Prediction of tread block forces for a free-rolling tyre in contact with a smooth road

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ABSTRACT

This paper presents a model for the contact mechanics of tread blocks on a free-rolling pneumatic tyre, of a complexity intermediate between sophisticated 3D finite element (FE) analyses and simple brush models. The model is able to capture the essential physics of the contact while being sufficiently simple to be incorporated into dynamic tyre models, such as can be used to predict hub forces. The tread blocks are represented by a ‘brush-type’ spring model, taking into account the geometry of the contact and shear deformations of the tread. The rubber is represented as a linear viscoelastic material. The spring model is found to give similar normal and shear force predictions to a more sophisticated finite element analysis undertaken in the paper, both for tread blocks mounted on a deformed tyre belt (with the geometry taken from measurements of a loaded tyre) and for blocks on a rigid cylinder, matching the geometry of a small-scale rolling contact test rig. Predictions compare reasonably with measurements on the rig of tread block normal and tangential forces.

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1. Introduction

Tyres on a passenger vehicle, besides generating noise outside the vehicle cabin, also transmit forces associated with the road conditions and their own vibrations into the vehicle hubs via the wheel rims on which they are mounted. As the input stimuli to the vehicle suspension system, the dynamic hub forces have been a topic of interest for automobile manufacturers. At the design stage of a passenger vehicle, it is desirable for the automobile manufacturers to have predictive tools to characterise the hub forces for frequencies up to 1 kHz [1]. Since the unsteady forcing of vehicle hubs is a direct result of the dynamic behaviour of tyres, an advanced tyre model has an essential role in developing such tools.

Recently, Lecomte et al. [2] have developed a theoretical tyre belt vibration model that shows excellent correlation with the experimental results below 300 Hz and satisfactory agreement up to 1 kHz. As part of the same project, we have presented a simplified yet robust tread block contact mechanics model to provide input stimuli to Lecomte et al.’s tyre belt vibration model [3]. The aim of the approach described in [3] and developed further in this paper is to provide as simple a description as possible to capture the essential physics of the contact and provide the necessary inputs to such tyre belt representations. In a similar vein Wullens and Kropp [4] describe a coupled contact and tyre structure model for prediction of tyre/road noise. The simplicity of the proposed contact model is a requirement of the computational intensity of the tyre vibration calculation, which requires a time-stepping simulation. The tread contact model is a relatively small part of the overall solution method and needs to be correspondingly straightforward to implement. (This rules out use of the well-established finite element (FE) method for describing the contact, although we recognise that such an approach can provide a useful tool to model the contact problem, and indeed is used as a benchmark in this paper. Choreisky [5] presents a comprehensive survey of finite element models of rolling tyres, while typical examples by Sujin et al. [6] and Hall et al. [7] show how details of the contact geometry can indeed be calculated.)

The simplest material model for rubber is linear elastic. This approach is used by Wullens and Kropp [4], taking advantage of analytical solutions for such a simple representation to allow inclusion of tyre–road interactions. Non-linear elastic effects associated with large strains can be accommodated using a hyper–elastic model, as adopted for example by Sujin et al. [6] and Hall et al. [7]. Rubber is a viscoelastic material so it is not surprising that various forms of viscoelastic model have also been used to describe tyre mechanics, either explicitly in the material model or implicitly via spring–damper formulations. Cesbron et al. [8] list a range of examples, noting that their experimental results can be explained in terms of such viscoelastic behaviour. Non-linear behaviour (i.e. a strain dependence) is also relevant to viscoelastic models [9]. This point may be particularly relevant to rough road contacts (not considered in this paper) where local strains in the tread may be higher than the corresponding smooth road contact. Standard methods
have been developed to characterise the viscoelastic properties of such materials, e.g. [10], taking advantage of dynamic mechanical analysis (DMA) and the Williams–Landel–Ferry (WLF) transform to relate changes in temperature and frequency. Indeed the WLF transform has also been successfully used by Grosch [11] to model changes in rubber friction with temperature and sliding speed. The viscoelastic spring model defines the tread material via standard data. This is a significant advantage for using the tread block model as a predictive tool, since the DMA data for tread rubbers are straightforward to obtain. As the loading geometry is well defined, extraction of material properties can be made with relative confidence. Other tread block contact models proposed, for example, by Andersson et al.[13] and Brinkmeier and Nackenhorst [15], require a more sophisticated extraction of effective material properties from non-standard test geometries.

In the contact mechanics model, the tread blocks have been represented by a series of discrete viscoelastic springs. This approach is essentially the brush model or Winkler foundation well known to tyre modellers, e.g. Kropp [12], Andersson et al. [13] and Holtschulze et al. [14]. For the free-rolling situation considered in this paper, a rather more sophisticated spring model is required than commonly used, including a more accurate representation of the impact and release geometry. As briefly discussed in [3], the viscoelastic spring model has also opened the way for predicting the tread block forces in the tangential direction if a tangential spring with appropriate stiffness is added to each radial spring. When coupled to the tyre belt model, the predicted tangential forces enable the investigation of tyre vibration taking the tangential response into account. This may help answer the question raised by Wul-...
Fig. 2. Tread block rolling test rig.

1. Big wheel
2. Locking bars
3. Small wheel
4. Rocking arm
5. Rigid base
6. Tread block sample
7. Sample holder
8. Bi-axial load cell
9. Load cell amplifier
10. Rubber belt

To 22 °C by performing a WLF (Williams–Landel–Ferry) transform with standard coefficients ($C_1 = 8.86, C_2 = 101.6$ and glass transition temperature $T_g = -40^\circ$C) [20]. A 24-term Prony series expansion was fitted to the data using the sign control method proposed by Bradshaw and Brinson [21]. As shown in Fig. 1, very good agreement between the fitted moduli and the measured data was found in the frequency range of 0.25–2500 rad/s. Despite concerns about the use of the WLF transform for filled polymers [22], this approach appears to capture the relationship between temperature and strain rate effectively for this material (as evidenced by the relative smoothness of the curves). When referred to the tyre operating temperature of 60 °C, the frequency range covered by the data is 3.8 Hz–38 kHz, more than adequate to represent the dynamic behaviour of tread blocks. The Prony series parameters can be used directly in commercial finite element (FE) packages such as ABAQUS/Explicit. The volumetric data required in the FE material definition can be simplified by assuming a Poisson’s ratio of 0.47, since tread rubber is nearly incompressible and the Poisson’s ratio varies little with frequency [23].

2.2. Experimental set-up for material model validation

To validate the linear viscoelastic tread model, it is important to design an experiment which can take into account both speed and temperature effects. In addition, with a peak contact stress on a tread block of 0.4 MPa [19] and an average tread elastic modulus of 20 MPa for our tread sample [23], the strain on a tread block during free-rolling is likely to reach 2%, higher than the value of 0.5% used in the DMA test. Therefore any strain-level dependence also needs to be investigated in the experiments.
As reported in [3], a high-speed rolling contact test rig has been built to simulate the impact and release mechanisms of a tread block, and used to conduct tread block contact experiments. A schematic of the rig is shown in Fig. 2. A small wheel of 0.2 m diameter is mounted directly on a solid base and a big wheel of 0.6 m diameter is connected to the base by a rocking arm. A bi-axial (normal and tangential) load cell, to which the tread block sample is attached via a sample holder, is embedded in the small wheel. The latter is driven by an electric motor with controlled speeds up to 600 rpm. A rubber belt is attached around the small wheel with a cut-out window to accommodate the tread block sample. The surface of the tread block sample is at the same level as that of the rubber belt. To create compression in the tread block sample, two locking bars are used to fix the shafts of the two wheels in position with a prescribed degree of interference between the rubber belt and big wheel. The rubber belt transmits torque to the big wheel. Once the big wheel is accelerated to the same tangential speed as the small wheel, only minimal power is drawn from the small wheel to overcome the frictional loss on the shaft and the damping loss in the contact area with the rubber belt. With its high inertia, and almost continuous contact with the rubber belt, the big wheel maintains a constant tangential speed, even at the moments when the tread block sample impacts. This ensures a nearly free-rolling condition for the investigation.

The force signals generated by the load cell are amplified by bridge amplifiers mounted on the small wheel and transmitted through a slip ring unit off the rotating wheel. In order to achieve temperatures higher than ambient, a hair dryer is held on top of the small wheel to blow hot air onto the tread block. A thermocouple in contact with the tread block sample provides in situ monitoring of its operating temperature. The motor speed is controlled by a Moeller DF51 electronic inverter control unit and all signals from the sensors were logged during the experiments using a computer equipped with an NI-6024 data acquisition card.

Samples with either rectangular or parallelogram tread block plan areas were manufactured by dicing a lump of Goodyear tread compound using a sharp razor. The nominal dimensions of the samples are shown in Fig. 3. The speeds used in the experiments varied from 150 to 600 rpm on the small wheel and the interference level between the two wheels ranged from 0.1 to 0.3 mm.

2.3. Validation results and discussion

A finite element model was developed in ABAQUS/Explicit [24] to replicate the contact kinematics of the rolling contact test rig. As shown in Fig. 4, the top nodes of the tread block samples are tied to a rigid ring. The tread block is meshed using the 8-node linear brick element C3D8R with reduced integration and hourglass control. The mesh is refined at the bottom contact surface of the tread block. The ring rotates at a prescribed speed to bring the tread block into contact with the opposing surface. A pressure and slip-rate dependent coefficient of friction, shown in Fig. 4, was obtained for the tread/steel contact pair in the rolling test rig, based on a measurement conducted using a linear tribometer [3]. Using the VFRIC subroutine in ABAQUS/Explicit, this coefficient of friction was applied to the contact surfaces in the model. The friction model is implemented in ABAQUS via contact elements defining contact constraints and over-closure at integration points. The constraints are imposed using Lagrange multipliers [24]. By comparing the FE and the experimental results from the high-speed rolling contact test rig, we have already shown in [3] that the linear viscoelastic model yields only 10% error for stresses up to 1.5 MPa at various rolling speeds. Considering that the measured tread block stress on a free-rolling tyre does not exceed 0.4 MPa, the linear viscoelastic model has been confirmed as adequate even though it ignores any strain-level dependence. Here we present further FE and experimental results to demonstrate the effectiveness of the tread model.

Note that a tread block of dimensions 20 mm × 25 mm × 8 mm has a mass of 4 g. To produce a force of 40 N, corresponding to a pressure of only 80 kPa (c.f. Fig. 9), one has to see an unfeasibly large acceleration of 1000 g, justifying omission of block inertial effects in the modelling.

During the experiments, it was observed that the maximum steady temperature achievable on the tread block samples using the hair dryer was 45 °C. Therefore, in the FE model, a temperature field of 45 °C was applied to employ the WLF transform in the material definition.
possible to adjust the creep in the simulation to match this effect, leading to a corresponding asymmetry in the simulation.

The FE results for the peak normal forces for the rectangular block at 22°C are summarised and compared with the experimental results in Fig. 6, with the error bars indicating the minimum and maximum forces obtained in the experiments for the given testing conditions. Similar results were obtained at 45°C for the rectangular block, and at both temperatures for the parallelogram block. Considering only the data for realistic load levels, corresponding to 0.1 and 0.2 mm interference, the mean errors in the FE predictions are 18 and 10% for the rectangular and parallelogram blocks, respectively. This level of accuracy is considered acceptable for the tread block model.

For each set of data at a constant temperature θ and interference δ (but neglecting the δ = 0.3 mm measurements), the effect of circumferential speed was found by linear regression, fitting a line of the form

\[ F = F_0(1 + a_v ν) \]  

with the coefficient \( a_v \) expressing the relative dependence of force on speed. Similarly a coefficient \( a_θ \) expressing the relative dependence of force on temperature was found by comparing data at the two temperatures but the same interference (grouping together sets with different speeds). Table 1 details the results, comparing the measured and predicted coefficients, expressed as a percentage for the rectangular and parallelogram blocks. The mean and standard deviation of the four speed coefficients \( a_v \) for the different interferences and temperatures are reported, and the mean and range for the two temperature coefficients \( a_θ \) at different interference values. For typical measured values of \( a_v \) and \( a_θ \) of 2.5%/m and −0.36%/°C, a 10% change in contact force corresponds to changes of speed of 4 m/s or temperature of 28°C, respectively. These results suggest that the effect of speed on tread block force is significant while the effect of temperature is less important, for a realistic variation in these parameters. This confirms the value of using a viscoelastic model in the analysis. The standard deviations/spreads of data are significant, partly as data for differing conditions have been grouped together and also due to experimental scatter. Predictions for the speed effect are in reasonable accord with measurements, for both shapes of tread. For both measurement and predictions, the sensitivity to speed is reduced for the parallelogram block as compared with the rectangular block, albeit with the changes smaller than the measured standard deviations. The effect of temperature is overestimated by the FE model by a factor of about two. Perhaps the experimental setup using the hair dryer was not able to cause a uniform raise in temperature in the tread. However, given the more minor role of temperature, this difference is of less concern. The effect of tread shape on the temperature effect is insignificant.

### 3. Tyre belt profile measurement

Having validated an appropriate tread material model, in the remainder of the paper we combine it with the geometric boundary conditions of a loaded tyre belt to generate a tread block contact simulation for a full-size tyre. Since a direct measurement of the profile of a deformed tyre belt is difficult, a silicone rubber cast was made of the contact patch of a passenger car tyre.

#### 3.1. Manufacture and measurement of the silicone cast

A Goodyear HPAW 235 tyre was loaded by a down-force of 500 kg into a metal tray filled with ACC SE2005 silicone potting compound. The low-viscosity of the compound allowed it to flow into the tread grooves and copy the details of the contact patch. The tyre was held loaded until the silicone compound cured and then slowly lifted out to release the cast. A photograph of the silicone cast
is shown in Fig. 7; the tread block details are clearly visible. The four longitudinal grooves in the tyre tread resulted in four ridges in the cast, which can be regarded as defining the deformed shape of the tyre belt.

The heights of the four ridges were measured using a Mitutoyo Series 196 digital coordinate measuring machine (CMM), with its horizontal and vertical resolutions set to 2 mm and 10 μm, respectively. Once the measurement was completed for each ridge, the curvature at any given point on the deformed belt was obtained by performing a 3-point circular curve-fit with its two neighbouring points. The measured heights and curvatures for the four ridges are shown in Fig. 8. The spike in curvature inside the contact patch was caused by a random variation in the unfiltered height trace and therefore should be disregarded. The positions of the ridges, as identified in the key, correspond to the ridge locations in Fig. 7 (i.e. from the top to the bottom). Outside the contact patch the curvature of the belt rises to a maximum value of about 5 m⁻¹ (0.2 m radius), in contrast to the unloaded curvature of 3 m⁻¹ (0.33 m radius). The curvature of the belt then decreases rapidly towards zero as the belt is flattened to form the contact patch. Immediately adjacent to the contact patch, the curvature of the belt is about 2 m⁻¹ (0.5 m radius). The profile of the mid-top trace is chosen as representative

<table>
<thead>
<tr>
<th>Ridge Diameter</th>
<th>PE (mm)</th>
<th>FE (mm)</th>
<th>P. E.</th>
<th>FE (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.047</td>
<td>0.048</td>
<td>0.048</td>
<td>0.048</td>
</tr>
<tr>
<td>0.2</td>
<td>0.074</td>
<td>0.074</td>
<td>0.075</td>
<td>0.075</td>
</tr>
<tr>
<td>0.3</td>
<td>0.101</td>
<td>0.101</td>
<td>0.101</td>
<td>0.101</td>
</tr>
</tbody>
</table>

Table 1

Results of a linear regression comparing measured and FE predictions for the coefficients \( a_v \) and \( a_\theta \) expressing as a percentage the relative dependence of normal force \( F \) on tangential rolling speed \( v \) and temperature \( \theta \), respectively. For \( a_v \) the standard deviations are included in brackets, for \( a_\theta \) the range (there were only two data points in this case).
of the tyre belt shape for use in establishing tread block contact mechanics models.

3.2. Implementation in finite element model

The measured tyre belt profile was combined with the tread rubber model defined by the Prony series in constructing an FE model in ABAQUS/Explicit. Instead of being tied to a rigid ring as in the test rig simulation, the top nodes of the tread block were prescribed with a moving path defined by the measured tyre belt profile, for a given velocity \( V_0 \). As in the contact rig experiment simulation, the tread block was meshed using the 8-node linear brick element C3D8R refined for the bottom contact surface. The opposing surface was modelled as a smooth and rigid plane.

Since the pressure and slip-rate dependent coefficient of friction shown in Fig. 4 was measured for the rolling contact test rig, i.e. tread rubber on steel, it is not suitable for modelling the tyre/road contact. In the FE model using the full-size tyre belt profile, an empirical model developed by Huemer et al. [25] was used to define the kinetic coefficient of friction \( \mu_k \) for tread rubber in contact with a concrete road surface. The empirical kinetic coefficient of friction model is of the form

\[
\mu_k = \frac{\alpha_p|p|^{n-1} + \beta_p}{a + b|v|^{1/m} + c|v|^{-2/m}}
\]

where \( p \) is the pressure, \( v \) is the sliding velocity, and the other variables are constants. Huemer et al. obtained all the parameters for a rubber/concrete surface contact through experiments [25]. The static coefficient of friction \( \mu_s \) was chosen as 0.9 in our work, which is a representative value for tread/smooth road contact in dry conditions [23]. Note that in this paper we are not considering road roughness, which would be expected to change the effective frictional conditions at the contact. By tracking for every time step the contact pressure and slip-rate of each contact element, this empirical coefficient of friction model was implemented in ABAQUS/Explicit using its VFRIC subroutine. The tread block forces predicted by the FE model are presented in Section 5.

4. Viscoelastic spring model

Although FE analysis has been widely used to model tyres under quasi-static conditions, the high computational cost limits its use in modelling the tyre belt dynamics. The advanced theoretical tyre belt model developed by Lecomte et al. [2] requires a simplified tread block model to be coupled with the tyre belt. We have established a computationally efficient discrete viscoelastic spring model in [3], taking into account both the tread properties defined by the Prony series and the geometrical complexity of a tread block, i.e. shape and sipes. The predicted normal forces have shown good agreement with the FE results and experimental results from the test rig experiment [3]. We are now in a position to apply the measured full-size tyre profile to the viscoelastic spring model and to extend the model to predict the tangential forces.

4.1. Mechanism of tangential force generation for a free-rolling tyre

It is well known that, on a free-rolling tyre, the tangential stresses across the contact patch have a sinusoidal-style distribution with the stresses in the leading half and the trailing half of the contact patch approximately equal but of opposite signs. The tangential force on a tread block first points to the leading edge of the contact patch as it impacts the road and gradually changes its direction towards the trailing edge. The magnitude of the tangential stress is approximately 1/3 that of the normal stress [26]. A measured contact stress history supplied by our industrial partners is presented in Fig. 9, which clearly shows the sinusoidal-style tangential force. It can be seen that the peak normal stress is higher than 0.4 MPa, probably due to an over-inflated cavity and the curvature effect of the test drum. It is also worth noting that the tangential force reverses its direction and forms a slip zone before the tread block completely loses contact with the road surface.
Although observed by many researchers, such as Lippmann and Oblizajek [28], no clear explanation for this slip zone is available.

The generation mechanism of the sinusoidal-style tangential force is generally believed to be related to the change of the tyre belt radius. Berger [29] analysed the generation mechanism based solely on a kinematics analysis involving both tyre belt and tread. However the modelling results were inconclusive as to whether it is the deflection of the tread or the in-plane deformation of the belt or both that contributes to the generation of tangential force. Klingbeil and Witt [30] provided a complete tyre model for predicting contact forces; their model has to be solved iteratively using a finite difference mesh and the results only proved to be qualitatively correct for a smooth tread tyre. To achieve quantitatively plausible data, both of the aforementioned approaches require the modelling of composite tyre belt deformation, which in itself is a very complicated problem. Recently Holtschulze et al. [14] have proposed two simplified assumptions, treating the tyre belt as either rigid or compressible in the longitudinal direction. It was argued accordingly that the tangential force could be due to either the deformation of the tread blocks or the shortening of the belt in the contact patch. Although both of these two assumptions have produced a qualitatively plausible sinusoidal-style tangential force, the assumption of a rigid tyre belt appears to be more reasonable than the compressible assumption. Firstly, tyre belts are reinforced by steel breakers longitudinally and nylon or polyester plies radially, both of which are much stiffer and subject to much smaller deformation than the tread. Secondly, recent studies by Matsuzaki and Todoroki [31] using a novel wireless strain sensor have shown that the stress imposed on a tyre belt inside the contact patch is in fact tensile rather than compressive, which contradicts Holtschulze et al.’s second assumption.

In the light of this discussion, we have adapted Holtschulze et al.’s rigid belt assumption to establish a quantitatively valid model for predicting the tangential force. The generation mechanism of tangential force based on this assumption is illustrated in Fig. 10. First, assume that: the belt in the contact patch is flat and has a radius of curvature approaching infinity; within the contact patch the belt and the road have the same constant velocity $V_0$; and there is no slip at the tread block/road interface. Under such conditions, as shown in Fig. 10(a), a tread block is sheared towards the leading edge of the contact patch as it enters the contact patch, due to the change of belt radius of curvature from $R_d$ to infinity (a proof is given later in Section 4.2). This results in a shear strain increasing from 0 to $\alpha$ on the tread block with respect to its neutral state delineated by the dashed line. Once the tread block is inside the contact patch, the bottom and top surfaces of the tread block have the same velocity $V_0$ and the shear strain $\alpha$ is maintained. As the tread block starts to be released, the shear strain increases again due to the restoration of the radius of curvature of the belt. Accordingly the shear force imposed by the road on the bottom of each tread block due to this mechanism points towards the leading edge of the contact patch. In order to have zero net force in the tangential direction for a free-rolling tyre, the road must move slightly faster than the tyre belt such that, assuming no slip, a static frictional force is imposed on each tread block pointing towards the trailing edge of the contact patch. As illustrated in Fig. 10(b), if the tyre is brought to rest, a creep velocity of $V_1$ induces a shear strain $\beta$ on each tread block which increases from 0 to its maximum value across the contact patch. Superposing the above two boundary conditions, the overall shear strain $\gamma$ on the tread blocks is shown in Fig. 10(c). While the belt and the road are travelling at $V_0$ and $V_0 + V_1$ respectively, the tread blocks are sheared outwards towards the edges of the contact patch, resulting in a sinusoidal-like shear force distribution. Although in reality a tensile strain is likely to be present in the tyre belt, this model uses the creep velocity $V_1$ to represent the overall velocity difference between the tyre belt and the road. Coupling the contact model with an elastic belt model, as envisaged, would allow correction of the creep velocity to allow for such elastic strains.

4.2. Formulation of equations

Although the contact force predicted by Holtschulze et al.’s mechanism is qualitatively correct, they claimed that the amplitude of the predicted tangential force was unrealistically small. One possible reason might be the over-simplification in the formulation of their model, in which the shear force due to belt radius change
is only considered as a function of the belt radius, without taking into account the actual deformation of tread blocks. The lack of a suitable material model might also have undermined the effectiveness of their numerical simulation. With the validated tread material model defined by the Prony series, the deformation of a tread block during contact can be investigated in detail, as we now show.

Following the normal force model reported in [3], the tread block is discretised into a series of viscoelastic spring elements; but this time a longitudinal spring is included in each element to account for the tangential force. The schematic in Fig. 11 illustrates the contact mechanics of a spring element when it impacts the smooth road. By introducing a creep ratio \( \xi = V_1/V_0 \), the road velocity in Fig. 10 can be expressed as \( V_0 + \xi V_0 \). Assuming that a tread element sticks to the road once it has made contact, the positions of \( B \) and \( C \) corresponding to the tread element top and bottom nodes at any arbitrary moment can be calculated, given \( V_0, \xi \) and the radius of curvature of the belt. Thus the distance \( L \) between points \( B \) and \( C \) can be obtained. At the same time, the angles \( \Phi \) between the vertical line and \( BC \), and \( \theta \), between the vertical line and \( BB' \), can be derived. The angle \( \psi \) between \( BB' \) and \( BC \), which is a function of time \( t \), is then given by

\[
\psi(t) = \Phi(t) - \theta(t)
\]

(6)

Therefore, the current lengths \( \delta \) and \( \chi \) of the radial and longitudinal spring elements can be expressed as

\[
\delta(t) = L(t) \cos \psi(t)
\]
\[
\chi(t) = L(t) \sin \psi(t)
\]

(7)

For a given tread block with surface area \( A_t \) and thickness \( h_t \), the time-dependent stiffnesses \( S_r(t) \) in the radial direction and \( S_l(t) \) in the longitudinal direction for a given tread block are given by

\[
S_r(t) = \frac{E(t)A_t}{h_t}
\]
\[
S_l(t) = \frac{G(t)A_t}{h_t}
\]

(8)

where \( G(t) \) is the time-dependent shear modulus; \( G(t) = E(t)/2(1 + v) \), with \( v \) the Poisson’s ratio for the tread rubber. If the tread block has been discretised into \( n \) elements, the springs in the radial and longitudinal directions of each element will then have time-dependent spring stiffness \( k_r(t) \) and \( k_l(t) \) given by

\[
k_r(t) = \frac{S_r(t)}{n}
\]
\[
k_l(t) = \frac{S_l(t)}{n}
\]

(9)

Therefore the radial force \( F_r \) and longitudinal force \( F_l \) on each tread element can be derived as

\[
F_r(t) = \int_{-\infty}^{t} k_r(t - \tau) \frac{d\delta}{d\tau} d\tau
\]
\[
F_l(t) = \int_{-\infty}^{t} k_l(t - \tau) \frac{d\chi}{d\tau} d\tau
\]

(10)

Finally, dynamic forces on a tread block as a lumped mass with slip can equivalently be regarded as normal and tangential forces inside the contact patch.

To apply the slip-rate and pressure dependent coefficient of friction defined by Eq. (5), the velocity of the bottom node of each tread element must be calculated in each time-step and a check made as to whether the tangential force exceeds the friction limit set by the product of the normal force and the static coefficient of friction \( \mu_s \). This can be achieved by assigning a lumped mass \( \bar{m} \) to the node, as shown in Fig. 12. The value of the equivalent lumped mass for a spring with distributed mass has been proved by Yan [32] to be half of the distributed mass. Therefore the lumped mass \( \bar{m} \) for each spring pair is given by

\[
\bar{m} = \frac{M}{2n}
\]

(11)

where \( M \) is the total mass of a tread block which has been discretised into \( n \) springs. Once a node is in contact with the road surface, its velocity can be calculated in each time-step by using the equation of motion in the tangential direction, which is

\[
\frac{d\chi}{dt} = V_1, \quad |F_l(t)| \leq \mu_s F_r
\]
\[
\bar{m} \frac{d^2\chi}{dt^2} + F_l(t) = \text{sign} \left( V_1 - \frac{d\chi}{dt} \right) \mu_s F_r, \quad |F_l(t)| > \mu_s F_r
\]

(12)

When slip is detected, the acceleration \( d^2\chi/dt^2 \) is integrated once to obtain the tangential velocity for the next time-step, which is subsequently used to calculate \( \mu_s \), through Eq. (5), and \( F_l \), according to Eq. (10). Further integration to find the tangential displacement then defines the radial displacement, \( \delta \), and hence \( F_r \) via Eq. (10).

4.3. Implementation of measured tyre belt profile

The analysis based on Fig. 11 assumes a constant radius of tyre belt outside the contact patch. In reality, the radius of a deformed tyre belt is a function of position, as shown in Fig. 8. To implement the measured tyre belt profile, i.e. the mid-top trace as defined in Fig. 8, the angle \( \theta \) in Fig. 11 for an arbitrary point on the path can...
be approximated by taking into account the positions of its neighbouring points. As shown in Fig. 13, the top node of a spring element follows the path prescribed by the deformed tyre belt profile with its position $J_i$ and corresponding coordinates $(x_i, y_i)$ at time-step $i$. Also, the positions $J_{i-1}$ and $J_{i+1}$ at the previous and following time-steps have coordinates $(x_{i-1}, y_{i-1})$ and $(x_{i+1}, y_{i+1})$, respectively. $\theta_i$ for position $J_i$ can then be approximated by

$$\theta_i = \tan^{-1}\left(\frac{y_{i+1} - y_{i-1}}{x_{i+1} - x_{i-1}}\right)$$  

(13)

Once $\theta$ is determined for every time-step, Eqs. (6)–(12) can be used to derive the normal and tangential contact forces, given a prescribed maximum compression $\Delta$ for the tread block. The creep ratio that corresponds to free rolling is then found by a direct search over trial values.

5. Modelling results and discussion

It should be noted that, although the high-speed rolling contact rig can simulate the impact and release mechanisms of a tread block, it is unable to provide a flat contact patch to test the tread block contact model for a full-size tyre directly. However, an FE tread contact model has already been validated in Section 2, using the test rig data, for predictions of both the normal and tangential contact forces. Therefore, an FE simulation of the tread block contact process for a tyre geometry can now be used with confidence as a yardstick against which to assess the validity of the viscoelastic spring model.

The same rectangular tread block with dimensions $25 \text{ mm} \times 20 \text{ mm} \times 8 \text{ mm}$ was used in both FE and viscoelastic spring models, coupled with a tyre belt geometry defined by the mid-top trace in Fig. 8. In the viscoelastic spring model, the tread block was discretised at a resolution of 0.5 mm. A temperature field of $60^\circ \text{C}$ was applied in the simulation and the speed of the tyre belt was set to 16.7 m/s (60 km/h). When the maximum compression is set to 0.2 mm, a normal contact force of about 200 N, corresponding to a realistic contact pressure of 0.4 MPa, is achieved. A creep ratio $\xi$ equal to 0.3% was found to bring the net tangential force close to zero. The modelling results from the FE and viscoelastic spring models are compared in Fig. 14, where very good agreement can be seen between the two models. The maximum tangential force magnitude is approximately 80 N, close to one third of the normal force magnitude, as reported in [26–30]. Note that the creep ratio found in this simulation is smaller than that expected in rolling contact. This is because the creep ratio has been adjusted in this case to bring the net force to zero. For practical free rolling there would be a net tangential force, equivalent to the rolling resistance, leading to a larger creep ratio. Inclusion of such a net frictional force would alter the details of the contact, but not the general form of solution nor the good agreement between the FE and spring models. Although it would be important to include such a feature in a full tyre model, for the purposes of validating the spring model, which is the key aim of this simulation, the difference is not significant.

The level of detail required for the belt shape description is also of interest. We therefore now consider the simplified profile of Fig. 15, which consists of a flat contact patch of length $\ell$ and two arcs of the same radius $R_d$. From the measurement in Fig. 8, the radius of the tyre belt at the contact patch edges is about 0.5 m. The results of a simulation with $R_d = 0.5 \text{ m}$ and $\ell = 0.17 \text{ m}$ are shown in Fig. 16. Predictions are close to those using the actual measured profile,
Given the pressure and slip-rate dependent coefficient of friction block, as predicted by both the FE and viscoelastic spring models. The tangential force during the release motion of the tread achieves the imposed shear rate. This results in the change of direction of the tangential spring means that, if the shear rate \( d\psi/dt \) is greater than \( +\psi \), the tangential force will remain negative and gradually reach zero as the tread element approaches its neutral axis upon exiting the contact patch. However, the viscoelastic nature of the tangential spring means that, if the shear rate \( d\psi/dt \) is greater than the self-recovery rate of the spring, a positive force is needed to achieve the imposed shear rate. This results in the change of direction for the tangential force during the release motion of the tread block, as predicted by both the FE and viscoelastic spring models. Given the pressure and slip-rate dependent coefficient of friction \( \mu_k \) determined by Eq. (5), it can also be seen that some slippage is inevitable at the end of the contact patch for the conditions considered as the positive tangential force reaches its friction limit (i.e. the product of the friction coefficient and the rapidly dropping normal force).

6. Conclusions

In this paper, we have presented a numerical investigation of the contact mechanics between the tread blocks and a smooth road for a free-rolling tyre. The tread material has been modelled as a linear viscoelastic material using a Prony series approximation. A small-scale rolling contact test rig has been used to validate the material model and an FE simulation by capturing the impact and release motions of a tread block. Both the material model and the FE contact model have shown reasonable agreement with the experimental findings. The effect of rolling speed has been shown to be more significant than temperature effects in changing the tread contact force. The simplified viscoelastic spring model reported previously in [3] has been extended to predict tangential tread block forces by analysing the spring element deformation during the impact and release motions. Using a measured tyre belt profile, the results from the viscoelastic spring model show very good agreement with FE predictions. The predicted normal and tangential forces are both qualitatively and quantitatively plausible compared to the measurement and modelling results of other researchers. The viscoelastic model will be used to provide input stimuli to the tyre belt vibration model [2] also developed in this project.

Limitations of the model include a lack of thermal modelling either of the tread block or belt; Lin and Hwang [33] show that these might well be important, particularly since sources of heat are likely to be relatively localised. However inclusion of such effects would require significant effort, which might be avoidable by a judicious choice of temperature. We also reiterate that the work presented in this paper relates only to smooth road conditions. Rough roads will contact only on part of a tread block, and then at different heights and intensities depending on the road profile. An alternative model, which accounts for such effects, is included in [18].

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