the soil is modelled as a pair of concentric cylindrical shells named the pipe-in-pipe model (figure 1) [5]. Thin shell equations are used for the tunnel (the inner pipe) whilst the surrounding soil (the outer pipe) is modelled as a three-dimensional elastic solid in the form of a thick walled cylinder, with an inner diameter equal to the diameter of the tunnel and an outer diameter of infinite extent. The analytical solutions for these two components of the physical model are coupled through the use of appropriate stress and displacement boundary conditions at the interface of the tunnel wall and the soil [5, 6, 7].



a. *b.*

Figure 1: The pipe-in-pipe model: (a) tunnel wall of outer radius r_1 modelled as a thin cylindrical shell and (b) surrounding soil modelled as a thick cylindrical shell with internal radius r_1 and external radius r_2 which is taken as infinity.

THE COUPLED FE-BE SUBDOMAIN FORMULATION

The three-dimensional dynamic soil-tunnel interaction problem is assumed to be periodic with period L in the longitudinal direction $\mathbf{e}_y$ along the tunnel axis and can be restricted to periodic fields of the second kind defined on a reference cell $\tilde{\Omega}$ (figure 2) using the Floquet transformation [4]. The boundary $\partial\tilde{\Omega}$ of this domain is decomposed into the free surface $\tilde{\Gamma}_{s\sigma}$ and the boundaries Σ_0 and Σ_L on which periodic conditions are imposed. The generic cell $\tilde{\Omega}$ is decomposed into two subdomains: the soil $\tilde{\Omega}_s$ and the tunnel $\tilde{\Omega}_t$. The interface between these subdomains is denoted by $\tilde{\Sigma}_{ts}$. The boundary $\tilde{\Gamma}_{s\sigma}$ is the free surface of the soil, while a surface force $\tilde{\boldsymbol{f}}_t$ is applied on $\tilde{\Gamma}_{t\sigma}$ (figure 2). The displacement field in the reference cell $\tilde{\Omega}$, is a function of the position vector $\tilde{\boldsymbol{x}}$ in the reference cell, the frequency ω, and the wavenumber κ.

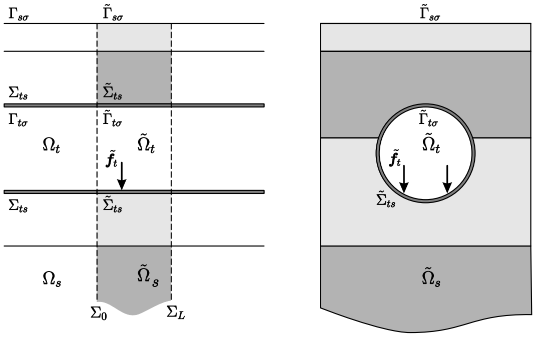


Figure 2: Problem outline and notations.

As the tunnel is bounded, the displacement field $\tilde{\boldsymbol{u}}_t(\tilde{\boldsymbol{x}},\kappa,\omega)$ in the tunnel can be decomposed on a basis of functions $\tilde{\boldsymbol{\psi}}_m(\tilde{\boldsymbol{x}},\kappa)$:

$$\tilde{\boldsymbol{u}}_t(\tilde{\boldsymbol{x}},\kappa,\omega) \quad = \quad \sum_{m=1}^{N} \tilde{\boldsymbol{\psi}}_m(\tilde{\boldsymbol{x}},\kappa)c_m(\kappa,\omega) \tag{1}$$

where $\tilde{\boldsymbol{\psi}}_m(\tilde{\boldsymbol{x}},\kappa)$ obeys the following periodicity condition of the second kind in the longitudinal direction:

$$\tilde{\boldsymbol{\psi}}_m(\boldsymbol{x}'+L\boldsymbol{e}_y,\kappa) = \exp(-i\kappa L)\tilde{\boldsymbol{\psi}}_m(\boldsymbol{x}',\kappa) \tag{2}$$

with $\boldsymbol{x}'$ the position vector in the plane perpendicular to the longitudinal direction $\boldsymbol{e}_y$. The modes $\tilde{\boldsymbol{\psi}}_m(\tilde{\boldsymbol{x}},\kappa)$ are constructed as follows from the eigenmodes $\tilde{\boldsymbol{\psi}}_m^0(\tilde{\boldsymbol{x}})$ of the reference cell of the tunnel that are periodic of the first kind in the reference cell:

$$\tilde{\boldsymbol{\psi}}_m(\tilde{\boldsymbol{x}},\kappa) \quad = \quad \exp(-i\kappa\boldsymbol{e}_y\cdot\tilde{\boldsymbol{x}})\tilde{\boldsymbol{\psi}}_m^0(\tilde{\boldsymbol{x}}) \tag{3}$$

The periodicity of the first kind is defined as:

$$\tilde{\boldsymbol{\psi}}_m^0(\boldsymbol{x}'+L\boldsymbol{e}_y) \quad = \quad \tilde{\boldsymbol{\psi}}_m^0(\boldsymbol{x}') \tag{4}$$

The coupled system of equations in the frequency-wavenumber domain is [2, 3, 4]:

$$\left[\mathbf{K}_t(\kappa) - \omega^2\mathbf{M}_t(\kappa) + \mathbf{K}_s(\kappa,\omega)\right]\mathbf{c}(\kappa,\omega) = \mathbf{F}_t(\kappa,\omega) \tag{5}$$

with $\mathbf{K}_t(\kappa)$ and $\mathbf{M}_t(\kappa)$ the projected stiffness and mass matrix of the tunnel, $\mathbf{K}_s(\kappa,\omega)$ the dynamic stiffness matrix of the soil and $\mathbf{F}_t(\kappa,\omega)$ the generalized force vector applied on the tunnel invert.

The soil impedance $\mathbf{K}_s(\kappa,\omega)$ is calculated with a periodic boundary element formulation with Green-Floquet functions defined on the periodic structure with period L along the tunnel [2]. When the displacements and the stresses on the tunnel-soil interface are known, the incident wave field is obtained by application of the dynamic representation theorem in the unbounded soil domain corresponding to the reference cell [2]. The incident wave field in the soil is obtained by evaluating the inverse Floquet transform.

NUMERICAL EXAMPLE

Problem outline

A concrete tunnel is considered, embedded in a homogeneous soil that is modelled as a fullspace without the presence of a free surface. The tunnel has a mean radius of 2.875 m and a wall thickness of 0.25 m. The concrete is assumed to have a Young's modulus $E^t = 50 \times 10^9$ Pa, a Poisson's ratio $\nu^t = 0.3$ and a density $\rho^t = 2500$ kg/m^3. The soil has a shear wave velocity $C_s = 309$ m/s, a compressional wave velocity $C_p = 944$ m/s, a density $\rho = 2000$ kg/m^3 and a hysteretic material damping ratio $\beta_s = \beta_p = 0.03$ in shear and volumetric deformation.

A concentrated harmonic load is applied on the tunnel invert. A harmonic analysis is performed in the frequency range between 1 and 80 Hz.

Kinematics of the tunnel

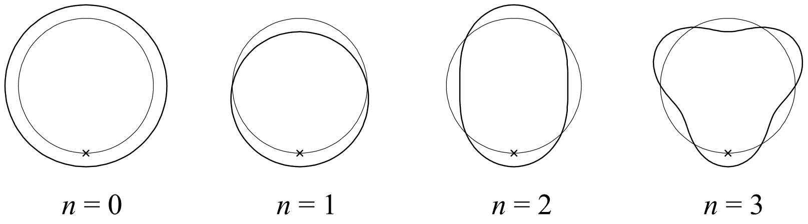
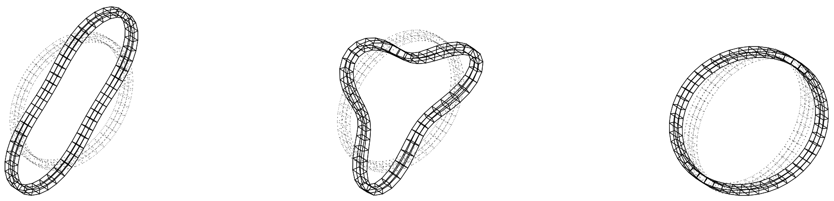


Figure 3: Fourier series decomposition of the load at the tunnel wall around the circumference.

The displacement of the pipe-in-pipe model due to any load is calculated by summing the displacements due to the load components. Any load at the inner surface of the tunnel can be decomposed spatially into its components around and along the tunnel. The load is decomposed around the circumference into the Fourier series components while it is decomposed along the tunnel into an infinite number of harmonics by using the Fourier transformation. Figure 3 shows the Fourier series components for a concentrated load applied at the tunnel invert (marked with a cross). In this case, only the cosine terms in the form $\cos n\theta$ appear in the decomposition, as the concentrated load is described as an even function around the tunnel invert.



a. Mode 6 at 17.6 Hz. *b. Mode 7 at 50.0 Hz.* *c. Mode 11 at 153.9 Hz.*

Figure 4: The first two in-plane and the first out-of-plane modes of the cell of the tunnel.

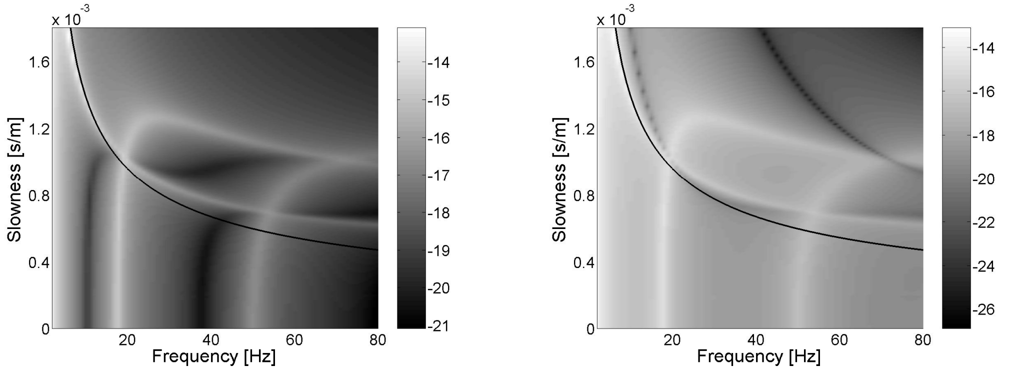
In the coupled FE-BE approach, the model is reduced to a single bounded generic cell, exploiting the periodicity of the tunnel in the y-direction. The length L of the reference cell is equal to 0.5 m, which is discretized using 8-node volume elements with incompatible bending modes. The kinematic basis for the tunnel consists of modes $\tilde{\boldsymbol{\psi}}_m(\tilde{\boldsymbol{x}}, \kappa)$ that are derived from the eigenmodes $\boldsymbol{\psi}_m^0(\tilde{\boldsymbol{x}})$ of the generic cell that satisfy free boundary conditions on $\tilde{\Sigma}_{ts}$ and periodicity conditions at both ends Σ_0 and Σ_L. Due to these constraints and the symmetry of the cell, displacements in the y-direction are decoupled from displacements in the x- and z-directions and only 4 rigid body modes are found. Figure 4 shows the first two in-plane and the first out-of-plane flexible modes $\boldsymbol{\psi}_m^0(\tilde{\boldsymbol{x}})$ of the cell.

Since the kinematic basis consist of the modes $\tilde{\boldsymbol{\psi}}_m(\tilde{\boldsymbol{x}}, \kappa)$, the equilibrium equations in frequency-wavenumber domain become coupled. In order to solve this system accurately, apart from the lowest modes, some higher modes should also be included. These higher modes can be found by studying the coupling parts of the full impedance matrix for a non-zero wavenumber and frequency.

Harmonic response of the free tunnel

First, a free tunnel, i.e. a tunnel without the surrounding soil, is subjected to a harmonic load on the tunnel invert. Figures 5a and 5b show the logarithm of the absolute value of the vertical displacement at the driving point and the tunnel apex as a function of the frequency ω and the slowness $p = \kappa/\omega$. The response is high (or infinite in the absence of damping) at wavenumbers and frequencies along the dispersion curves. Superimposed on the same graphs is the dispersion curve of a bending wave in an equivalent beam. At a zero slowness value, the cut-on frequencies of the first two flexural modes can be identified at 17.6 Hz and 50.0 Hz, respectively. Two dispersive modes propagate along the tunnel and become non-dispersive at limiting high frequencies. The compression mode has insignificant contribution to the response due to the weak coupling between the longitudinal and radial response for $n = 0$.

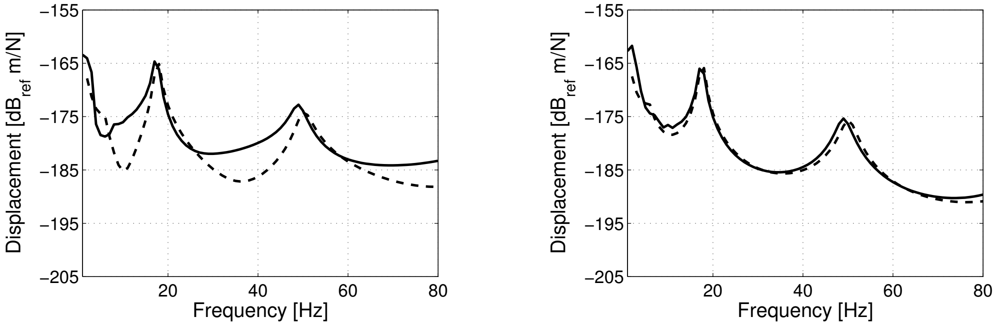
The inverse Floquet transform with respect to the wavenumber gives the response in the spatial domain. Figures 6a and 6b compare the response of the free tunnel at the driving point and at the tunnel apex obtained with both models. The peaks in the



a. *b.*

Figure 5: Logarithm of the absolute value of $\tilde{u}_t$ as a function of ω and p at (a) the tunnel invert and (b) the tunnel apex, computed with the periodic FE model. Superimposed on the same graph is the dispersion curve for the bending wave in an equivalent beam (solid line).

response are found at the eigenfrequencies of the first two flexural modes of the generic cell or a two-dimensional ring structure. The driving point response is very sensitive to the finite element mesh and is prone to discretisation errors. Therefore, the driving point response obtained with the periodic FE model does not correspond very well with the analytical solution. However, the response at the tunnel apex shows a good agreement between the results of both models.



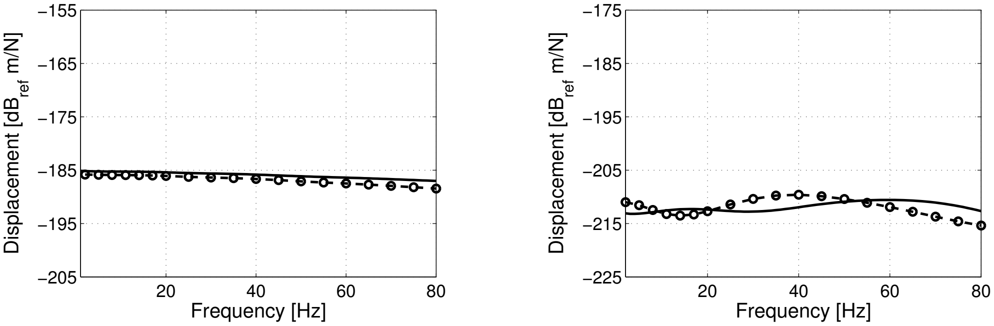
a. *b.*

Figure 6: Transfer function at (a) the tunnel invert and (b) the tunnel apex, computed with the insertion gain model (solid line) and the periodic FE model (dashed line).

Harmonic response of the tunnel-soil system

In the insertion gain model, elastic continuum theory is used to model the soil as a fullspace with a cylindrical cavity. The analytical solutions for the soil and the tunnel are coupled using the appropriate boundary conditions in the frequency-wavenumber domain.

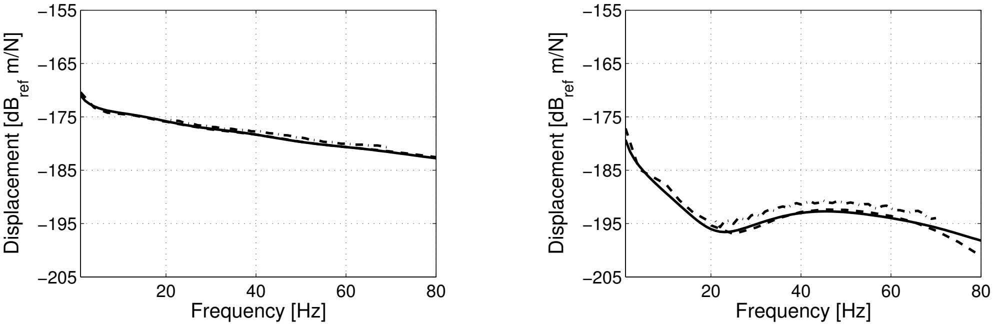
In the coupled FE-BE approach for the soil, the boundary element mesh is derived from the exterior surface of the finite element mesh. The Green-Floquet function



a. *b.*

Figure 7: Transfer function at (a) the driving point and (b) the tunnel apex, computed with the insertion gain model (solid line) and with the coupled periodic FE-BE model (circles on dashed line).

for the homogeneous fullspace is used to determine the impedance of the soil. The dynamic soil-structure interaction problem is solved on the generic cell in the frequency-wavenumber domain. Figures 7a and 7b compare the response at the driving point and the tunnel apex computed with both approaches for selected frequencies in the frequency range between 1 and 80 Hz. Both methods give a good estimation of the driving point response. Unlike the response of the free tunnel, there are no peaks in the driving point response of the coupled system. The presence of soil as an infinite fullspace around the tunnel suppresses the resonance frequencies of the tunnel. Energy is dissipated due to the radiation and material damping in the soil.



a. *b.*

Figure 8: Plane strain transfer function at (a) the driving point and (b) the tunnel apex, computed with the insertion gain model (solid line), the coupled periodic FE-BE model (dashed line) and the ANSYS FE model (dashed-dotted line).

The results are also compared with a two-dimensional plane strain finite element calculation. A plane strain problem arises when a line load is applied on the infinite tunnel. In the insertion gain model, the zero wavenumber term directly corresponds to the two-dimensional plane strain transfer functions, whereas in the periodic FE-BE approach first the Fourier transform has to be obtained from the Floquet transform.

A plane strain finite element model is built in ANSYS and local absorbing boundary condition are used to account for Sommerfeld's radiation conditions. Figures 8a and 8b show the plane strain transfer function at the tunnel invert and the tunnel apex. The response at the driving point and the tunnel apex shows a good agreement between the results of the insertion gain model, the coupled periodic FE-BE model and the ANSYS FE model.

CONCLUSIONS

The main objective of the work presented in this paper is to compare the two modular prediction tools for estimating vibrations due to underground railway traffic. This study prospers a better understanding in the context of vibration from underground railways. The insertion gain model is an analytical formulation, and hence computationally more efficient. However, it has the disadvantage that it can only be used for a deep bored tunnel with a simple geometry. It cannot take into consideration the presence of a free surface or layering in the soil. In contrast, the coupled periodic FE-BE approach can deal with more general periodic tunnel-soil systems, although it is computationally very expensive. Both models give a good match of the driving point response when a tunnel embedded in a homogeneous fullspace is considered. The insertion gain model allows to make rational decisions about the track design for reducing vibration transmission at an early phase without need for extensive site investigations or structural details. For the accurate prediction of vibrations due to underground railways with a complex geometry and inhomogeneous soil characteristics, the coupled FE-BE approach should be preferred.

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