1 Introduction

Friction models are used in the cold metal rolling industry to increase productivity and improve surface quality via better online control and off-line scheduling. Cold rolling mills use oil as lubricant and coolant to reduce the friction and temperature in the roll bite. To achieve a satisfactory friction level and good surface finish, lubrication during rolling is generally in the “mixed” regime, in which there is someasperity contact between the roll and strip surfaces but also significant oil entrained into the bite due to hydrodynamic action. In many rolling processes, the roll grinding process leads to a roll roughness with a pronounced lay, with the ridges and valleys of the asperities running along the direction of rolling. This longitudinal roughness is in turn transferred to the strip. When the strip and roll come into contact, the surface asperities are crushed, leading to finger-like areas of close contact extending into the bite. These are separated by valleys containing oil, which is drawn through the bite under pressure due to the entraining action of the roll and strip. The contribution to friction from the areas ofasperity contact is generally much higher than from viscous shearing of oil in the intervening valleys. Therefore it is important to evaluate accurately the ratio of the true to nominal contact area (the “contact area ratio”) and the corresponding boundary friction coefficient on these asperity contacts.

During the last decade several models have been developed for cold rolling in the “mixed” lubrication regime [1–4], which predict the contact area ratio and frictional stresses in the bite. In these models, the surface roughness is simplified as a uniform array of asperities. Models for flattening of the strip asperities need to take into account the effect of plastic deformation of the underlying strip, which greatly increases the conformity between roll and strip [5–6]. Modifications to Reynolds’ equation are used to account for the effect of roughness on the hydrodynamic pressure generated in the lubricant. These are based either on an “averaged Reynolds equation” (Christensen [7]), or the “flow factor” approach developed by Patir and Cheng [8–9]. However predictions of friction from these models are significantly higher than values measured by Tabary et al. [10] on an experimental mill. Although this discrepancy could arise from either errors in the estimate of the contact area ratio or of the boundary lubrication coefficient, the surface analysis showed that the conformity was significantly lower than that predicted by a single-wavelength model, suggesting that the contact area ratio is not being modelled satisfactorily.

Steffensen and Wanheim [11] show how the inclusion of short wavelength asperities, superimposed on long wavelength asperities, can reduce significantly the predicted contact area ratio in the case where there is no bulk deformation. Since the ground surface roughness normally present on the rolls (and the resulting finish on the strip) contains a spectrum of wavelength components, this effect is expected to be significant in cold rolling. Sutcliffe [12] has developed a two-wavelength asperity model for dry rolling and finds that the prediction of the contact area ratio is greatly reduced when the short wavelength component is included. Predictions of the change in amplitude of both roughness wavelengths show excellent agreement with experiments.

Recent measurements of changes in surface roughness during cold rolling with mixed lubrication by Sutcliffe and Le [13–14] show that short wavelength components persist longer than long wavelength components, mirroring the results for dry rolling. These results support the inference that difficulties with predicting the friction measurements of Tabary et al. [10] may have arisen from errors in estimating the area of contact ratio. Hence a two-wavelength surface-flattening model was developed by Le and Sutcliffe [15] to consider this effect. Longitudinal roughness is assumed, with ridges and valleys running along the rolling direction. The hydrodynamic pressure in the oil is found using the averaged Reynolds equation. For simplicity the oil pressure is assumed to be the same in each valley at a given location through the bite. The predicted amplitudes of the two wavelengths of roughness have reasonable agreement with measurements, but the contact area ratio is still much higher than that inferred from friction measurements. The idea of a model that allows different hydrodynamic pressures in each valley was raised in that paper but not pursued. In this paper that approach is revisited. The hydrodynamic theory and the surface flattening models are described in the next section and the numerical scheme briefly outlined. These results are used to infer the friction variation through the bite. The model is verified by a comparison with experimental measurements of surface roughness and friction coefficient.

Oil may be applied neat or in an emulsion with water to in-
crease its effectiveness as a coolant. A review of the tribological conditions for emulsions is given by Schmid and Wilson [16]. Quantitative differences between neat oils and emulsions arise from the effectiveness with which the oil can be delivered to the roll bite. In this paper only lubrication with neat oil is considered. Nevertheless the factors shown in this paper to be important for neat lubrications are expected to retain their importance for lubricant emulsions. Further extensions to rolling models include work-hardening of the strip and elastic deflections of the rolls, which significantly affect the geometry of the bite during foil rolling [17–19]. Again the roughness effects discussed in this paper will have relevance to rolling models which include these effects.

2 Theoretical Analysis

The model described in this paper is similar to the two-wavelength model outlined by Le and Sutcliffe [15]. The main difference arises in the hydrodynamic model, as described in section 2.2, while there are detailed differences in the asperity-flattening model, section 2.3.2. An overview of the rolling process is illustrated in Fig. 1, where the strip is reduced in thickness from \( t_1 \) to \( t_2 \) as it passes through rolls of equal radius \( R \). The length of the roll bite \( b \) is derived from the circular roll arc as \( b = \sqrt{t_1 R} \), where \( t \) is the reduction in strip thickness, and the inlet angle \( \delta \) is equal to \( b/R \). The strip material is taken as rigid ideally plastic. The roll bite can then be divided into three zones according to the nature of deformation: an inlet zone with no bulk deformation of the strip, a transition zone with bulk deformation and significant asperity crushing due to the difference in pressure between the asperity tops and the valleys, and a work zone in which the pressure between the asperity tops and valleys has equalized [3].

2.1 Roughness Geometry. As with the earlier two-wavelength model [15], the roughness geometry is modelled by two wavelengths of longitudinal roughness with the ridges and valleys of the roughness running in the direction of rolling. The roughness geometry is shown in Fig. 2, illustrating a section taken transverse to the rolling direction containing a “unit cell” with one long wavelength asperity onto which a number of short wavelength asperities are superimposed. The rolling direction is out of the plane of the figure, so that the strip elongates out of the plane of the figure and is compressed in the vertical direction as indicated. A plan view of the strip surface is illustrated in Fig. 3, with areas of contact shown shaded. As the strip and roll come into contact, the longitudinal roughness gives rise to “fingers” of asperity contact running into the bite, separating isolated valleys. For cold strip rolling, plane strain conditions apply, so that there are no transverse displacements across the vertical symmetry lines at the center and edges of the roughness profile shown Fig. 2. The wavelengths of the short and long wavelength components are \( \lambda \) and \( \lambda \), respectively. Their corresponding peak-to-peak amplitudes are \( Z \) and \( z \), with initial values \( Z_0 \) and \( z_0 \). The asperity geometry is characterized by the valley depth \( \delta \) which varies both across the width of the contact and in the rolling direction. Associated with each short wavelength asperity is a local mean film thickness \( \bar{h} \), surface roughness variance \( s^2 \) and area of contact ratio \( a \). These can be expressed as explicit functions of the valley depth \( \delta \) [12]. The mean of the local surface roughness variances gives the overall short wavelength variance and corresponding short wavelength roughness amplitude \( s \). The long wavelength asperity also has an area of contact ratio \( A \) associated with it, giving the proportion of

![Fig. 1 Schematic of the strip rolling process](image)

![Fig. 2 Cross-section of the contact between a smooth roll and strip with a two-wavelength surface roughness. The rolling direction is out of the plane of the figure: (a) when roll and strip first contact, and (b) after some asperity flattening.](image)

![Fig. 3 Plan view of the geometry of the contact patch between a smooth roll and rough strip with a two-wavelength surface roughness](image)
the contact over which the small wavelength asperities make contact. The long wavelength amplitude \( \Sigma \) is found from the roughness profile after subtracting off the short wavelength profile. A triangular cross-section is used for both wavelengths of roughness in this paper. A pseudo-Gaussian shape [7] has also been examined, but no significant difference was observed. Although both strip and roll surfaces are rough, it is assumed that the roll is smooth and rigid, while the strip surface assumes the combined surface roughness of both the strip and roll. With the assumed longitudinal roughness, there is no ploughing action so that this assumption is appropriate.

2.2 Hydrodynamic Pressure. The averaged Reynolds equation [7] is used to derive the variation in hydrodynamic pressure of the lubricant in the inlet and transition zones. We assume that, far ahead of the bite, where there is no asperity contact, the oil pressure is uniform across the contact. As asperities progressively make contact, an increasing number of isolated valleys are formed, as illustrated in Fig. 2. It is expected that the shallower valleys would develop a larger hydrodynamic pressure than the deeper valleys, at a given position through the bite. We consider this effect by applying Reynolds’ equation in two ways. Once a valley has become isolated (e.g., valleys 1 and 2 in Fig. 2), Reynolds’ equation is applied directly to that valley to track its continuing increase in pressure. The valleys which remain connected, and which together constitute the valley of the long wavelength component of roughness (i.e., valleys 3, 4, and 5 in Fig. 2), are treated together as all having the same hydrodynamic pressure. This approach contrasts with the previous paper in which it is assumed that all valleys have the same hydrodynamic pressure [15], whether isolated or not. For the isolated valleys, Reynolds’ equation for the variation in the rolling direction \( x \) of the reduced pressure \( q \) in the lubricant becomes [20]

\[
\frac{dq_i}{dx} = 12 \eta_0 \alpha \bar{u} \left( h_i - h_i^* \right)(1 - a) / \bar{h}_i^2 \left( 1 + 3 \bar{h}_i^2 / \bar{h}_{\infty}^2 \right)
\]

where the reduced pressure \( q \) is related to the pressure in the valley \( p_c \) by \( q = 1 - \exp(-ap) \), \( \eta_0 \) is the viscosity at ambient pressure, \( \alpha \) is the pressure viscosity coefficient in the Barus equation \( \eta = \eta_0 e^{ap} \) and \( \bar{u} = (u_+ + u_-)/2 \) is the entraining velocity. The subscript \( i \) is used to emphasize that the reduced pressure, mean film thickness and roughness variance are evaluated for each of the individual valleys. The term \( (1 - a) \) is introduced to allow for the reduction in valley width as asperity crushing proceeds. This expression is valid for the case where pressure variations in the transverse direction are small and flow rates in the transverse direction may be neglected compared to flow rates in the entraining direction. The derivation by Baglin [20] assumes that the roughness is symmetrical about the nominal film thickness and it is not restricted to small roughness amplitudes. This is true for the isolated triangular valleys and is a good approximation for the connected valleys. To solve for the values of the flow rate constant \( h_i^* \) in each valley it is assumed that the gradient of the hydrodynamic pressure \( dq / dx \) is equal to zero in each valley at the end of the transition region where the pressures on the asperity tops and in the valleys have equalized. This assumption is adequate under normal rolling condition where the hydrodynamic pressure builds up fast and the asperity crushing process is confined to short inlet and transition zones. For the results presented here, this assumption is good, but it would be less good for “low speed” rolling, where the hydrodynamic pressure builds up slowly and asperity crushing extends through the bite [21].

For the connected valleys (e.g., valleys 3, 4, and 5 of Fig. 2) the averaged Reynolds equation becomes,

\[
\frac{dq_i}{dx} = 12 \eta_0 \alpha \bar{u} \left( h_e - h_i^* \right)^2 / \bar{h}_i^2 \left( 1 + 3 \bar{h}_i^2 / \bar{h}_{\infty}^2 \right)
\]

where the subscript \( c \) is used to emphasis that the reduced pressure, mean film thickness and surface roughness variance are evaluated over all the connected valleys. The flow rate constant \( h_i^* \) is given by the sum of the flow rates \( h_c^* \) for the individual valleys which make up this connected region. Well ahead of the bite there is no asperity contact so that Eq. (2) applies, with \( h_c^* \) equal to the sum of all the local flow rates \( h_i^* \).

2.3 Asperity Crushing. In this section we describe equations associated with the mechanics of asperity crushing to find the variation in valley depth \( \delta \) in the inlet and transition zones.

2.3.1 Inlet Zone. In the inlet zone there is no bulk deformation of the strip. We assume the inlet zone is very short compared to the roll radius. Hence the roll profile in this region can be taken as a straight line inclined at an angle \( \phi \) to the horizontal. By choosing the origin for the co-ordinate \( x \) in the rolling direction as the point where first contact is made between the roll and strip surfaces, the valley depth \( \delta \) is given by

\[
\delta = \delta_0 - \phi x
\]

where \( \delta_0 \) is the initial asperity depth at the point of first contact. It is assumed that elastic deformation of the strip can be neglected, and that individual asperity contacts behave as mini-indentations. Hence, the Prandtl field applies and the difference between the pressures \( p_a \) and \( p_c \) on the top of the asperity and in the adjacent valley is equal to the hardness of the asperity, taken as 2.57 times the plane strain yield strength \( Y \) of the material (see [22]), i.e.,

\[
p_a - p_c = 2.57Y
\]

It is further assumed that the tops of the asperities are crushed as if the material on the top is removed. Although in fact the material will tend to be displaced into the adjacent valley, experiments by Sutcliffe [5] suggest that this is not a significant effect. The local mean contact pressure \( P \) averaged over each short wavelength asperity can be expressed as

\[
P = p_a + p_c (1 - a)
\]

Note that \( P \) varies over the width of the contact. When the pressure \( \bar{P} \) averaged over the whole surface reaches the yield strength \( Y \), bulk deformation occurs in the underlying material. This marks the end of the inlet zone. Here the effects of inlet tension and frictional stress on the yield criteria are ignored, though these could easily be included where necessary.

Elastic deformations of the asperities is not included in this model. Models of microelastohydrodynamic lubrication (e.g., [23]) show that asperities can undergo significant elastic deformation under the high pressures of concentrated contacts. However it is expected that this effect will not be important in metal rolling, where severe plastic deformation of the asperities is observed during rolling, and where the asperities do not become rounded due to running in, as for elastic contacts. Nevertheless it remains an interesting avenue of future enquiry.

2.3.2 Transition Zone. In the transition zone bulk deformation of the strip takes place. The hydrodynamic pressure \( P_r \) in the valley of the long wavelengthasperity is simply the pressure in the connected small wavelength valleys. The pressure \( P_p \) on the contact area of the long wavelength asperity can be found from the yield criterion as

\[
P = AP_p + (1 - A) P_c - Y
\]

It is assumed that the mean pressure on the long wavelength asperity contact is uniform and equal to \( P_r \), from which the local asperity pressure \( p_c \) on the short wavelength contact area is found via Eq. (5). As with the inlet zone it is assumed that the transition region is short, so that the roll profile can again be taken as straight with a slope \( \phi \). The effect of bulk deformation on asperity flattening is modelled using the method described by Sutcliffe [5].
The relative velocity $V_f$ between the long wavelength asperity contact and valley is found from a dimensionless flattening rate $W_A$ given by

$$W_A = \frac{2V_f}{\bar{\kappa}} = W(A, \Delta P)$$  \hfill (7)

This flattening rate $W_A$ is a function of the long wavelength area of contact ratio $A$ and the dimensionless pressure difference $\Delta P = (P_p - P_\infty)/Y$ between the asperity and hydrodynamic pressures $P_p$ and $P_\infty$. The function $W$ is taken from the finite element solutions for an infinite array of similar asperities by Korzekwa et al. [24], fitted using polynomials as detailed in the Appendix. Note that, although the dimensionless crushing rate $W$ has been expressed using a bulk strain rate $\bar{\epsilon}$, this is only included as a convenient way of normalizing the asperity flattening rate $V_f$. The evolution of the contact area does not depend on bulk strain rate, per se.

For the short wavelength asperities in contact, the local flattening velocity $v_f$ is given by a similar non-dimensional flattening rate $W_s$ given by

$$W_s = \frac{2v_f}{\bar{\kappa}} = W(a, \delta p)$$  \hfill (8)

The flattening rate $W_s$ is given by the same function $W$ detailed in the Appendix, but now depends on the local contact area ratio $a$ and local pressure difference $\delta p = (p_a - p_p)/Y$ between the short wavelength asperity contact and valley pressures.

Sutcliffe [12] details continuity relations, which relate the local and long wavelength flattening rates $v_f$ and $V_f$, needed to find a solution for the local variation in pressure and flattening rate $v_f$ across the contact. These local flattening rates can then be used to calculate the rate of change of valley depth $\delta$ with position in the rolling direction as

$$\frac{d\delta}{dx} = \frac{v_f}{\bar{\kappa}} \frac{dx}{dx}$$  \hfill (9)

where the variation in strain in the rolling direction $d\varepsilon/dx$ is determined from the inlet angle and inlet strip thickness. As the asperities are progressively flattened in the transition zone, the hydrodynamic pressure in the oil rises until it reaches the pressure on the asperity tops. The location where this has occurred in every valley marks the end of the transition region. It is assumed that, during the crushing process, material displaced from under the valley marks the end of the transition region. It is assumed that, on the asperity tops. The location where this has occurred in every valley marks the end of the transition region. It is assumed that, during the crushing process, material displaced from under the asperity tops does not alter the valley profiles. This assumption is supported by the finite element results of Korzekwa et al. [24].

### 2.4 Work Zone Analysis

Although there is no pressure difference between the contact areas and the valleys in the work zone, the asperity valleys move up relative to the plateaux in this zone due to stretching of the strip associated with elongation of the strip. Oil mass conservation requires that the flow rate $\bar{u}h$ of the lubricant in each valley must be constant, where the variation in the mean velocity $\bar{u} = (u_r + u_\infty)/2$ is found as before from mass continuity of the strip. The implied reduction in mean film thickness $\bar{h}$ with increasing mean velocity $\bar{u}$ implies a reduction in valley depth $\delta$ in the work zone. Assuming that the contact area ratio does not change in the work zone and that there is a uniform strain distribution through a cross-section of the strip, this leads to the condition $\Delta \delta / \bar{h} = -\Delta \bar{u}/\bar{u}$. Hence the variation in valley depth $\delta$ through the work zone can be found directly from the mean velocity $\bar{u}$.

### 3 Solution Method

A flow chart of the solution method is included in Fig. 4. Due to symmetry, only half of a long wavelength asperity is considered, on to which are superposed five short wavelength asperities. The program starts with an assumed set of $h^*_t$, one for each of valleys. The overall flow rate constant in the connected region $h^*_t$ is equal to the sum of the values of $h^*_t$ for each of the valleys. As valleys become isolated, their contribution to the flow rate in the connected region is subtracted from $h^*_t$.

In the inlet zone before the strip and roll make contact, all the valleys are connected and Eq. (2) is integrated to solve the hydrodynamic pressure, starting from a position far ahead of the entry. When the strip and roll make contact, Eq. (3) is used to calculate the depth of each valley $\delta$, and hence the mean film thickness $\bar{h}$ and variance $\sigma^2$ for each valley. Equation (1) is used to solve the hydrodynamic pressure in the isolated valleys, while the hydrodynamic pressure in the connected region is solved by Eq. (2). Equations 4 and 5 are then used to calculate the asperity pressure and mean contact pressure of each asperity.

When the yield criterion is satisfied (Eq. (6)), bulk deformation occurs, marking the boundary between the inlet and transition regions. Equations governing the asperity crushing rate (Eqs. (7), (8) and (9)) and the oil pressure (Eqs. 1 and 2) are integrated simultaneously by the 2nd/3rd order Runge-Kutta method, to give the variation in hydrodynamic pressure and the depth of each valley in the transition zone. The yield condition, Eq. (6), is used to find the long wavelength asperity contact pressure $P_p$ at the end of each step, and hence the local pressure on each small wavelength asperity. After each integration values of $h^*_t$ are adjusted using a standard equation solver to satisfy the boundary condition of zero pressure gradient in each valley at the end of the transition zone. The deformation of the asperities in the work zone is then continued following the model described in section 2.4.

### 4 Friction Model

The contact area and the valleys are treated separately. The shear stress $\tau$, associated with viscous shearing of the lubricant in the valleys is calculated by the Eyring viscous model

$$\tau_v = \tau_0 \sinh^{-1} \frac{\Delta \bar{u}}{\tau_0}$$  \hfill (10)

where $\tau_0$ is the Eyring shear stress, $\Delta \bar{u}$ is the sliding speed, and $\bar{h}$ is the mean film thickness in the valley.
Friction on the contact area is generally regarded as due to boundary friction. Here the friction coefficient \( \mu_e \) is taken as constant, so that the shear stress \( \tau_e \) on the tops of the small wavelength asperities is given by

\[
\tau_e \approx \mu_k p \frac{d}{l} \tag{11}
\]

Although the factors governing the value of the boundary friction coefficient \( \mu_k \) are not well understood, this is likely to depend strongly on the physico-chemical behavior of the strip and lubricant additives in the bite. It appears from the experimental data that the boundary friction increases with increasing reduction, perhaps associated with the greater area of new surface created at a lower reduction [10]. The results presented in this paper, values of 0.09 for strip reductions of 25 percent, are used. To some extent this parameter can be used as a fitting parameter. However, reducing the degrees of freedom associated with boundary friction to a single fitting parameter seems more physically reasonable than the variation in boundary friction coefficient with rolling speed required by previous models (Marsault et al. [4], Le and Sutcliffe [25]). The shear stress averaged over each short wavelength asperity is given by

\[
\tau = \tau_e a + \tau_s (1 - a) \tag{12}
\]

This local variation in shear stress is then averaged across the width to give the mean shear stress \( \bar{\tau} \) and hence a friction coefficient:

\[
\mu = \frac{\bar{\tau}}{P} \tag{13}
\]

The friction coefficient \( \mu \) is then averaged through the roll bite by taking a mean value with respect to position to give an overall average friction coefficient \( \bar{\mu} \).

### 5 Results and Discussions

Two sets of parameters used in the calculations are extracted from measurements on cold rolling of aluminum alloy by Tabary et al. [10] and Sutcliffe and Le [13], as outlined in Table 1. Oil pressures are taken from the respective references. Viscosity either comes from manufacturers’ data [10] or direct measurements [13]. The pressure viscosity coefficient and Eyring shear stress for Tabary et al.’s data are estimated according to Johnson and Tevararkw [26], while for Sutcliffe and Le’s measurements the same values are assumed.

The roughness of the strip and rolls are given in Table 1. In both cases the roll is much smoother than the strip and its roughness has been ignored in the calculations. One uncertainty in the model is how to split the surface spectrum into two wavelengths. Here we have split the roughness spectrum at a wavelength which gives a simple ratio of amplitudes for the two components. The two characteristic wavelengths are estimated from the spectral density of the roughness. For the experiments of Tabary et al. [10], measurements on their samples were used to extract the spectral density of the ingoing strip. The roughness spectrum is split at a wavelength of 10 \( \mu \)m so that the long-wavelength components have a r.m.s amplitude of 0.5 \( \mu \)m and the short-wavelength components 0.1 \( \mu \)m. The maximum spectral density of the long-wavelength components occurs at a wavelength of 30 \( \mu \)m. Though the spectral density of the short-wavelength components is rather scattered, the maximum appears to be around a wavelength of 6 \( \mu \)m. For the results of Le and Sutcliffe [13], the same procedure is followed, splitting the spectrum at 10 \( \mu \)m, with the long and short wavelengths again taken as 30 and 6 \( \mu \)m. A physical basis for making this choice would require more understanding about the mechanisms of boundary friction than is currently available.

#### 5.1 Contact Pressure and Asperity Depth

In this section the form of solution with the parameters taken from the second column of Table 1 is presented (Tabary et al. [10]). For this set of calculations the reduction in strip thickness is 25 percent and the rolling speed is equal to 0.1 m/s. Figure 5 shows the variation of hydrodynamic pressure in the valleys through the bite, while Fig. 6 shows the associated change in valley depth. In the plot of pressures, Fig. 5, the bottom curve tracks the pressure in all the connected valleys. Once contact is made, for a valley depth \( \delta \) so that \( \delta / \tau_0 < 1 \), the pressure in an isolated valley leaves this lower line. In the very short inlet zone the pressure in valley 1 (shallowest valley shown in Fig. 2) rises most steeply, until it reaches the hardness of the asperity 2.571 at which point local asperity deformation stops. Proceeding through the bite the valley pressures rise until bulk yield occurs at a value of \( x/h \) approximately equal to 0.02. In the subsequent transition region the top pressure curve of Fig. 5 is equal to the mean pressure on the contact area of the long wavelength asperity, as determined by the yield condition for the strip. Local asperity flattening then occurs for each short wavelength asperity as it comes into contact (i.e., starting from the shallowest valley number 1 and proceeding through to the deepest valley number 5), until it is sufficiently flattened that the local hydrodynamic pressure reaches the mean pressure on the contact area. At this point there is no difference between the hydrodynamic pressure in the valley and the pressure on the contact area, so no further flattening occurs, as shown in Fig. 6. This is indicated by the way that the curves for each valley rise from the "connected valley" lower curves to the upper curve corresponding to the pressure on the long wavelength asperity contact. The hydrodynamic pressures in all the valleys converge to the yield stress of the strip at the end of the transition zone. Although it

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**Table 1: Parameters used in the theoretical model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Tabary et al. [10]</th>
<th>Sutcliffe and Le [13]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll radius ( R )</td>
<td>30 mm</td>
<td>25 mm</td>
</tr>
<tr>
<td>Strip material</td>
<td>1200AA aluminum</td>
<td>5052AA aluminum</td>
</tr>
<tr>
<td>Inlet gauge ( t_i )</td>
<td>0.35 mm</td>
<td>0.82 mm</td>
</tr>
<tr>
<td>Reduction in strip thickness ( r )</td>
<td>25–50%</td>
<td>25–50%</td>
</tr>
<tr>
<td>Rolling speed ( u )</td>
<td>0.05–5.0 m/s</td>
<td>0.003–1.0 m/s</td>
</tr>
<tr>
<td>Lubricant viscosity ( \eta_0 )</td>
<td>0.077 Pa.s (35°C)</td>
<td>0.35 Pa.s (20°C)</td>
</tr>
<tr>
<td>Viscosity pressure index ( \alpha )</td>
<td>10 ( \times 10^{-6} ) Pa(^{-1})</td>
<td>2.2 ( \times 10^{-6} ) Pa(^{-1})</td>
</tr>
<tr>
<td>Eyring shear stress ( \sigma_0 )</td>
<td>2.0 MPa</td>
<td>2.0 MPa</td>
</tr>
<tr>
<td>Yield stress ( Y )</td>
<td>200 MPa</td>
<td>214 MPa</td>
</tr>
<tr>
<td>Long wavelength</td>
<td>( \Sigma_l = 0.5 \mu m )</td>
<td>( \Sigma_l = 0.375 \mu m )</td>
</tr>
<tr>
<td>Short wavelength</td>
<td>( \sigma_l = 0.1 \mu m )</td>
<td>( \sigma_l = 0.075 \mu m )</td>
</tr>
</tbody>
</table>

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**Fig. 5 Evolution of hydrodynamic pressure in valleys across the width as for conditions of [10], \( r=25 \) percent, \( u_r=0.1 \) m/s**
appears that there is a discontinuity in pressure gradient as the curve for each valley meets the top curve, in fact an enlarged view shows a smooth transition as per the imposed boundary condition. Some lack of smoothness in the curves is associated with the numerical approximations, and with the discrete nature of the solution containing only a few valleys. It can be seen that the asperity flattening process occurs in a short region in the inlet and transition zones as assumed.

5.2 Comparison With Measurements. Two sets of measurements are used to compare the predictions: either measurements of surface roughness [13] or measurements of friction [10].

5.2.1 Surface Roughness Measurements. For comparison of the surface roughness, the rolling conditions are taken from [13] as outlined in the third column of Table 1. Lubrication conditions are characterized by the speed parameter $\Lambda_s$, equal to the ratio of the “smooth” film thickness [27] to the initial strip r.m.s surface roughness $\sqrt{\Sigma_0^2 + \sigma_0^2}$:

$$\Lambda_s = \frac{6 \eta_0 \alpha \bar{u}}{\phi(1 - e^{-\alpha}) \sqrt{\Sigma_0^2 + \sigma_0^2}}$$

Large $\Lambda_s$ corresponds to high speed conditions when the surfaces are well separated by oil, while for mixed lubrication conditions $\Lambda_s$ is below about 1. The predicted surface roughness is compared with the measurements in Figs. 7 and 8 for strip reduction of 25 percent and 50 percent, respectively. It is found that the predictions for the long wavelength component have good agreement with measurements, while the short wavelength component is slightly underestimated at low rolling speeds. One possibility for this discrepancy at short wavelengths is that the stylus technique may not be able to pick up the very short wavelength components after the strip is rolled. Alternatively it may be that the burnishing of the roll on the strip, during relative sliding between the surfaces, enhances the removal of the short wavelength components. This is not included in the surface flattening models.

5.2.2 Friction Measurements. Average friction coefficients predicted by the current two-wavelength model are compared with measurements by Tabary et al. [10] in Fig. 9. The parameters are taken from Table 1 for Tabary et al. [10]. The boundary friction coefficient $\mu_b$ is constant and treated as the single fitting parameter for all speeds. For the theoretical predictions shown in Fig. 9, choosing a plausible boundary friction coefficient of $\mu_b = 0.09$ for a reduction of 25 percent gives reasonable agreement with the measurements except at higher speeds. This implies that the short wavelength component plays an important role in determining friction. Friction is slightly overestimated at high speeds as hydrodynamic conditions are approached. This may be due to errors in...
estimating the Eyring behavior of the oil, errors in measurements of friction (which will become greater at lower values of friction) or due to inaccuracies in the theoretical models at these high speeds. The hydrodynamic friction coefficient \( \mu_h \) for full film lubrication is estimated using Eqs. (10) and (13) and is included on Fig. 9. Measured values fall below this at high speeds, indicating errors in either the hydrodynamic friction model or the inferred friction coefficient under these conditions.

Figure 9 also includes two other theoretical curves: (i) using the approach described above but including only a single asperity wavelength and a constant boundary friction coefficient, (ii) a semi-empirical model [25], again using a single wavelength but taking a variation in boundary friction coefficient with speed from experimental measurements on a strip drawing rig. The single-wavelength model with a constant boundary friction coefficient \( \mu_h = 0.09 \) significantly overestimates the friction coefficient. Choosing a significantly lower and probably implausible value of boundary friction coefficient (e.g., \( \mu_h = 0.05 \)) would improve the fit, but the shape of the curve would still not reflect the measured variation in friction. Indeed it is the difficulty of fitting friction data using such a model that has motivated the inclusion of a second wavelength of roughness in our modelling work. The semi-empirical friction model, in which the boundary friction factor varies with rolling speed, has rather similar predictions to the two-wavelength model, giving a reasonable fit to the measured data. However its accuracy relies on appropriate measurements of the boundary friction factor, which vary with oil properties, strip material and initial surface roughness of the die and strip used. Figure 10 suggests that this relatively simple semi-empirical model might instead be “calibrated” using the two-wavelength model described in this paper.

6 Conclusions

The paper describes a two dimensional model of mixed lubrication in metal rolling which contains two wavelengths of surface roughness. The model assumes that the roughness lay is entirely longitudinal, containing ridges and valleys running in the rolling direction. The following conclusions can be drawn from the model.

1. Valleys become progressively isolated from each other as contact takes place between asperities on the roll and strip. Results show that the hydrodynamic pressure in the isolated valleys is greater than in the remaining connected valleys. This increase in pressure inhibits asperity crushing, leading to a reduction in predicted contact area ratio, as compared with the conventional one-wavelength models [1–4].

2. Predictions of the change with rolling speed of the amplitude of the strip surface roughness and the mean friction coefficient using the current model are in reasonable agreement with published measurements.

Acknowledgments

The authors wish to thank Dr. K. Waterson at Alcan International Ltd. and Dr. P. Reeve at ALSTOM Power Conversion Ltd. for their advice. The financial support from the Engineering and Physical Sciences Research Council, Alcan International Ltd. and ALSTOM Power Conversion Ltd. is gratefully acknowledged.

Nomenclature

\[ A(a) \] = contact area ratio (i.e., the fraction of the nominal area in asperity contact), for long (short) wavelength components of roughness
\[ b \] = roll bite length
\[ h \] = mean film thickness in each isolated valley
\[ h_c \] = mean film thickness in the connected valleys
\[ h^* \] = constant in Reynolds’ equation for isolated valleys
\[ h_c^* \] = constant in Reynolds’ equation for a connected valley
\[ P(\bar{P}) \] = contact pressure on each short-wavelength component (averaged across the whole long-wavelength component)
\[ P_e(P_v) \] = contact pressure averaged on the top (in the valley) of the long wavelength roughness component
\[ P_s(P_v) \] = pressure on the top (in the valley) of the short wavelength roughness component
\[ q \] = reduced hydrodynamic pressure, \( q = 1 - \exp(-\alpha p_v) \)
\[ R \] = roll radius
\[ r \] = reduction in strip thickness
\[ S_c(s) \] = mean surface variance for the connected valleys (isolated valley)
\[ \bar{u} \] = mean entraining velocity at entry
\[ W_L(W_s) \] = flattening rate of long (short) wavelength roughness component
\[ x \] = co-ordinate in the rolling direction, measured relative to the point of first contact between the roll and strip
\[ Y \] = plain strain yield stress of the strip
\[ Z(z) \] = amplitude of long (short) wavelength roughness component
\[ Z_0(z_0) \] = initial amplitude of long (short) wavelength roughness component
\[ \delta L_0 \] = (initial) depth of crushed short wavelength roughness component
\[ \delta p \] = difference in pressure between the peak and valley of the short wavelength roughness component
\[ \Delta P \] = difference in pressure between the peak and valley of the long wavelength roughness component
\[ \varepsilon \] = strain rate of the underlying strip
\[ \eta(\eta_0) \] = viscosity of lubricant (at ambient pressure)
\[ \lambda(\lambda) \] = wavelength of long (short) wavelength roughness component
\[ \lambda_s \] = ratio of ‘smooth’ film thickness to combined strip and roll roughness
\[ \mu \] = friction coefficient averaged across the width of the bite
\[ \bar{\mu} \] = friction coefficient averaged through the roll bite
\[ \mu_b \] = boundary friction on the areas of contact
\[ \mu_s \] = hydrodynamic friction coefficient estimated from full film theory
\[ \Sigma(\Sigma_0) \] = (initial) r.m.s. amplitude of long wavelength roughness component
Appendix

Here we describe the details of the curve fit of the finite element solutions by Korzекwa et al. [24] for the crushing rate \( W \) for an infinite array of similar asperities. We follow the suggestion of Sutcliffe [12] of fitting the asperity crushing finite element results using polynomial functions, but use the slightly different fit given below, which is more accurate for the range of values of \( A \) and \( \Delta \) considered here:

\[
W(A, \Delta) = f(A) \left( C_1(\Delta) + C_2(\Delta)A + C_3(\Delta)A^2 \right) \tag{A1}
\]

where \( f(A) = (1.0657 + 0.3538A)(1 - A) \) and the functions \( C_1(\Delta), C_2(\Delta), C_3(\Delta) \) are given by:

\[
C_1 = 0.8960\Delta - 0.4812\Delta^2 + 0.3812\Delta^3
\]

\[
C_2 = 6.6107\Delta + 0.3069\Delta^2 - 1.7752\Delta^3
\]

\[
C_3 = 0.5229\Delta - 8.6318\Delta^2 + 4.4320\Delta^3
\]

A comparison of the curve fit with the finite element solutions is given in Fig. 10.

References