MICROBUCKLE PROPAGATION IN FIBRE COMPOSITES

M. P. F. SUTCLIFFE and N. A. FLECK
Cambridge University Engineering Department, Trumpington St, Cambridge, CB2 1PZ, U.K.

(Received 15 February 1996; accepted 2 July 1996)

Abstract—The propagation of a microbuckle in a unidirectional long fibre composite has been investigated by the finite element method. The tip region of the microbuckle is modelled using alternating layers of fibre and matrix, while the microbuckle at the macro scale is treated as a sliding mode II crack. By assuming that the composite deforms in accordance with a deformation theory of plasticity, material history effects are neglected. The microbuckle propagation direction is predicted for a wide range of material properties by finding a correlation direction of the deflected fibres ahead of the microbuckle: propagation angles of between 5 and 30° are predicted, depending on the matrix shear yield strain and the strain hardening exponent. For the case of a carbon fibre-epoxy composite, the predicted value of 19° is in good agreement with measured values in the range 20–30°. A predicted value of tip toughness $G_{\text{p}}/d$ of 25 is also in reasonable agreement with experimental measurements of 32–55, where $G_{\text{p}}$ is the mode II strain energy release rate, $\tau_s$ is the longitudinal shear yield strength of the composite and $d$ is the fibre diameter. An alternative couple-stress finite element calculation, in which the bending resistance of the fibres is incorporated directly into the element formulation, predicts propagation angles in reasonable agreement with those found from the above layer finite element model over the range of material parameters considered, with excellent agreement for carbon fibre-epoxy composites. Copyright © 1997 Acta Metallurgica Inc.

1. INTRODUCTION

Carbon-fibre epoxy composites are increasingly used in applications where weight is an important factor, for example in aircraft and sports equipment. In order to reduce safety factors to a minimum, a detailed understanding of the main failure mechanisms is needed. Compressive failure is of particular concern since the compressive strength is usually less than the tensile strength.

Long fibre composite laminates generally fail in compression by plastic microbuckling—the formation of a localised band of buckled fibres [1, 2]. In unnotched specimens, microbuckling initiates at a region of misaligned fibres [3]. Figure 1(a) illustrates a microbuckle band whose normal is inclined at an angle $\beta$ to the fibre direction. With brittle fibres, fracture of the fibres occurs on at least two planes as illustrated in Fig. 1(b). The direction transverse to the microbuckle may be either the through-thickness direction of the panel, in which case Fig. 1 represents an 'out-of-plane' microbuckle, or it may lie in the plane of the panel; Fig. 1 then represents an 'in-plane' microbuckle. A major aim of the current paper is to explore the dependence of the inclination of the microbuckle $\beta$ upon the constitutive properties of the composite. Typically, the fibres in the kink band rotate by an angle $\phi$ of twice the inclination angle $\beta$ of the kink band, and then lock up. Kinematic arguments [4] suggest that a fibre rotation $\phi$ in the range of zero to $\beta$ is associated with dilatation within the microbuckle band by the mechanism of tensile microcracking. The component of hydrostatic strain rate is tensile for $\phi$ in the range of $0-\beta$ and compressive for $\phi$ in the range of $\beta-2\beta$; the net hydrostatic strain is compressive for $\phi$ exceeding $2\beta$. Thus, it is imagined that the tensile microcracks shut and produce a volumetric locked-up state within the band at the critical point where $\phi$ attains the value $2\beta$.

Soutis et al. [5] have studied compressive failure from holes in carbon fibre-epoxy laminates; they find that failure is governed by microbuckling in the $0^\circ$ plies. The geometric inhomogeneity induces fibre rotation under increasing applied load; deformation localises within a band and a microbuckle is initiated [5]. The microbuckle then propagates in a stable manner and the component fails at a higher load than the initiation load. Microbuckle propagation and final failure of the component are modelled using a large-scale bridging analysis. This model has been extended to other notch geometries and has been incorporated into a user-friendly computer program [6, 7]. Soutis et al. use coupon tests to measure both the unnotched strength and the compressive 'toughness' of the laminate. From a design point of view it is desirable to predict these laminate properties from the mechanical properties of the fibres and matrix and from the lay-up geometry. Fleck and Shu [3] have recently developed a finite element code incorporating couple stresses associated with fibre bending to predict the unnotched strength of unidirectional material. Wisnom [8] and Kyriakides et al. [9] use commercial finite element codes to investigate the initiation of microbuckling in specimens of finite
width. In this paper we take the first step in predicting the laminate compressive toughness by predicting the propagation toughness for an in-plane microbuckle in unidirectional material.

Sutcliffe et al. [10] have predicted the initiation compressive toughness by addressing the initiation of microbuckles from sharp notches. Sutcliffe and Fleck [11] investigated experimentally their subsequent propagation in a carbon fibre-epoxy composite. They observed both in-plane and out-of-plane microbuckles. There exists a short region at the tip of the propagating microbuckle, of about 20 fibre diameters in length, in which the initially straight fibres buckle and then break to form a kink band. Figure 2(a) and (b) shows an in-plane microbuckle under load and after unloading, respectively. The fibres behind the microbuckle tip seen in Fig. 2(a) have straightened because they have 'popped-up' out of the plane of the section. Evans and Adler [12] observe similar tip zones in specimens which were sectioned after unloading.

Sutcliffe and Fleck [11] and Sivashanker et al. [13] observed that both in-plane and out-of-plane microbuckles in a carbon fibre-epoxy composite have a tip toughness of 10–17 kJ/m². The flanks of the in-plane and out-of-plane microbuckles were treated as sliding cracks, and the friction stress on the equivalent crack was estimated as \( T_b = 40-90 \text{ MPa} \). This estimate for the shear stress on the microbuckle is comparable to the shear yield strength of the composite of 62 MPa.

There have been at least two approaches to predicting the inclination \( \beta \) of a microbuckle. Budiansky [16] calculated the fibre rotation around a point defect of infinite fibre waviness for an elastic composite. He found that the orientation of peak fibre rotation is dependent upon the degree of elastic orthotropy of the composite, and he argued that subsequent propagation of the microbuckle is locked-in to this orientation. Alternatively, Moran et al. [14] predict the propagation direction \( \beta \) of a microbuckle from an infinite band model. They assume that the matrix locks-up in shear by strain hardening and use an energy calculation to predict the orientation \( \beta \) of the microbuckle. While both these attempts to model propagation directions are plausible, experimental evidence suggests on the one hand that microbuckle directions are not locked in as soon as they initiate and on the other hand that large matrix strains in the microbuckle band are accompanied by considerable void growth and cracking which restrict the amount of strain hardening found in practice. In addition, it is not clear that a microbuckle would propagate in a direction corresponding to the minimum energy for transverse propagation of an infinite band microbuckle.

In this paper we model a propagating microbuckle using a finite element formulation with a tip process zone and a sliding crack behind the tip. We predict the tip toughness and propagation direction for in-plane microbuckling in uni-directional material. To verify the assumptions used in this finite element model, an alternative finite element formulation of the problem is described in Section 3.6. These predictions are compared with infinite band models in Section 4 and with experimental measurements in Section 5. It is intended that the predictions of toughness for uni-directional material presented in this paper should be part of a laminate model of toughness which in turn would be used in a large-scale bridging model of failure from notches.
Fig. 2. The tip of a propagating in-plane microbuckle: (a) under load; (b) after unloading.
2. THEORETICAL MODEL

2.1. General description

The composite is modelled using a finite element analysis (ABAQUS implicit formulation [17]), similar to that described by Sutcliffe et al. [10]. A brief outline of the model is given in this section as full details have already been described by Sutcliffe et al. [10]. In-plane deformation is considered, since this two-dimensional deformation field is considerably simpler than three dimensional out-of-plane deformation. Sutcliffe and Fleck [11] have shown experimentally that in-plane and out-of-plane microbuckles behave in a similar way. The overall geometry of the problem is shown in Fig. 4(a). We consider propagation of a microbuckle which is long compared with the fibre diameter \( d \), and which has an initial orientation angle \( \beta_0 \). Behind the tip, the microbuckle is treated as a mode II crack with crack-face friction. The overall strategy is to argue that the correlation direction of fibre rotation ahead of the microbuckle tip provides the steady state inclination angle \( \beta \), and the applied mode II energy release rate at which the fibres lock-up provides an estimate for the mode II tip toughness.

\[
\gamma = \frac{\tau}{\gamma_y} + \alpha \left( \frac{\tau}{\gamma_y} \right)^n
\]  

where \( \tau \) is the shear yield strength, \( \gamma_y \) is the shear yield strain and \( n \) is the strain hardening exponent. The shear modulus \( G \) equals \( \frac{E}{2(1+\nu)} \), and \( \nu \) is taken as 0.37. For the calculations in Section 3 we select the material properties to represent those of uni-directional Toray T800 carbon fibres in a Ciba-Geigy 924C epoxy matrix. This material has been intensively examined within the authors' laboratory [18] and its Ramberg-Osgood parameters have been measured by Jelf and Fleck [19]. Material parameters used in our calculations are summarised in Table 1.

The mesh has two regions. In an inner region each fibre and the intervening matrix are modelled by alternate plates, as illustrated in Fig. 4(b). The volume fraction of fibres is set at the typical value of 0.64 and is not changed in this study. The fibres are modelled by 8-noded linear isotropic elastic elements, and the matrix is modelled using 4-noded elements made from an isotropic, deformation theory solid with Ramberg-Osgood strain hardening behaviour. The fibres are taken to be isotropic and linear elastic, with a Young's modulus of \( E_f = 240 \) GPa and a Poisson's ratio of \( \nu_f = 0.32 \). These values are typical of Toray T800 carbon fibres, which have a fibre diameter \( d = 5 \) µm. The in-plane response of the matrix is modelled by the Ramberg-Osgood non-linear deformation theory solid model of equation (1),
The appropriate relationship between the composite properties and the properties for the matrix elements (in particular $n$ and $\sigma$) is described in detail in [10].

In the outer region, the composite is modelled using two types of element. The shear and transverse response is again modelled using 4-noded elements made from an isotropic, deformation theory solid with the Ramberg-Osgood strain hardening behaviour described by equation (1). The appropriate values of the material properties used in these elements differ from those in the inner mesh (see [10] for details), and are again summarized in Table 1. Between these elements are elastic beam elements to give the required axial and bending stiffness for the composite. By using beam elements it is possible to coarsen the mesh while still retaining appropriate shear and bending stiffness properties. Because the fibres are modelled as plates in the inner region, they have a slightly larger bending stiffness for the same volume fraction than the outer region where the beam properties were chosen to match the circular geometry of the fibres. Although the behaviour of the two types of mesh is not significantly different, the inner mesh allows a more accurate assessment of the dilation of the matrix at this critical region. 8-noded elements were used for the elastic elements to ensure high accuracy for these elements which undergo elastic bending deformation, while the 4-noded elements used for the matrix material can accommodate the large shear strains in the matrix material. Although the elements are then incompatible, errors will be small. Since the matrix exhibits void growth at large volumetric strains, the effective strain hardening in the microbuckling band will be somewhat less than that assumed using the Ramberg–Osgood relationship derived from shear tests without dilation. The effect of this is not considered in this paper.

We assume that the composite behaves as a deformation-theory solid. Consequently, material history effects can be disregarded and the steady state microbuckle propagation problem can be modelled by that of a stationary microbuckle. There exists some experimental evidence to support the assumption that an epoxy matrix composite behaves as a deformation theory solid at the strains experienced in the material outside the microbuckle band itself, where the effects of non-proportional loading can be expected to be significant. Compression tests on a small specimen of 924C Ciba–Geigy epoxy matrix show a non-linear elastic behaviour for strains of less than about 8% (Sutcliffe, private communication, 1995). Fleck and Jelf [20] have conducted non-proportional loading tests on carbon fibre epoxy matrix tubes and found that the response more closely matches that of a deformation theory solid than that of a flow theory solid.

In the finite element analysis it is assumed that the stress state is plane stress. The rationale is as follows. In the physical problem of interest with in-plane microbuckling, the extent of the microbuckle zone is assumed to be much less than the thickness of the composite so that in-plane deformations dominate. Since the fibres are much stiffer than the matrix, fibre rotation is associated with a volumetric strain in the matrix. Experimental observations show that this volumetric straining is accommodated by voiding within the epoxy matrix. In the finite element model the constitutive description adopted for the matrix is taken to be incompressible for convenience. By assuming that the stress state is plane stress, in-plane hydrostatic strains and fibre rotations are achievable within the matrix. In this way we model in an approximate way volumetric expansion of the matrix by void growth.

The composite is loaded by a uniform normal traction applied to the top and bottom boundaries of the mesh, see Fig. 4(a). The flanks of the microbuckle are modelled as a mode II sliding crack, and interface elements are used to carry a constant frictional shear stress $\tau_s$ (implemented in ABAQUS by a large value of Coulomb friction coefficient and a limiting shear stress $\tau_s$). As suggested by experimental measurement [11, 13], it is assumed that the frictional shear stress $\tau_s$ equals the shear yield strength of the composite. While it would be consistent with infinite band results to assume a higher frictional shear stress than this at low values of the strain hardening index $n$, the approach taken here reflects the limit on strain hardening associated with matrix damage.

The microbuckle length and mesh size are sufficiently great for the remote field to be elastic and for the non-linear zone at the microbuckle tip to satisfy the small scale yielding approximation. Thus it is convenient to express the results in terms of a

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Composite</th>
<th>Inner mesh: fibres</th>
<th>Inner mesh: matrix</th>
<th>Outer mesh: 4-noded elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_t$ (GPa)</td>
<td>9.25</td>
<td>240</td>
<td>6.4</td>
<td>18</td>
</tr>
<tr>
<td>$E_r$ (GPa)</td>
<td>162</td>
<td>162</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G$ (GPa)</td>
<td>6.8</td>
<td>6.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.0195</td>
<td>0.36</td>
<td>0.40</td>
<td>0.34</td>
</tr>
<tr>
<td>$\gamma (= e/3)$ (MPa)</td>
<td>6.24–624</td>
<td>6.24–624</td>
<td>6.24–624</td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>3.7</td>
<td></td>
<td>0.041</td>
<td>3.7</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>3.5–100</td>
<td>3.5–100</td>
<td>3.5–100</td>
<td></td>
</tr>
</tbody>
</table>

Unless otherwise stated, $n = 3.5$ and $\tau_s = 62.4$ MPa ($\gamma = 9.2 \times 10^{-3}$), when material properties are typical of uni-directional T800/924C carbon fibre-epoxy material.
microbuckle tip strain energy release rate $G$, in preference to the remote applied stress $\sigma$. The connection is achieved by considering the auxiliary problem of a mode II sliding crack in an orthotropic elastic body of identical dimensions to that defined for the microbuckle calculation, Fig. 4(a). Specifically, the elastic body is subjected to a crack shear traction $t_b$ and to a remote axial stress $\sigma_1$; the associated strain energy release rate $G$ is calculated by the virtual crack extension method, implemented within ABAQUS.

3. THEORETICAL RESULTS

In this section we present detailed results for a particular set of material properties corresponding to a carbon-fibre epoxy composite T800/924C. Figure 5(a) shows the deformed mesh at the tip of a microbuckle with an initial orientation $\beta_0$ of 5° at an applied tip strain energy release rate $G/\tau_0d$ of 28. A number of fibres ahead of the microbuckle tip have been filled in to identify them more clearly. This figure shows that the microbuckle has begun to propagate. It also shows a small split above the tip of the crack. This split arises because the crack model for the propagating microbuckle concentrates the sliding displacements behind the crack tip on the plane of the crack, rather than being spread over the width of the microbuckle. The split which is formed leads to a lack of support for the fibres at the tip of the microbuckle. To avoid including this spurious effect we present results for fibres slightly ahead of the crack tip at the tenth fibre ahead of the microbuckle tip. This position was chosen as being just ahead of the point at which the smooth change in deformed shape ahead of the microbuckle tip appeared to be affected by the proximity of the crack tip. In Section 3.6 we describe calculations which confirm the validity of this approach.

Figure 6 plots the peak rotation in the 10th fibre ahead of the tip of the crack (labelled A in Fig. 5(a)) as a function of load. The strain transverse to the fibre direction $\varepsilon_T$ in the matrix material just behind fibre A at the centre of the microbuckle band is plotted as a function of load in Fig. 7. In approximate terms, the value we calculate for $\varepsilon_T$ in our plane stress problem provides a useful measure of the volumetric strain in the matrix in the more realistic plane strain problem. Figure 7 shows that $\varepsilon_T$ is initially positive (corresponding to volumetric expansion in the microbuckle band in the plane strain case) as the fibres rotate, but that at an applied strain energy release $G/\tau_0d$ of 28 the volumetric strain returns to

![Fig. 5. The deformed mesh at the load corresponding to lock-up. No amplification of deformation is displayed. The line ahead of the microbuckle tip joins points of inflection in fibres and beam elements: (a) $\beta_0 = 25°$, $\gamma_0 = 9.2 \times 10^{-3}$, $n = 3.5$, $G/\tau_0d = 28$, $\beta = 21°$; (b) $\beta_0 = 10°$, $\gamma_0 = 9.2 \times 10^{-4}$, $n = 3.5$, $G/\tau_0d = 9.5$, $\beta = 28°$; (c) $\beta_0 = 7°$, $\gamma_0 = 9.2 \times 10^{-1}$, $n = 15$, $G/\tau_0d = 3.9$, $\beta = 7°$.](image-url)
zero. In practice the fibres would lock-up at this point and resist further rotation. In the finite element calculation this lock-up does not occur and there is considerable further volumetric compression.

3.1. Propagation direction

To predict the direction of microbuckle propagation, we calculate the direction of propagation of microbuckles with four initial orientations $\phi_0$ of 7°, 10°, 25° and 40° and then interpolate to estimate the stable propagation direction.

Experimental observation of a propagating microbuckle (Fig. 2(a)) shows that the propagation direction is fixed by the correlation direction of fibre rotation ahead of the propagating microbuckle, rather than being determined by the successive positions of fibre fracture ahead of the microbuckle. This suggests that we do not need to model accurately each individual fibre fracture process but need instead to look at the 'correlation direction' for self-similar growth of the microbuckle. To find a correlation direction we map the position of the inflection points in the deformed fibres ahead of the tip as shown on Fig. 5(a). We note that, although the first few fibres ahead of the crack tip are affected by the split formed at that point, a little ahead of the crack tip the microbuckle orientation only varies slowly. At the lock-up value for $G/\gamma_d$ of 28 the correlation direction equals 21° (taken as the tangent at fibre A to the line of inflections). This correlation direction only increases slightly at higher loads. Calculations with the initial microbuckle oriented at angles of 10° and 40° give propagation directions of 14° and 19°, respectively. These calculations suggest that there exists a stable propagation direction of about 19° by linear interpolation. This interpolation method minimises any effect arising from the fact that the mesh is oriented along the initial microbuckle orientation $\beta_0$. In this ABAQUS calculation, we cannot model lock-up behaviour, so that tracking the propagation of the microbuckle over a finite length would be invalid. However Section 3.6 describes finite element calculations where this lock-up process is included, confirming the approach used in this section.

3.2. Calculation of tip toughness

Sutcliffe and Fleck [11] divide the propagating microbuckle into a tip process characterised only by a tip toughness $G_t$ and a sliding region behind the tip modelled by a constant bridging stress $\tau$. In the finite element calculation we chose to make this division at the point where lock-up occurs (i.e. at the point where $\theta_0$ equals zero). After lock-up, band broadening occurs at a constant axial stress; this is represented in the large scale bridging model by the constant friction stress on the crack flanks.

The tip toughness $G_t$ for microbuckle propagation in a typical carbon fibre composite (T800-924C) is estimated as follows. We take as representative material properties $\gamma_s = 9.2 \times 10^{-3}$ and $n = 3.5$. Then, Fig. 7 shows that lock-up occurs at $G/\gamma_d = 28$, for an initial propagation angle $\phi_0$ of 25°. In similar fashion, lock-up occurs at $G/\gamma_d = 20$ for $\beta_0 = 10°$. Hence, linear interpolation implies that the non-dimensional tip toughness $G_t/\gamma_d$ equals 25 for the stable propagation direction $\beta$ of 19°.

3.3. Effect of material properties upon predicted $\beta$ direction and tip toughness

Next, we investigate the sensitivity of the predicted $\beta$ angle and the tip toughness $G_t$ to the shear yield strain $\gamma_s$ and the hardening exponent $n$ of the composite. The effect of the magnitude of $\gamma_s$ upon the collapse response is made clear from a comparison of Fig. 5(a)–(c), all for $n = 3.5$. The deformed shapes are shown in Fig. 5(a) for $\gamma_s = 9.2 \times 10^{-3}$; this value of $\gamma_s$ is decreased by a factor of ten in Fig. 5(b) and is

---

†In Ref. [21] Fleck uses a cohesive zone model to predict a toughness $G$ for microbuckle propagation. In his model the tip of the microbuckle is easily identified. However this is not the case in the model described in this paper, and Fleck's definition of $G$ cannot be used here.
propagation can be judged by comparing the deformed mesh at lock-up for the case $n = 3.5$ in Fig. 5(a) with the case $n = 15$ in Fig. 5(d). In both cases $\gamma_y = 9.2 \times 10^{-3}$. A comparison of the two figures shows that the predicted propagation angle $\beta$ decreases with increasing $n$, with little concomitant effect upon the curvature of buckled fibres. The dependence of $\beta$ and the tip toughness $G_t/\tau_d$ upon $n$ is shown explicitly in Fig. 9 for this layer model, for the case $\gamma_y = 9.2 \times 10^{-3}$. We deduce from Fig. 9(a) that $\beta$ increases monotonically with decreasing $n$, from a value of about 5° as $n \to \infty$ to $\beta = 19°$ at $n = 3.5$. The tip toughness shows a similar dependence upon $n$: $G_t/\tau_d$ increases from about 2.5 as $n \to \infty$ to a value of about 25 at $n = 3.5$ (see Fig. 9(b)).

3.4. Effect of loading details

We have characterised the applied load in the finite element model using the mode II energy release rate at the microbuckle tip associated with a stress applied to the top and bottom faces of the mesh. In this section we consider the results for an alternative loading scheme, wherein the outer boundaries of the mesh are subjected to the displacements associated increased by a factor of ten in Fig. 5(c). In all cases the deformed meshes are shown for values of $\beta_0$ closest to the predicted stable propagation direction, at the load when the matrix behind fibre A has just locked up.

A comparison of Fig. 5(a)-(c) shows that the curvature of microbuckled fibres increases with increasing $\gamma_y$; this is in qualitative agreement with previous one-dimensional calculations for an infinite microbuckle band [15]. A number of additional simulations were conducted to explore the dependence upon $\gamma_y$ of the predicted inclination $\beta$ and tip toughness $G_t$ for microbuckle growth. The results of this layer model are plotted in Fig. 8. It is clear from Fig. 8(a) that the stable inclination $\beta$ increases monotonically from about 10° to about 27° as $\gamma_y$ is increased from $\gamma_y = 9.2 \times 10^{-4}$ to $\gamma_y = 9.2 \times 10^{-2}$. In contrast, the dependence of $G_t/\tau_d$ upon $\gamma_y$ shown in Fig. 8(b) is not monotonic: $G_t/\tau_d$ has a peak value of about 25 at $\gamma_y \approx 3 \times 10^{-3}$. The maximum is not a strong one, however: $G_t/\tau_d$ exceeds a value of 20 for $\gamma_y$ in the range $1.4 \times 10^{-3}$ to $2 \times 10^{-2}$.

The effect of strain hardening upon microbuckle
bending strain of 1.8%. We predict that microbuckling is accompanied by fibre fracture in agreement with experimental observation.

Table 2. Peak tensile strain in fibre A at lock-up

<table>
<thead>
<tr>
<th>Hardening exponent n</th>
<th>3.5</th>
<th>3.5</th>
<th>3.5</th>
<th>15</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composite shear yield strain (\gamma_i)</td>
<td>(9.2 \times 10^{-4})</td>
<td>(9.2 \times 10^{-3})</td>
<td>(9.2 \times 10^{-2})</td>
<td>(9.2 \times 10^{-1})</td>
<td>(9.2 \times 10^{-1})</td>
</tr>
<tr>
<td>Peak tensile strain</td>
<td>0.35%</td>
<td>1.8%</td>
<td>6.5%</td>
<td>0.42%</td>
<td>0.28%</td>
</tr>
</tbody>
</table>

3.6. Comparison with couple-stress finite element model

A number of modelling assumptions have been made in the foregoing sections, both to simplify the problem and to be able to use commercial software to predict a propagation direction and tip toughness. In particular we model the pre-existing microbuckle by a crack, and deduce the microbuckle propagation direction from a correlation direction ahead of the crack when the volumetric strain in the matrix just a little ahead of the crack returns to zero. In this section we use an alternative finite element formulation to examine the quality of these assumptions. In this alternative model, the pre-existing microbuckle is represented by continuous material rather than by a crack, and lock-up of matrix material is included. However limitations on mesh size, and consequent inaccuracies in calculating the traction across the pre-existing microbuckle prevent an accurate determination of the effective tip toughness of the microbuckle.

The alternative model uses the finite element code described by Fleck and Shu [3, 23]. Couple stresses associated with fibre bending are included in the analysis. As for the ABAQUS model, the composite response in shear is modelled by the Ramberg-Osgood behaviour (equation 1) while the fibre bending and axial response is linear elastic. Composite material properties used by the model are summarised in the first column of Table 1. The shear modulus of the fibres is also required in this calculation; the value shown in Table 1 is used. In these calculations plane stress conditions are assumed, and Poisson’s ratio effects are not included. Lock-up is included in this model by assuming that the material behaves in a linear elastic manner when the volumetric strain of the material is less than zero. The transverse modulus of locked-up material is taken to be the same as the transverse modulus of the composite.

The mesh used for these calculations is similar to that used for the layer model. Fig. 4(a). The height and width of the mesh are typically 2000 \(d\) and 1000 \(d\), respectively. However the pre-existing microbuckle is now modelled by a band of material containing initially misoriented fibres. Typically this band has a length of 400 \(d\), a height of 20 \(d\) and a peak fibre misorientation of \(10^\circ\). The band is inclined at an angle \(\beta_h\) to the fibre direction (cf. Fig. 4(a)). Fibres outside this region are initially straight. The composite is loaded by a uniform normal traction on the top and bottom boundaries of the mesh. Studies confirm that the direction of microbuckle propagation is similar in broad terms for the two loading types, with a similar propagation direction. However, the value of the applied load \(G/\gamma_i\) at which lock-up occurs is 100 for the \(K\)-field loading with \(\beta_0 = 25^\circ\) and \(\gamma_i = 9.2 \times 10^{-3}\). This is significantly larger than the corresponding value of 28 found using the stress loading in Section 3. With the \(K\)-field loading, significant crack opening behind the microbuckle tip occurs; alternatively, with remote stress-loading the microbuckle flanks are closed along their length. The associated modification to the geometry of the fibres at the microbuckle tip significantly increases the predicted tip toughness. Since stress-loading with a frictional shear stress equal to the shear yield stress is the stress field found in practice, the values of tip toughness using this loading arrangement are taken as representative; results presented elsewhere in this paper are based on this loading condition.

3.5. Fibre fracture

As the fibres rotate and bend, significant tensile strains are generated in the fibres (cf. Ref. [10]). Representative values for the peak fibre strains in the 10th fibre ahead of the microbuckle tip at lock-up are shown in Table 2, interpolated to the stable propagation direction as for the tip toughness estimates. If the fibre failure strain is less than this peak, then fibre fracture will occur. The experimental measurements of tip toughness in unidirectional material by Sivashanker and Fleck (private communication, 1995) suggest that fibre fracture has relatively little effect on the tip toughness. We note from Table 2 that the maximum tensile strain in the fibres increases from 0.35 to 6.5% in a monotonic fashion as the shear yield strain of the composite \(\gamma_i\) is increased from \(\gamma_i = 9.2 \times 10^{-3}\) to \(\gamma_i = 9.2 \times 10^{-2}\).

For the case of T800-924C carbon fibre-epoxy composite, with \(\gamma_i = 9.2 \times 10^{-2}\) and \(n = 3.5\), the tensile failure strain for the fibres is about 1.5%. The finite element calculation suggests a maximum fibre bending strain of 1.8%; we predict that microbuckling is accompanied by fibre fracture in agreement with experimental observation.
propagation does not change with an increase in mesh size or density.

As the load is first applied, the rotations in the pre-existing microbuckle increase, until this material locks up. The microbuckle then propagates forwards. Contours of fibre rotation $\phi$ for a composite with the representative material properties $\gamma_Y = 9.2 \times 10^{-3}$ and $n = 3.5$ are shown in Fig. 10, at two load steps 180 and 300. As the microbuckle propagates, the tip of the microbuckle moves forward and the region of locked-up material (i.e. $\phi = 40^\circ$) extends at an inclination $\beta$ of $20^\circ$. This propagation direction is insensitive to the orientation of the initial microbuckle, and is in good agreement with the corresponding value of $19^\circ$ estimated in Section 3.1 using the ABAQUS finite element code. A comparison between Fig. 10 and the corresponding Fig. 5(a) for the layer model show that the shape of the microbuckle tip is similar in both cases, but that the extent of the tip region is significantly greater for the couple-stress formulation.

Predictions of the propagation direction using this couple stress model are compared with the finite element layer model in Figs 8(a) and 9(a), showing the dependence on composite shear yield strain and strain hardening exponent. The predictions are broadly similar, confirming the quality of the assumptions made in Section 3.1 to estimate $\beta$ using the layer model. Differences probably arise at small values of $\beta$ from the sensitivity of the calculations to the details of the lock-up behaviour.

4. COMPARISON WITH TOUGHNESS ESTIMATES WITH AN INFINITE BAND MODEL

Shu and Fleck [24] have modelled microbuckling using an infinite band model, as illustrated in Fig. 1(a). They use the finite-element approach outlined in Section 3.6 above to include large rotations and calculate the collapse response as a function of remote applied stress $\sigma_0$. The deformation in the kink band is expressed in terms of the extra shortening $A\nu$ in the fibre direction associated with fibre rotation. (This shortening is calculated from the remote displacements $\Delta \nu$ by subtracting off the linear elastic response of the material above and below the microbuckle.) A typical collapse response using the calculation method described by Shu and Fleck is shown in Fig. 11 with a kink band orientation of $\beta = 19^\circ$. The remote stress is normalised by the yield strength of the material $\gamma_Y$ and the end-shortening is normalised by the fibre diameter $d$.

Following the J-integral argument presented by Rice [25] the tip toughness for a propagating microbuckle in a deformation theory elastic is determined by calculating the work done in taking the infinite band from the initial unbuckled state to the point of lock-up. The work done equals the area under the curve of applied axial stress vs extra displacement $A\nu$ shown in Fig. 11 until the point of volumetric lock-up. (This method of predicting the tip toughness differs from that used by Fleck [21]—see footnote on page 927.)

Values for the tip toughness from this infinite band prediction are included in Figs 8(b) and 9(b). In each case the value assumed for $\beta$ in the infinite band calculation is that derived from the corresponding full 2D calculation using the layer finite element model. There is good quantitative agreement between the infinite band predictions and the 2D finite element calculations of tip toughness for the full range of $n$ with the typical value of $\gamma_Y = 9.2 \times 10^{-3}$. At much smaller values for $\gamma_Y$ with $n = 3.5$, the infinite band prediction of tip toughness is too high by a factor of about two, see Fig. 8(b).

5. COMPARISON WITH EXPERIMENTAL MEASUREMENTS

Calculations presented in Section 3 have been made using material properties corresponding to the T800/924C carbon fibre epoxy composite studied experimentally by Sutcliffe and Fleck [11] and
on the crack faces is associated with the resistance to
this stress when resolved to a shear component.

This good agreement provides some justification for
the use of the plane stress analysis; calculations using
the model described in Section 3.6 predicts a larger region
of fibre rotation ahead of the microbuckle than found experimentally.

A comparison of Figs 2(a) and 5(a) show that the
defor­mation at the tip of the microbuckle predicted
by the layer finite element model is in good agreement
with observation. Material parameters for the finite
element model match that of the experimental
observations and the scale bar of 100 μm in Fig. 2(a)
corresponds to the scale bar of 20 μm given in Fig. 5(a).
This good agreement provides some justification for
the use of the plane stress analysis; calculations using
plane strain were not found to model well the
geometry at the microbuckle tip. The couple stress
model described in Section 3.6 predicts a larger region
of fibre rotation ahead of the microbuckle tip than found experimentally.

6. APPLICATION OF THE ANALYSIS
TO PROPAGATION OF AN OUT-OF-PLANE MICROBUCKLE

Sutcliffe and Fleck [11] observe that microbuckles
in a thin plate grow in an out-of-plane manner rather
than an in-plane manner, see Fig. 12. For the
out-of-plane microbuckle, the normal to the microbuckle plane makes an angle β to the normal of
the plane of the plate. They show that this can be
modelled as a mode I compressive crack, with a
compressive tip toughness G and a constant
compressive bridging traction σ, in a direction normal
to the faces of the crack, as illustrated in
Fig. 12 (cf. the mode II model for the in-plane
microbuckle, Fig. 3). The out-of-plane displacement
across the microbuckle flanks appears as an apparent
overlap of the crack. The normal bridging traction σn
on the crack faces is associated with the resistance to
sliding of the microbuckle faces. Experimental
measurements by Sutcliffe and Fleck [11] and
Sivashanker et al. [13] show that σn is effectively constant; when this stress is resolved to a shear
traction τb on the flanks of the microbuckle, a typical
value for the shear traction equals the shear yield
strength of the composite. Sutcliffe and Fleck [11]
show experimentally that the microbuckle orientation
β is the same for in-plane and out-of-plane
microbuckling in a carbon fibre-epoxy composite.
Estimates for the mode II toughness for in-plane
microbuckling and the mode I toughness for
out-of-plane are also the same. Hence the two
dimensional finite element calculations for the
microbuckle propagation direction β and the tip
toughness G for in-plane microbuckling in the
current study may be reinterpreted as the microbuckle orientation β and the mode I compressive
toughness for propagation of an out-of-plane
microbuckle. Similarly, the mode I bridging stress σt
across the flanks of the out-of-plane microbuckle can
be estimated in terms of the shear yield strength of
the composite as follows. The resolved shear component
of σt on the microbuckle flanks is given by
\[ τ_b = σ_t / 2 \sin 2β / \sin 2β. \]

7. CONCLUSIONS

The propagation of in-plane microbuckles in long
fibre composites has been investigated using a finite
element model. The tip of the microbuckle is
modelled using alternate layers of fibre and matrix
material, while the existing microbuckle is modelled
as a sliding crack, with sliding resisted by a shear
traction equal to the shear yield strength of the matrix.
The deformation pattern at the microbuckle tip is
found to agree well with experimental observations
for a carbon fibre-epoxy composite (T800/924C).

The microbuckle propagation direction for a wide
range of material properties has been predicted by
finding the direction of the inflection points in fibres
ahead of the microbuckle. Calculations predict that
the microbuckle propagates at an angle of between
about 5° and 30°. The predicted value of 19° is in good
agreement with the measured values of 20-30° for a
carbon fibre-epoxy composite. An alternative couple
stress finite element formulation is also presented, in
which the lock-up of matrix material is included and
the microbuckle is allowed to propagate a short
distance. Predictions of the microbuckle propagation
direction from this model are found to agree
reasonably with predictions using the layer model
over the range of material parameters considered,
with excellent agreement in the region corresponding
to polymer matrix composites.

A tip toughness is found by calculating the work
done in rotating fibres to a lock-up angle. Lock-up is
identified by finding when the volumetric strain in the
matrix material, which is positive in the early stage of
deformation, returns to zero. The predicted tip
toughness G/τc,d is relatively insensitive to the shear
yield strain of the composite, but decreases from 25
to 2.5 as the matrix hardening exponent n is increased.
from 3.5 to 100 with $\gamma_c = 9.2 \times 10^{-3}$. The predicted and experimental measurements of tip toughness $G_{\text{tip},d}$ for a carbon fibre-epoxy composite of 25 and 32-55 respectively are in reasonable agreement. Section 6 describes how the results calculated in this paper for in-plane microbuckle propagation can be applied to out-of-plane microbuckles.

Acknowledgements—The authors are grateful for helpful discussions with Dr J. Y. Shu and Prof P. T. Curtis and for financial support from the Procurement Executive of the Ministry of Defence, contract 2029/267, from the U.S. Office of Naval Research grant 0014-91-J-1916, and from the Nuffield Foundation. They would like to thank a reviewer for a number of helpful comments.

REFERENCES