Failure of sandwich honeycomb structures under indentation loading is considered. A failure criterion for Nomex honeycombs subjected to combined compressive and shear stresses is determined using biaxial tests. By combining this with a theoretical calculation of the stress distribution in the core due to indentation loading, found from a high-order sandwich beam theory (HOSBT), the indentation failure load of the sandwich beam due to core failure can be predicted. It is assumed that both the skin and core are elastic up to failure, which is a reasonable approximation for the GFRP skins and Nomex cores considered. Short beam 3-point bending tests are used to validate the theoretical predictions, using beams made with GFRP skins and Nomex cores with densities between 29 and 128 kg/m^3. Theoretical predictions of indentation failure load are in excellent agreement with measured values. Inclusion of shear stresses in the failure criterion significantly improves the predictions, correctly modelling the observed stronger behaviour of cores with a longitudinal ribbon direction. © 2000 Published by Elsevier Science Ltd.

Keywords: Sandwich beam; Indentation; High order sandwich beam theory; Arcan test; GRRP; Nomex
using elastic high-order sandwich beam theories (HOS-BTs), such as those proposed by Frostig and Baruch [8] and Reddy [9]. Recent work by Petras and Sutcliffe [4] applies Frostig’s model to GFRP/Nomex sandwich panels. This paper develops their work further to include failure prediction.

Indentation loading gives rise to both shear and out-of-plane compressive stresses in the core. Failure due to these components is depicted schematically in Fig. 1. To model this, the behaviour of the core under combined compressive and shear loading is required. References to biaxial loading tests for cellular materials are not common in the literature. Gibson et al. [10] have performed a series of tests on a variety of foams under biaxial, axisymmetric and hydrostatic loading conditions. Zhang and Ashby [11] have investigated the in-plane biaxial buckling behaviour of Nomex honeycombs. More recently, Stronge and Klintworth [12] have studied the yield surfaces for honeycombs under biaxial macroscopic stresses. However, the required combination of shear and compressive loading for Nomex cores has not been investigated.

The low-velocity or quasi-static impact behaviour of sandwich panels has been investigated experimentally by several authors. The lack of accepted test methods for measuring impact damage resistance of composite sandwich structures lead Lagace et al. [13] to propose a new methodology based on static indentation and impact tests. Mineset al. [14,15] have tested the dropped weight impact performance of sandwich beams with woven/chopped strand glass skins and polyester foam and aluminium honeycomb cores and have simulated the upper skin post-failure energy absorption behaviour with an elastic–plastic beam bending model. Low velocity damage mechanisms and damage modes have been investigated in sandwich beams with Rohacell foam core by Wu and Sun [16] and with Nomex honeycomb core by Herup and Palazotto [17].

In this paper, we use higher order beam theory to model the stress in the honeycomb core due to indentation loading under 3-point bending. This uses a failure criterion derived for combined shear and compressive loading of Nomex honeycomb core. Predictions are compared with measurements in which short beam sandwich panels are loaded under 3-point bending. Although this paper focuses on core failure, skin failure should also be considered in the final beam design. The mode of failure expected for a given beam geometry can be identified using failure mode maps, as given by Petras and Sutcliffe [18].

2. Failure criteria for Nomex honeycomb under combined shear and compressive loading

In this section, we use the Arcan test rig [19,20] to investigate the failure behaviour of Nomex honeycombs under shear and out-of-plane compression loading and so derive a new failure criterion for the core. The rig consists of two pairs of plane semi-circular loading

<table>
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<tr>
<th>Nomenclature</th>
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<tbody>
<tr>
<td><strong>Greek symbols</strong></td>
</tr>
<tr>
<td>( \delta )</td>
</tr>
<tr>
<td>( \sigma_{cc} )</td>
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<tr>
<td>( \sigma_{zz} )</td>
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<tr>
<td>( \tau_{cs} )</td>
</tr>
<tr>
<td>( \tau_{s} )</td>
</tr>
<tr>
<td>( \varphi )</td>
</tr>
<tr>
<td><strong>Latin symbols</strong></td>
</tr>
<tr>
<td>( A_f ), ( C_f ), ( D_f ), ( E_c ), ( R_1 ), ( R_2 )</td>
</tr>
<tr>
<td>( b )</td>
</tr>
<tr>
<td>( c )</td>
</tr>
<tr>
<td>( C_m )</td>
</tr>
<tr>
<td>( L )</td>
</tr>
<tr>
<td>( m )</td>
</tr>
<tr>
<td>( M )</td>
</tr>
<tr>
<td>( q(x) )</td>
</tr>
<tr>
<td>( W, (W_0) )</td>
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</tbody>
</table>
plates with antisymmetric cutouts, as illustrated in Fig. 2. The specimen is attached to these plates via loading platens. The two pairs of loading plates are loaded by a servohydraulic testing machine through standard grips. The array of bolt holes in the loading plates allows the loading plates to be attached to the grips at different orientations. This allows a range of values of the angle \( \phi \) between the plane of the specimen and the loading direction, and hence a variation of the ratio \( \tan \phi \) of compression to shear loads applied to the specimen.

Specimens were prepared by cutting rectangular pieces from each of the available sandwich panels of four different densities, 29, 48, 64 and 128 kg/m\(^3\), with a 3 mm cell size. The specimens are rectangular plates typically 46 mm long and 30 mm wide, with a core thickness of 9.4 mm. Although the aim of this test is to examine the behaviour of the honeycomb core, the skins are left on the test specimen during the test. The skins do not affect the behaviour of the core in the rig, but they do provide a useful base for attaching the core to the rig using standard acrylic adhesive. For each core density, tests were conducted for both orientations of the honeycomb ribbon (i.e., with the long axis of the specimens coincident with the ribbon longitudinal and transverse direction). Loading was monotonic up to failure, with a constant displacement rate of 0.3 mm/min. In order to cover a wide range from nearly pure shear to nearly pure compression, tests were made at three angles of \( \phi = 21.5^\circ, 51.5^\circ \) and \( 81.5^\circ \). Fig. 3 shows load–deflection curves for the three loading angles \( \phi \) for a core of density 128 kg/m\(^3\). The deflection is measured from the cross-head displacement of the machine.

Fig. 3 shows that the cores behave in a relatively brittle manner. The peak load \( P \) is used to extract the combination of normal compressive load \( P \sin \phi \) and shear load \( P \cos \phi \) at failure. Assuming that these loads are distributed uniformly over the external surfaces of both skins, the compressive and shear stresses \( \sigma_{zz} \) and \( \tau_{i} \) in the core at failure are given by \( P \sin \phi / (bl) \) and \( P \cos \phi / (bl) \), respectively, where \( b \) and \( L \) are the breadth and length of the specimen. Plots of the combination of shear and compressive loading at failure are derived from tests at the three loading angles \( \phi \) for each core density and for both honeycomb ribbon directions, as shown in Fig. 4. The applied shear and compressive stresses are normalised by the corresponding core compressive or shear stresses. 

![Fig. 2. The modified Arcan rig.](image)

![Fig. 3. Load–deflection curves for specimens of core density 128 kg/m\(^3\), with the three loading angles \( \phi \).](image)
strength $\sigma_{cc}$ or $\tau_{cs}$, taken from the manufacturer’s data sheet and given in Table 1. Except for the honeycombs with density 128 kg/m³, the failure envelopes are well approximated by the linear failure criterion

$$\frac{\sigma_{zz}}{\sigma_{cc}} + \frac{\tau_{x}}{\tau_{cs}} = 1, \quad (1)$$

which is included as dashed lines in Fig. 4. The inconsistency observed in the honeycombs with a core density of 128 kg/m³ is due to a difference between our measurements and the data sheet value of failure stress under pure shear loading. This does not significantly affect the accuracy of our calculations.

3. Failure analysis using high-order sandwich beam theory (HOSBT)

In this section, the failure criterion established for the core in the previous section is combined with a model of the stress distribution in the core to derive a failure model for indentation loading. The HOSBT described by Frostig and Baruch [8] is used to model the core stresses. The details of how this can be applied to 3-point bending of sandwich beams is described in detail by Petras and Sutcliffe [4] and only an outline is given here. In their work, Petras and Sutcliffe [4] show that it is essential to model the local deflections under the indenter to make a sensible estimate of the stress distribution in the core, and that HOSBT does this reasonably well. ‘High-order’ refers to the non-linear way in which the in-plane and vertical displacements are allowed to vary through the height of the core. The core vertical displacement is assumed to have a quadratic

<table>
<thead>
<tr>
<th>Density (kg/m³)</th>
<th>$\sigma_{cc}$ (MN/m²)</th>
<th>$\tau_{cs}$ (MN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>0.90</td>
<td>0.50</td>
</tr>
<tr>
<td>48</td>
<td>1.79</td>
<td>1.22</td>
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<tr>
<td>64</td>
<td>3.00</td>
<td>1.83</td>
</tr>
<tr>
<td>128</td>
<td>11.00</td>
<td>3.50</td>
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variation in the through-thickness direction. Other core
displacements also vary in a non-linear way, with the
exact variation being expressed in terms of a Fourier
series. This contrasts with simple beam theory, where
the core in-plane displacements are assumed to vary in a
linear way through the depth, and the out-of-plane
displacements are assumed to be constant. These high-
order variations allow modelling of the more compli-
cated changes in core geometry which occur at loading
points. Thus the basic assumptions of HOSBT are:

- The shear stresses in the core are uniform through the
  height of the core.
- The core vertical displacement varies as a quadratic
  polynomial in the through-thickness direction, allow-
ing the core to distort and its height to change.
- The core is considered as a 3-D elastic medium, which
  has out-of-plane compressive and shear rigidity, but
  negligible resistance to in-plane normal shear stresses.

To simplify our calculations we assume that the inden-
tation line load $W$ is applied to the beam of length $L$
and core thickness $c$ as a uniform pressure over a given width
$\delta = 1$ mm at the midspan of the beam. This assumption
will be good for sandwich panels typically used in indus-
try, and it’s validity is discussed in detail by Petras
and Sutcliffe [4]. The variation of applied load distri-
bution $q(x)$ with position $x$ on the top skin is described
by a Fourier series of $M$ terms

$$q(x) = \sum_{m=1}^{M} C_m \sin \left( \frac{m\pi x}{L} \right)$$

with Fourier coefficients $C_m$ given by

$$C_m = \frac{4W}{m\pi \delta} \sin \left( \frac{m\pi}{2} \right) \sin \left( \frac{m\pi \delta}{2L} \right) ,$$

where $m$ is an index for the wavelength of the Fourier
term. For the failure analysis, we need to know the stress
field in the core-top skin interface; i.e., the out-of-plane
normal stresses $\sigma_{zz}$ and $\tau_z$. Petras and Sutcliffe ([4],
equation (13)) show that the normal stress component
can be expressed in terms of the equivalent Fourier se-
ries as

$$\sigma_{zz}(x) = \sum_{m=1}^{M} C_m \left( \frac{E_c}{D_l - E_c} \right)$$

$$- \frac{cm\pi}{2L} \frac{A_tC_t}{R_1 + R_2} \sin \left( \frac{m\pi x}{L} \right) ,$$

where the bold symbols in the above equation are
combinations of the material and geometric properties
of the beams, as detailed by Petras and Sutcliffe [4]. The
equivalent Fourier series for the shear stress component
is given by Petras [21] as

$$\tau_z(x) = \sum_{m=1}^{M} C_m \frac{A_tC_t}{R_1 + R_2} \cos \left( \frac{m\pi x}{L} \right) .$$

The line load $W_0(x)$ which would cause failure in the core
at a position $x$ along the beam is found by substituting
the variation in shear and compressive stress in the core–
skin interface (Eqs. (5) and (4)) into the core failure
criterion, Eq. (1)

$$\frac{\sigma_{zz}(x)}{\sigma_{cc}} + \frac{\tau_z(x)}{\tau_{cs}} = 1.$$

The location of first failure and the corresponding crit-
ical line load $W_0$ is found from the minimum value of
$W_0(x)$. To examine the importance of including the shear

![Fig. 5. Experimental setup.](image1)

![Fig. 6. Variation of line load and midspan core compression with top skin deflection for a sandwich beam with a core density of 29 kg/m$^3$, loaded by a central roller of diameter 6 mm. Legend: (---) longitudinal and (– – –) transverse honeycomb ribbon direction.](image2)
stresses, a failure line load is also predicted as when the maximum compressive stress in the core (directly under the indentor) equals the core compressive strength.

4. Experimental measurements of indentation failure

To validate the theoretical model of indentation failure, 3-point bending tests were performed on short sandwich beams of width 30 mm and length 70 mm. Panels had the same 9.4 mm thick cores tested in Section 2, with nominal densities 29, 48, 64 and 128 kg/m$^3$, and 0.38 mm thick skins of cross-ply GFRP laminate. Further details of the material properties are given in [4]. The experimental setup is depicted in Fig. 5. The specimens were loaded under 3-point bending using two outer rollers spaced 60 mm apart and a central roller of three different diameters (6, 10 and 20 mm). The central roller was displaced relative to the outer rollers at a rate of 0.5 mm/min. A clip gauge on one side of the specimen recorded the midspan core compression during loading while a video camera recorded images of the other side of the beam.

Fig. 6 plots typical variations with the top skin deflection of the applied line load and midspan core compression (i.e., the difference in out-of-plane displacements on the top and bottom of the beam measured at the midspan). These results are for beams with a core density of 29 kg/m$^3$, loaded by a central roller of diameter 6 mm. Separate curves for longitudinal and transverse ribbon directions are included. Very similar curves are observed for the whole range of core densities, ribbon orientations and roller diameters. Fig. 6 shows that there is significant core compression, explaining the need for a high-order beam theory to capture the local indentation response. The variation of applied load and core compression with top skin deflection is almost linear up to the peak load, which is taken as the failure load. This justifies the use of an elastic model of the beam to predict failure. The core compression is greater for beams with a longitudinal than with a transverse honeycomb ribbon direction, at a given line load, because of the higher shear stiffness of the Nomex honeycomb with this ribbon orientation.

Fig. 7(a) and (b) illustrate the portion of the sandwich beams under a central roller of diameter 10 mm, just before failure and well after the peak load is reached, corresponding to a top skin deflection of approximately 2 mm. The cores have densities of 29 and 64 kg/m$^3$, for Fig. 7(a) and (b), respectively. For the low density core, Fig. 7(a), there is significant shear in the core and widespread elastic buckling of the cell walls before failure. Fig. 7(b) shows that the high out-of-plane stiffness of the high density core tends to prevent deformation of the core away from the loading point. The core

<table>
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<th>Just prior to peak load</th>
<th>Just prior to peak load</th>
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<tr>
<td>Well after the peak load</td>
<td>Well after the peak load</td>
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Fig. 7. Photographs of a sandwich beam under a 10 mm diameter central roller just prior to peak load and well after the peak load. The core density is (a) 29 kg/m$^3$ and (b) 64 kg/m$^3$ with a longitudinal ribbon direction.
eventually fails by plastic buckling of the cell walls directly under the indentor. The complex deformation fields in the core explain the need for high order beam theory and inclusion of both shear and compressive stress components when predicting indentation failure.

5. Comparison between predictions and measurements of indentation failure load

Fig. 8 shows a comparison between the predicted and measured indentation failure loads for four different core densities, two core ribbon directions and three roller diameters. Theoretical predictions are included, which either consider both shear and compressive stresses in the core, or just the compressive component. Predictions are in good agreement with measurements. Measured failure loads do not vary significantly with the indentor size, as indicated by the results of Petras and Sutcliffe [4], who show that the core stresses for this material and beam configuration are relatively insensitive to the details of the load distribution. This also justifies the assumption made in the theoretical modelling that the load is applied as a uniform pressure over a fixed width δ. Fig. 8 shows that predictions using the failure criterion which includes shear and compressive stresses in the core are significantly better than using a criterion which considers only the compressive stresses. Moreover, the combined failure criterion successfully predicts the higher indentation resistance of sandwich beams with a longitudinal honeycomb ribbon direction.

6. Conclusions

A systematic approach has been developed to predict failure of sandwich honeycomb structures under indentation loading. It is assumed that both the skin and core are elastic up to failure, which is a reasonable approximation for the GFRP skins and Nomex cores considered. A failure criterion for Nomex honeycombs loaded simultaneously by out-of-plane compressive and shear stresses has been determined using biaxial tests with an Arcan rig. A linear dependence on the compressive and shear components is found to approximate well the measured response. HOSBT is used to predict the compressive and shear stress distribution in the sandwich beam core under indentation loading. By combining this stress distribution with the failure criterion determined from the biaxial tests, the failure load of the sandwich beam due to core failure can be predicted. This criterion can be used to model failure as the failure mode switches from pure core crushing to pure core shear, depending on the core and skin material properties and the beam geometry. Skin failure should also be considered separately, following the approach suggested by Petras and Sutcliffe [18].

Short beam 3-point bending tests are used to validate the theoretical predictions. Measurements of the core compression under the central indentor demonstrate the need for HOSBT in predicting the core deformation, while video images illustrate the importance of core shear, particularly for low density cores. Theoretical predictions of indentation failure load are in excellent agreement with measured values. Inclusion of a shear component in the failure criterion significantly improves the predictions. Moreover, this model is able to predict the observed stronger behaviour of cores with a longitudinal ribbon direction.

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