MODELLING GENERATION OF FIBRE WAVINESS DURING PROCESSING

B.B Qu, M.P.F. Sutcliffe*

Cambridge University Engineering Department, Trumpington Street, Cambridge, CB2 1PZ, United Kingdom
*mpfs@eng.cam.ac.uk

Abstract

The generation of fibre waviness during processing is explored via a mixture of experiments and theory. The experimental model system of fishing line embedded in honey is used to mimic the fibre and resin. The use of such large-scale fibres makes visualisation of the deformation easier, while honey is a relatively clear fluid with well-established viscous properties. A beam buckling model is used to predict the effect of load and time on the buckling behaviour of a single fibre. Experiments confirm the validity of the model. A multi-fibre array is tested and it is seen that the interactions of the fibres and resin is critical in controlling the cooperative buckling behaviour. Preliminary finite element calculations using a porous medium to model the resin flow show promise in modelling such multi-fibre arrays and ultimately the real composite geometry.

1 Introduction

The detrimental effect that fibre waviness has on the compressive strength of composites is well known. For example Fleck [1] provided a list of the main compressive failures of composites with aligned and continuous fibres that are often encountered in engineering practice. In the presence of fibre waviness, plastic buckling is the dominant failure mechanism. The understanding of the generation and impact of waviness defects controlling plastic microbuckling is therefore of great interest.

When the processing temperature of a composite is higher than the glass transition temperature of the matrix, the whole composite can be deemed as free of residual stress [2]. However, as the material is cooled, the coefficient of thermal expansion (CTE) mismatch between the fibre and the matrix will induce residual stresses within the composites in both the longitudinal and the transverse directions [3]. Studies by Bhalerao [4] and Chan and Wang [5] have observed that such residual stresses contribute to the initiation of fibre waviness. In general, researchers concluded that pressure has no effect on waviness generation. As long as the hold temperature is high enough to turn the matrix of the prepreg into flowing resin and the hold time is long enough to relax all the residual stresses within the panel, they have no other effects on the waviness initiation [6]. Kugler et al. [7] indicate that, with a given combination of fibre, matrix and tool plate, there is a critical part length below which waviness does not appear. Therefore the main parameters generating waviness in a given part are the cooling rate and the tool plate material. The cooling rate of the part plays an important role in waviness generation as it determines the temperature gradients experienced by the tool and the part [6]. The choice of tool plate material is also an important parameter as it determines the CTE mismatch between the tool plate and the matrix, which would tend to
generate fibre waviness. Olsen [8] showed that, when a tool plate with low CTE was used, the waviness can be significantly reduced and even eliminated.

Despite the many observations made of fibre waviness generation in processing, few models have been developed. This paper aims to develop such models to assist the accurate prediction of compressive failure of composite components. The paper first develops an understanding of the process via modelling and experimental study of a single fibre loaded compressively in a viscous medium. The work is then extended to multi-fibre arrays, both with look-see experiments and numerical modelling.

2. Buckling of a single fibre in viscous medium

In this section the key aspects of the fibre generation process are explored via a simplified model of the process. A single fibre subject to end load is modelled, see Fig. 1, representing part of a fibre within a structure subject to compressive stresses due, for example, to residual stresses. In this idealisation the fibre is immersed in a Newtonian viscous fluid, representing amorphous polymer matrix at high processing temperatures [9].

2.1. Theoretical model

The fibre is modelled as an Euler beam of length $L$ subject to a compressive load $P$ with pinned end conditions, as shown in Fig. 1, with $x$ and $y$ coordinates along and transverse to the beam. There is a transverse force $f(x)$ due to viscous effects which varies along the length of the beam. There are induced shear force and bending moments $S$ and $M$ in the beam. Equilibrium of a beam element can be used to derive the governing equation for small deflections of the beam as

$$D_f \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} = f(x)$$

(1)

where $D_f=EI$ is the bending stiffness of the beam.

Letting $w$ be the deflection of the fibre, and assuming that the transverse load depends linearly on the transverse speed via a viscosity $\eta$, equation 1 becomes

$$D_f \frac{d^4 w}{dx^4} + P \frac{d^2 w}{dx^2} = -\eta \frac{dw}{dt}$$

(2)
Using the substitution \( w = \hat{w} \), \( x = \hat{x}L \) and \( t = \eta L^2 \dot{\hat{t}} / E \) equation 2 takes the dimensionless form

\[
D \frac{d^4 \hat{w}}{d\hat{x}^4} + \frac{d^2 \hat{w}}{d\hat{x}^2} = \frac{d\hat{w}}{d\hat{t}}
\]

(3)

with \( D = EI / L^3 \).

For the boundary conditions of the problem this equation can be solved by separation of variables, to give a variation of displacement with position and time of the form

\[
\hat{w} = \sum_{n=1}^{\infty} A_n e^{\lambda_n} \sin(n\pi \hat{x})
\]

(4)

where the \( nth \) buckling mode is

\[
\hat{w}_n = A_n e^{\lambda_n} \sin(n\pi \hat{x})
\]

(5)

with \( \alpha_n = n\pi \) and with coefficients \( \lambda_n \) given by

\[
\lambda_n = n^2\pi^2 - Dn^4\pi^4
\]

(6)

The general solution of equation 3.15 is hence

\[
\hat{w} = \sum_{n=1}^{\infty} A_n e^{(n^2\pi^2 - Dn^4\pi^4)\hat{t}} \sin(n\pi \hat{x})
\]

(7)

The exponential term \( e^{\lambda_n} \) in equation 7 indicates how the deflection grows from the initial imperfection to the final waviness shape. If \( \lambda_n \leq 0 \), the \( e^{\lambda_n} \) term indicates decay while \( \lambda_n > 0 \) is required for the deflection of the \( nth \) mode to grow. For the lowest mode \( n=1 \) to grow, it can be shown that

\[
P > EI \left( \frac{\pi}{L} \right)^2
\]

(8)

This is exactly the same as the critical buckling load of a beam without viscous fluid. Therefore viscous damping does not alter the buckling criterion of the fibre. It only affects the process during which the fibre buckles to its final shape. The initial fibre shape has to be used to determine the constants \( A_n \).

\subsection*{2.1.1. Effective viscosity}

The transverse force per unit length \( f(x) \) in Fig. 1 takes the form

\[
f(x) = -\eta \frac{dy}{dt}
\]

(9)

Here \( \eta \) is a constant which has the same units as the viscosity of the fluid \( \mu \).
The drag on the fibre is modelled as if the fibre a cylinder moving horizontally in the resin at some average velocity. In this case, the drag force per unit length is \( \eta \vec{v} \). The total drag force is therefore

\[
F_D = L \eta \vec{v} \tag{10}
\]

The Reynolds number of the fibre \( \text{Re} = 2 r \rho v / \mu \) is in the order of \( 10^{-4} \). Therefore Stoke’s flow can be assumed for which [10]

\[
F_D = \frac{24}{\text{Re}^2} \rho \vec{v}^2 r L \tag{11}
\]

Comparing equation 10 with equation 11 gives

\[
\eta = 12 \mu \tag{12}
\]

2.2. Experimental validation

This section presents an experimental validation of the single fibre model. The set-up is illustrated in Fig. 2, showing a fibre immersed in fluid. The fibre used is nylon polymer fishing line of diameter 0.75 mm. Because this was obtained on a reel, it had a curvature which was largely eliminated by heating and stretching individual strands. The bending stiffness \( EI \) of the fibre was determined by loading a small section as a beam in bending, to give a modulus \( E \) equal to 1.32 GPa.

The viscous fluid used was acacia honey (supplied by Marks and Spencer), chosen because of its transparent nature, and well-documented viscous properties. To determine the honey viscosity, a rigid solid sphere was dropped into the honey and the process recorded using a video camera. The viscosity estimates obtained using different size spheres were consistent for a given temperature. The average viscosity was 9 kg/m/s at 26.5°C and 10 kg/m/s at 24.1°C, confirming the sensitivity of viscosity to temperature.

A fibre is held stationary at the bottom and clamped, via a steel rod, into a moving top end. Both ends of the fibre are effectively constrained to prevent rotation. The load is applied via a mass at the top of the fibre, which runs in a track. The effect of friction was estimated and found to be small [11]. Table 1 details the experimental parameters used for two sets of experiments.
Figure 2. Experimental set-up for buckling of a single fibre in viscous fluid.

Table 1. Experimental conditions.

<table>
<thead>
<tr>
<th></th>
<th>Experiment 1</th>
<th>Experiment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of the fibre (mm)</td>
<td>0.375</td>
<td>0.375</td>
</tr>
<tr>
<td>$E$ (GPa)</td>
<td>1.32</td>
<td>1.32</td>
</tr>
<tr>
<td>$L$ (m)</td>
<td>0.16</td>
<td>0.20</td>
</tr>
<tr>
<td>Effective $L$ (m)</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>Mass weight (g)</td>
<td>16.73</td>
<td>16.73</td>
</tr>
<tr>
<td>Initial max deflection (mm)</td>
<td>2.30</td>
<td>10.30</td>
</tr>
<tr>
<td>Honey viscosity (26.5°C, kg/m/s)</td>
<td>9.00</td>
<td>9.00</td>
</tr>
</tbody>
</table>

The change in shape of the fibre with time was derived from a video of the loading process, using image analysis within the Matlab [12] programming language. Figure 3 shows the fibre shape initially and at a typical deflected position, showing that a sinusoidal function fits the data.

Figure 3. The measured fibre shape: (a) initial imperfection (b) a typical deflected shape of the fibre plotted against position. The dark (blue) lines are sinusoidal curve fits (a) and model predictions (b).
2.3. Comparison with theoretical model

The analysis detailed in section 2.1 is applied to the experimental conditions, with the minor adjustment that the clamped end conditions in the experiments give an effective length of the buckled fibre as $2L$. Figure 4 compares measurements and predictions, showing that the growth of the waviness is well modelled using the theory, at least for the initial stages of growth. Further analysis in [11] shows that the discrepancy at larger deflections can be well fitted by including a large deflection beam analysis.

![Diagram](image)

**Figure 4.** Comparison between experiment measurements and theoretical predictions of the growth of the maximum fibre transverse deflection with time: (a) experiment 1; (b) experiment 2.
### 2.4. Application to real composites

To correlate the modelling work with practical applications, in this section we consider the time scale during which typical waviness is generated in composites. To do so, an approximation for the permeability of the material is needed. The Kozeny–Carman equation has been extensively used in the literature for calculating permeability of composites [13]

\[ \kappa_f = \frac{(1-f)r_h^2}{K_{CK}} \]  

(13)

where

\[ r_h = r \frac{(1-f)}{2f} \]  

(14)

where \( r \) is the fibre radius, \( f \) the fibre volume fraction and \( K_{CK} \) is the Kozeny constant, which is about 5 when the fibre volume fraction is between 0.3 and 0.8. Using this model, the time scale of deflection for a typical composite laminate can be determined. Consider a sample of unidirectional graphite/PMR-15 polyimide composite, of which the compressive load due to CTE mismatch is estimated to be 50 MPa by Benedikt, et.al [14]. A typical carbon fibre has a radius of 3.5 μm and modulus of 350 GPa. The viscosity of the polyimide resin is about 2 Pa.s at the processing temperature. By using a fibre volume fraction of 0.5 and an initial imperfection 10 times the fibre radius, the model predicts that it takes about 15 minutes for the fibre to generate a typical waviness of 200 times the fibre radius (0.7 mm). The same process requires about 30 minutes if the fibre volume fraction is 0.6.

### 3. Multi-fibre modelling

The previous section has established the modelling and experimental techniques for a single fibre. In this chapter, multi-fibre arrays are considered. Some look-see experiments are described in section 3.1 while a finite element analysis is detailed in section 3.2.

#### 3.1 Observations of multi-fibre buckling

Figure 5 illustrates the setup used for these multi-fibre experiments. Again nylon line and honey were used. The experiment was contained in a purpose-built metal box of height 26 cm and width 10 cm. The fibres were placed in honey in the middle of the box with both ends free to move. The top ends of the fibres were placed in a slot cut on the loading plate while the bottom ends are supported in another slot at the bottom of the box. A transparent plastic plate was used to cover the front of the box. The depth of the gap cavity containing the fibres is 1 mm so that the fibres cannot roll over each other. Finally, the load is applied by a loading metal plate falling into the honey from the top of the gap. Further details of the arrangement are given in [11].
Figure 5 (a) shows the initial position of 10 fibres in the experiment. The fibres were purposely placed in two bundles with initial imperfections in opposite directions. Soybean sauce was injected between the bundles using a hypodermic needle, the sauce was used to track honey flow. The two circled regions in Fig. 5 (b) show the movement of honey tracked by the soybean sauce. As seen from the shape of the soybean region, the honey was “dragged” away by the deflected fibres, with a similar displacement profile to the fibres.

In general, the deflection of each bundle of fibres behaves in a similarly to the single fibre case. However, the interaction among the bundles causes unpredicted changes to the shape, rate and direction of deflections. It can be observed in Figure 5 that the deflection of each bundle induces resin flow, which in turn causes deflection of other bundles of fibres. Such behaviours were not predicted in the single fibre model. As a result, the fluid flow is an extra factor which needs to be considered in the multi-fibre case.

3.2 Finite element modelling multi-fibre buckling

Preliminary calculations have been undertaken using Abaqus finite element software [15] to model the generation of waviness. The porous material capability of Abaqus is used to model the flow of resin via a permeability coefficient. The host material for the resin (in effect a dummy material) was modelled as a compliant linear elastic material, while beams were embedded in the resin material. In these preliminary calculations just three beams (each representing a bundle of fibres) were included. The boundary of the model is given the default porous boundary condition corresponding to no-flow there. The beams were loaded axially via a displacement loading. To initiate an imperfection the central of the three beams was given a small curvature, while the two adjacent beams were straight. The ends of the beams were restrained from rotating. A simplified model of just the beam was used to confirm the
Euler buckling load. When the surrounding resin and beams was added the structure was found to collapse at a similar load, with the two straight beams ‘failing’ soon after the central one. Figure 6 illustrates the pore pressure distribution in the resin at this point. The displacement of the beams has caused a redistribution of pore pressure which will resist further deformation. Further work on this model is needed to define boundary conditions and material properties. Nevertheless it shows promise as a method to capture the key features seen in the experiments and explore the influence of the flow boundary conditions and fibre interactions.

![Finite element model of multi-fibre waviness generation. The contours plot the pore pressure associated with the resin.](image.png)

**Figure 6.** Finite element model of multi-fibre waviness generation. The contours plot the pore pressure associated with the resin.

**Conclusions**

A beam buckling theory including transverse viscous forces associated with resin has been used to model generation of waviness in composites processing. The solution is given by a separation of variables. The model shows the same critical buckling load as a standard Euler strut, but now the increase of buckling deformation with time is controlled by the external load and viscous forces.

Experiments have been carried out using polymer fishing line as the fibre and acacia honey as the viscous fluid. The fibre was then deflected in the honey, with a compressive load applied by dropping a metal mass down a vertical track. The model provides reasonable agreement with the experimentally measured maximum deflection data at the early stage of the experiment. However, as fibre deflects in the honey, the predictions made using the simple buckling model start to deviate from the experimental observation, while a large-deflection model has an improved prediction [11].

Experimental observations have been obtained for the multi-fibre buckling problem. It was observed that the interactions among the fibres and between the fibres and the honey introduce additional factors into the single fibre model. A preliminary finite element model of multi-fibre waviness generation has been build, using the permeability capabilities of Abaqus. The model shows promise to be able to consider the effects of geometry and fluid boundary conditions on waviness generation.
References