EXPERIMENTAL MEASUREMENTS OF LOAD AND STRIP PROFILE IN THIN STRIP ROLLING

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Abstract—Experiments are described in which plasticine strips are rolled using elastomer rolls. Conditions cover the range from "thick strip" behaviour, in which roll elastic deformations are small, to "thin strip rolling", in which elastic deformations of the rolls are very significant. Results provide the first direct experimental confirmation of the thin strip rolling model proposed by Fleck et al. (Proc. Instn Mech. Engrs B 206 (1992) 119-131). Strip profiles clearly show a short region of reduction at the inlet to the bite and a central region which is relatively flat, in accord with the theory. The profiles do not however show a short region of reduction at the exit as predicted. For intermediate strip thicknesses the measured loads are in reasonable agreement with theory. For the thinnest strips, although the form of the dependence of load on reduction and inlet strip thickness is as predicted by theory, the measured loads are almost an order of magnitude lower than predicted. It is suggested that this is caused either by differences between the assumed rigid-perfectly plastic strip and the real constitutive behaviour of the plasticine, or by errors in treating the rolls as elastic half-spaces, an approximation which is accurate for industrial metal rolling, but is not good for the conditions of these experiments. © 1998 Elsevier Science Ltd. All rights reserved

Keywords: metal rolling, roll deformation, thin strip, foil, plasticine.

NOTATION

- a semi-contact width of contact
- \( b_0 \) (initial) semi-thickness of strip
- \( B_0 = b_0 E_e / Y \) dimensionless inlet semi-thickness
- \( B_0 = b_0 U^{-0.24} \) dimensionless inlet semi-thickness group
- \( E_e = E_e / (1 - \nu^2) \) plane strain Young's modulus of rolls
- \( F \) load between rolls
- \( L \) length of contact line between rolls
- \( m \) friction factor, strain-rate hardening exponent
- \( p \) normal pressure between rolls and strip
- \( P \) preload
- \( r \) overall reduction in strip thickness
- \( R \) roll radius
- \( U = \mu E_e / Y \) dimensionless friction
- \( W \) line load on strip
- \( W = W E_e^2 / Y \) dimensionless load
- \( W = W U^{-0.70} \) dimensionless load group
- \( x, y \) coordinates in the rolling and transverse directions
- \( Y \) plane strain yield stress
- \( \varepsilon \) true strain
- \( \mu \) coefficient of Coulomb friction
- \( \nu \) Poisson ratio
- \( \sigma \) true stress
- \( \sigma_f \) flow stress
- \( \tau \) frictional traction

1. INTRODUCTION

The need for higher quality and productivity and reduced wastage in the metal rolling industry has driven the development of increasingly sophisticated models of metal rolling, both for mill set-up and for on-line control. One important area in which these models can be improved is in the mechanics of the roll and strip profile in thin strip rolling. In traditional models of cold rolling the
roll shape is assumed to remain circular in profile, either with its original curvature [1] or with a radius of curvature slightly increased as a result of elastic deformation [2]. However, in thin strip, foil and temper rolling, the elastic deformation of the roll becomes significant compared with the plastic deformation of the strip and models based on a circular roll arc break down. Fleck and Johnson [3] overcome this deficiency using a "mattress model" of the roll. Fleck et al. [4] refine this approach by modelling the rolls as elastic half-spaces. In this analysis the strip is taken as elastic-perfectly plastic. The form of solution derived by Fleck et al. for the pressure distribution and the strip semi-thickness through the bite is illustrated in Fig. 1 for a reduction in strip thickness \( r \) of 50\%, as a function of an inlet strip semi-thickness group \( \hat{B}_0 \) (further defined in Section 3). For a large strip inlet thickness, Fig. 1(a), the roll profile remains nearly circular. The pressure profile is given by a friction hill, with a peak at the neutral point where the direction of slip reverses. As the inlet thickness decreases (b) the roll deformation becomes more noticeable. At the thinnest inlet gauges (c-f) the roll profile is markedly different from the circular assumption assumed by conventional theory. In these latter cases, there is a flat neutral zone of finite length where there is no reduction in strip thickness and no slip between roll and strip, separating two regions of reduction at the inlet and exit. The pressure profile is no longer a friction hill, but is approximately given by the Hertzian pressure distribution for contact between two elastic rollers, modified at the inlet and exit where the regions of reduction occur.

The model described by Fleck et al. has been supported by finite element calculations [5], and it has been extended to temper rolling by Domanti et al. [6] and Yuen et al. [7]. Early experimental evidence of non-circular roll deformation is provided by Orowan [8], who measured the profiles of brass and steel strips cold-rolled with steel rolls. In one test all the reduction in thickness was concentrated over the first 20\% of the contact. The remainder of the contact then showed some thickening in the central section and at the exit, separated by a small reduction region. Although this profile provides useful support for the model by Fleck et al., the results are somewhat inconclusive.

Fig. 1. Effect of strip thickness upon pressure distribution and deformed shape of the roll [4]. \( R = 89 \) mm, \( U = 30 \), \( r = 0.5 \): (a) \( \hat{B}_0 = 16.4 \), (b) \( \hat{B}_0 = 7 \), (c) \( \hat{B}_0 = 5.6 \), (d) \( \hat{B}_0 = 3.5 \), (e) \( \hat{B}_0 = 2.34 \), (f) \( \hat{B}_0 = 1.5 \).
and do not explore the thinnest foil regime. Moreover, there is insufficient information on the frictional conditions to make a direct comparison with the theoretical model.

Experimental verification in the thin foil regime has relied on the results of industrial mill trials [6, 7]. However, these comparisons require estimates of friction in situations where friction cannot be predicted with any confidence, leading to considerable uncertainty about interpretation of the data. It is the aim of this paper to provide an experimental verification of the thin strip model in a situation where friction is relatively well defined, by measuring both the load and the deformed shape of the strip as it is being rolled. This is achieved by rolling plasticine strip with elastomer rolls, lubricated with French chalk. Section 2 describes the details of the experimental setup. Section 3 describes the application of the model of Fleck et al. to the experimental conditions, and Section 4 makes a comparison between experimental results and predictions.

2. EXPERIMENTAL APPARATUS AND METHODS

2.1. Material choice

Elastomer rolls were used to roll plasticine strip, lubricated by a covering of French chalk. By using this material combination, two areas of experimental difficulty were overcome. Firstly an accurate measurement of the strip profile is required. The elastic deformations of metallic rolls are too small to be easily measured. However, deformations of elastomer rolls can be significantly larger, while still remaining within the elastic range, without making the rolls of unrealistic proportions. To prevent yield in the rolls, the yield strength of the workpiece material must be somewhat lower than that of the rolls. Plasticine has been used as the workpiece in many metal working simulations and is a suitable candidate here.

Secondly, to make a comparison between theory and experiments, it is necessary to estimate the frictional tractions in the bite. In thin strip rolling, lubrication is often in the "mixed regime" where friction cannot easily be predicted and where it also depends on speed [9, 10]. By using a material which can be rolled without hydrodynamic lubrication, the frictional tractions can be estimated relatively accurately and are insensitive to rolling speed. Hence it is possible to stop the rolls and examine the strip profile in the bite without substantially changing friction during slow-down.

2.2. Roll details and elastic modulus

Rolls were constructed by moulding a layer of 35 Shore-Hardness silicone rubber on a central steel shaft. Two pairs of rolls were made, with nominal radii 24 and 50 mm. They were ground to circularity to within 50 µm. Both sets of rolls had a steel core of radius 12.5 mm; the larger rolls had a solid core while the smaller rolls had a hollow core of wall thickness 2 mm. Simple beam bending calculations showed that deflections due to roll bending were negligible.

A key part of the rolling model is the roll elastic deformation. In metal rolling, the rolls are generally of steel, and the length of the contact arc is small compared with the roll radius. Hence the elastic deformations in the bite may be calculated using a linear elastic half-space approximation. In the experiments there are three complicating factors. Firstly silicone rubber behaves in a non-linear elastic fashion at the strains experienced in the tests. Secondly the region of contact is of the order of the roll radius, so that the semi-infinite half-space approximation is no longer accurate. Finally, the rubber is mounted on a steel shaft, so that calculations assuming a single material for the rolls may be in error. To overcome these difficulties, an effective Young’s modulus for the rolls was estimated by comparing the roll contact behaviour with Hertzian theory for a linear-elastic material. A roll was compressed between rigid flat platens in a tensile test machine and the variation of contact width 2a with line load W’ was measured. An effective plane strain Young’s modulus $E^*_{R}$\( (= E_R/(1 - v^2)) \) for the roll is determined by matching the measured semi-contact width a to that for an equivalent Hertzian contact [11]

$$ a = 2 \left( \frac{W' R}{\pi E_R^*} \right), $$

where $R$ is the roll radius. The deduced values of $E^*_{R}$ for the two sizes of roll are given in Fig. 2 as a function of line load $W'$. The estimates of effective Young’s modulus from measurements with the two sets of rolls are close at small loads. At larger loads the effective modulus rises. The major contribution to this stiffening is probably due to the presence of the central steel shaft. For example,
at a load of 20 kN m⁻¹, the ratio of the semi-contact length to the thickness of the rubber layer on the roll equals 0.91 and 0.61 for the small and large rolls, respectively. Poisson’s ratio for subsequent theoretical calculations was taken as a typical value for silicone elastomers of 0.47 [12]. Roll deflections are most significant when in the thin strip regime. For these conditions the pressure profile tends to Hertzian and the contact length in a rolling experiment and a platen test are similar at the same line load $W'$. Hence an effective value of roll modulus $E_k^*$ is estimated, for each plasticine strip, as the corresponding value of $E_k^*$ in the platen test, Fig. 2, at the measured rolling load $W'$.

2.3. Plasticine constitutive model
Plasticine is composed of a mixture of micro-wax (an oil by-product), natural resin and kaolin. Detailed measurements of the stress-strain behaviour of FILIA plasticine have been given in Ref. [13]. Flow stress was found to be strongly dependent on strain rate and temperature, and extensive creep and stress relaxation was exhibited at room temperature. Finer et al. [13] show that, at constant temperature, the strain and strain rate dependence of FILIA plasticine can be modelled by the expression

$$\sigma = K e^m t^n,$$  \hspace{1cm} (2)

where $K$ is a constant and $m$ and $n$ are strain-rate and strain hardening exponents, respectively. FILIA showed strain hardening behaviour for $e < 0.3$ and strain softening behaviour at higher strains. Since FILIA plasticine was not easily obtainable it was necessary to investigate differences in the response of the Harbutt plasticine that was used, and in particular to estimate a yield stress $Y$ for comparison with the rigid–perfectly plastic theory.

The plastic response of the Harbutt plasticine was first measured by compressing a cube, of initial side-length 60 mm, in a tensile testing machine at the maximum achievable cross head speed of 45 mm min⁻¹ and at the temperature used for the rolling experiments. Talc was applied between the faces of the cube and the rigid platens. Assuming plane stress conditions in the compression test with the von Mises yield criterion, and making a first order correction for friction [14], the flow stress $\sigma_Y$ in plane strain can be estimated from the applied true stress $\sigma$, as

$$\sigma_Y = \frac{2\sigma}{\sqrt{3(1 + \mu)}},$$  \hspace{1cm} (3)

where $\ell$ and $h$ are the length and height of the cube. The friction coefficient $\mu$ is taken as 0.35 (see Section 2.4). The variation of flow stress with true strain for a typical cube test is plotted in Fig. 3.

![Fig. 2. Variation of effective roll plane strain Young's modulus $E_k^*$ with line load $W'$.](image-url)
Significant strain hardening is observed at small strains. At a strain of 0.3, typical of the rolling experiments, the flow stress of the material approximately equals 95 kPa, although there was considerable scatter between tests of the order of ±50 kPa. At a strain of 0.3, the strain rate in the compression test is 0.017 s⁻¹ while a typical strain rate in the rolling experiments, estimated using the measured length of the deformation regions, is approximately 0.2 s⁻¹. Using the value of strain-rate exponent \( m = 0.17 \) appropriate to FILIA plasticine, this suggests an effective yield stress in the rolling experiments of 145 kPa.

An alternative estimate of the yield stress was made by fitting the measured rolling loads to the theoretical values for thick strip conditions, where roll deformation and frictional effects are small. This resulted in an estimate of plane strain yield strength \( Y \) of 215 kPa, although again there was significant scatter between tests. Since conditions in these thick strip tests are closer than those of the cube compression test to the conditions in thin strip rolling, this estimate of yield strength was considered to be more reliable than the cube data and a value of \( Y \) equal to 215 kPa is used in subsequent analyses. Although the effective yield stress can be expected to vary with rolling conditions (e.g. reduction and strip thickness), given the uncertainty in the estimate of the yield strength, no correction was made for this.

2.4. Friction estimate

French chalk was used as a lubricant in the rolling experiments to prevent sticking between the rolls and the strip. Because this is a solid lubricant, we can expect no dependence of frictional traction on sliding speed, although there may be some variation of friction through the bite as new surface is created and the layer of chalk is thinned out. In most of the calculations we assume a constant coefficient of Coulomb friction \( \mu \), relating the local frictional traction \( \tau \) to the local pressure \( p \) by

\[
\tau = \mu p.
\]  

To estimate the friction coefficient, the ring test method was used [15]. In this test a toroidal specimen of the plasticine of rectangular cross-section is compressed between flat steel platens. A 5 mm layer of the elastomer used for the rolls was placed between the platens and workpiece to match the interfacial conditions during rolling. The rubber layer and the specimen were lubricated as for the rolling experiments. The frictional traction at the interface is estimated from the change in geometry of the specimen as it is compressed. In particular the ratios of the internal diameters \( D_i/D_o \) and the heights \( h/h_0 \) of the specimen before and after the test are used to estimate the frictional traction.
coefficient, using calibration curves given in Ref. [15]. This test has the advantages that it is straightforward, that the tribology in the test is similar to that during rolling, and that estimates of friction are relatively insensitive to errors in the measurements of the geometrical changes. The initial internal and external diameters of the ring were 40 and 20 mm, and the initial height was 14 mm. The coefficient of friction was found to be 0.35. Scatters in ten tests suggested an accuracy in \( \mu \) of \( \pm 0.03 \).

2.5. Strip preparation

Specimens were prepared by rolling a strip of plasticine repeatedly at an elevated temperature of 40°C to remove air pockets. To produce a uniform thickness, the strips were rolled on the final pass between steel rolls, lubricated with a little talc. Finally, the strips were trimmed to a length of 200 mm and a constant width \( w \), typically of 50–100 mm. The strips had a uniform thickness across the strip, to within a tolerance of 0.1 mm.

2.6. Rolling test method

Rolling was performed using a two-high mill driven by a variable speed motor. Load was applied via screws on either side of the roll and the rolls were geared together to drive at the same speed. Prior to rolling the rolls were lubricated with a covering of French chalk. Since the thin strip regime is of interest, many of the tests were performed in the closed gap regime, where there is contact between the rolls outside the strip width during rolling. In this case a preload was applied to the rolls. Care was taken to ensure that the rolls were parallel by checking that the centre to centre distances of the rolls at each end of the roll width were the same, and that the loads in the two load screws were equal. For both roll sizes the peripheral speed of the rolls was 0.008 m s\(^{-1}\). During rolling the ambient and roll temperatures were between 20.5 and 21.5°C, measured with a small electronic probe thermometer. When only a part of the strip, typically of length 50 mm, had emerged from the exit of the bite, the rolls were stopped and the load then removed. While plasticine has been used with good success to simulate cold metal forming, it does exhibit time-dependent creep at room temperature. To limit any strip deformation after the rolls had been stopped, the load screws were undone relatively quickly using an air tool, taking between 5 and 10 s to release the load. Finally the inlet and exit strip thicknesses were measured along the centreline of the strip. In total 28 tests were performed to cover the range of rolling behaviour. Further details of the experimental method and tabulated test data are included in Ref. [16].

2.7. Estimate of load on the strip

Load cells were fitted between the load screws and the chocks at each end of the upper roll. These cells were calibrated in a standard tensile-testing machine. During rolling the load in each load cell was noted to give the total rolling load \( F \). In open gap rolling, where there is no contact between the rolls outside the strip width, the line load on the strip \( W' \) is simply given by

\[
W' = \frac{F}{w},
\]

where \( w \) is the strip width. In the closed gap regime, the contribution due to the contact between the rolls outside the strip width must be accounted for. As the rolls began to rotate (before feeding in the strip) the applied preload dropped by about 5%. The value after this settling occurred was taken as the effective preload \( P \). To estimate the line load \( W' \) on the strip, it is assumed that the line load outside the contact width during rolling of the strip is the same as the line load without the strip present. Hence the load taken by the contact outside the strip width during rolling is equal to \( P(L - w)/L \), where \( L \) is the length of contact between the rolls, and the line load \( W' \) on the strip is given by

\[
W' = \frac{F}{w} - \frac{P(L - w)Lw}{L}.
\]

This approximation ignores the small effects due to mill stretch and edge effects at the edge of the strip.

2.8. Measurement of strip profiles

An important aim of the project was to measure the strip profile in the bite. Direct contact measurements of the strip profile were not feasible at room temperature due to the softness of the plasticine. Initial trials using liquid nitrogen to increase the hardness of the strip were unsuccessful
due to the tendency of the plasticine to shatter. Instead a small sliver was removed from the strip, after first cooling the strip in a freezer to increase its hardness and so prevent burring. This section was then mounted on a toolmaker’s microscope, and its profile measured optically. Accuracy was estimated to be within 20 μm both in the thickness and rolling directions.

3. PRESENTATION OF THEORETICAL RESULTS

Fleck et al. show that the dimensionless roll load \( \bar{W} = W' E'_{k} / R Y^{2} \) and strip semi-thickness \( b \) are given by the functional form

\[
(\bar{W}, b/b_{0}) = f(B_{0}, U, r)
\]

where elastic deformations in the strip can be neglected and no end tensions are applied to the strip, \( b_{0} \) is the initial strip semi-thickness, \( B_{0} = (b_{0} E'_{k} / R Y^{2}) \) is the dimensionless inlet strip semi-thickness, \( U = (\mu E'_{k}) / Y \) is the dimensionless friction and \( r \) is the overall reduction in strip thickness. To present results they further define a dimensionless load group \( \bar{W} = (\bar{W} U^{-0.70} r^{-0.70}) \) and a dimensionless semi-thickness group \( \bar{B}_{0} = (B_{0} U^{-1.24} r^{-0.50}) \).

The friction group \( U \) varies between 3 and 8 in the experiments described in this paper, because of the change in effective roll modulus with load. The computer program described in Ref. [4] has been used to calculate theoretical results for a typical value of \( U = 5.5 \). The strip was taken as rigid-perfectly plastic. However, a contact region between the rigid strip and the elastic rolls was included at the exit of the bite to allow the pressure to drop off smoothly. Results are presented in Fig. 4 for strip thickness reductions of 0.3 and 0.5 using the load and semi-thickness groups \( \bar{W} \) and \( \bar{B}_{0} \) described above. Calculations using the circular arc model of Bland and Ford with Hitchcock flattening are also presented in this figure [3, 4] using the roll and strip material properties appropriate to the model experiments with \( U = 5.5 \). \( B_{0} \) characterises the importance of roll flattening. For large \( B_{0} \) there is little roll elastic deformation. For \( B_{0} \) less than about 8, theory predicts a central flat region and the circular arc model of Bland and Ford is significantly in error.

Although the results of Fleck et al. were presented as appropriate to aluminium foil rolling, their use of dimensionless groups means that their calculations are also applicable to the present situation where values of yield stress and roll elastic modulus are three orders of magnitude lower. Fleck et al. performed calculations for values of \( U \) between 15 and 45 and for reductions \( r \) between 10 and 50%. When plotted using the load and semi-thickness groups \( \bar{W} \) and \( \bar{B}_{0} \) of Fig. 4 all their results fell on a single “master curve” to within an accuracy of about ± 10%. Their master curve lies on the curve shown on Fig. 4 for \( U = 5.5 \) and \( r = 0.5 \) to within this same accuracy of 10%.

![Graph showing variation of dimensionless load group \( \bar{W} \) with inlet semi-thickness group \( \bar{B}_{0} \).](image)

Fig. 4. Variation of dimensionless load group \( \bar{W} \) with inlet semi-thickness group \( \bar{B}_{0} \).
In the above calculations of load, the standard parabolic approximation to the undeformed roll shape is used, i.e.

\[ y = x^2/2R, \]

where \( x \) and \( y \) are coordinates of the roll position with an origin on the line joining the centres of the rolls. For some calculations described in Section 5.4 a circular profile is used for the undeformed roll,

\[ y = R - \sqrt{R^2 - x^2}. \]

4. EXPERIMENTAL RESULTS

Experiments were performed using rolls of radius \( R = 24 \) and \( 50 \) mm. The inlet strip semi-thicknesses were between 2 and 14 mm and the strip reduction was varied between 0.08 and 0.61. Experiments covered the range of behaviour from thick strip, in which elastic deformations were small, to thin strip, in which elastic deformation of the roll greatly affects the rolling behaviour. Two areas of comparison between theory and experiments can be made, in the rolling load and in the deformed strip profile in the bite.

4.1. Rolling load

Figure 4 compares the experimental load group \( \bar{W} \) with theory. The friction group \( U \) lies between 3 and 8 in the experiments. The value of yield stress \( Y \) has been chosen so that theory and experiments fit for the four data points in the thick strip regime with the inlet strip semi-thickness group \( B_0 \) is greater than 9. The conventional theory of Bland and Ford [3, 4] would predict a limiting gauge (where the load group \( \bar{W} \) tends to infinity) for \( B_0 \) around 5. This is not observed. There appears to be no significant difference between results for the two sizes of rolls and specimens with reductions between 8 and 61\% have been well grouped using the variables \( B_0 \) and \( \bar{W} \) proposed by Fleck et al. The one data point significantly above the trend, with \( \bar{W} \approx 40 \), corresponds to the most heavily loaded case with the small roll. For this data point the ratio of the Hertzian semi-contact width to the rubber layer thickness equals 0.95 and the stiffening effect of the steel core has probably increased the load. Scatter in the data will arise due to differences in temperature and friction between tests and to inaccuracies in the measurements of inlet and exit thickness.

The experiments are in reasonable agreement with theory for \( B_0 \) greater than about 5. For \( B_0 \) less than this, experimental loads are significantly below those predicted by theory. A number of factors were investigated to understand possible causes for this discrepancy, as discussed in section 5 below.

4.2. Deformed strip shape

Measurements of the deformed strip shape for six specimens are given in Figs 5–7.* Figures 5 and 6(c) show strips rolled with the 24 mm radius rolls, while the other specimens are rolled with the 50 mm radius rolls. For the thick strip case \( B_0 = 15 \), Fig. 5, the strip profile is close to the undeformed circular roll profile. Some thickening appears to have occurred near the exit. At an intermediate strip thickness \( B_0 \approx 3 \), Fig. 6, there is a sharp region of reduction at the inlet. The thickness changes relatively little through the rest of the contact, in some cases showing significant thickening either in the centre or at the end of the contact. This profile is strikingly similar to that measured by Orowan [8] for the cold-rolling of brass. Profiles for the thinnest strip with \( B_0 \approx 1.2 \), Fig. 7, are similar to those with \( B_0 \approx 3 \), Fig. 6, but the inlet reduction region is even shorter, there is a better defined central flat region and a gentle region of plastic reduction at the exit. Again there is some thickening at the exit.

Theoretical pressure distributions and strip profiles for each case have been calculated, matching the contact length between theory and experiments. This effectively matches the load, so that differences between theory and experiments give rise to differences in the reduction through the bite. In general the standard approximation of a parabolic shape for the undeformed roll [Eqn (8)] has been used, but some calculations are included in which the initial roll shape was taken as circular,

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*The values of inlet semi-thickness and reduction quoted in the figure captions are taken from independent measurements on the strip away from the contact and so differ slightly from values suggested by the profiles.
Eqn (9). Theoretical strip profiles and typical theoretical pressure distributions are included in Figs 5–7. As $\Phi_0$ decreases, the theoretical pressure distribution changes from a friction hill to a Hertzian profile. Note that there is a significant region of contact beyond the exit reduction region in all cases. For the smallest strip thicknesses, Fig. 7, calculations using the parabolic roll profile suggested that there would be no reduction for the measured load. However, predictions using the circular arc did predict a reduction for one of these cases, Fig. 7(a). A comparison of Fig. 6(c) with Figs 6(a) and (b), which have similar values of $\Phi_0$ but are for rolls of radius 24 and 50 mm, respectively, show that the form of the strip profile does not depend on roll radius.

5. DISCUSSION

A comparison of the loads and strip shapes has confirmed the general picture of thin strip rolling predicted by Fleck et al. No limiting gauge is observed and the picture of a short region of reduction in the inlet is clearly observed. The strip profiles also show a central region which is relatively flat, in accord with the theory. However, the profiles do not have the short region of reduction at the exit which is predicted by theory. Although the form of dependence of load on friction, reduction and inlet strip thickness parameters is as predicted by theory, the measured loads are significantly lower than predicted for the thinnest strips. A number of factors are discussed below which may lead to differences between theory and experiments. To understand the discrepancies observed at the thinnest gauges, those factors which have a greater impact on the thin strips are highlighted.

5.1. Elastic model of the roll

In many cases, particularly at the higher loads, the length of the contact between rolls and strip is approximately equal to the roll radius (see Figs 6 and 7). Clearly, the theoretical assumption that the roll can be modelled as an elastic half-space is no longer good. It is beyond the scope of this paper to derive an accurate model of the roll elastic response. However, the sensitivity of the calculations to

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1 Careful numerical investigation suggested that the length of the exit reduction region tended to zero for this solution. The numerical procedure described by Fleck et al. has been adapted to allow for solutions with no exit reduction region.
Fig. 6. Intermediate strip thickness results. (a) (i) Theoretical pressure profile, (ii) Strip profile, $B_0 = 3.0$, \( W = 24, r = 0.47, U = 4.7, b_0 = 3.85 \text{ mm}, R = 50 \text{ mm}, W' = 33 \text{ kN m}^{-1} \) (b) Strip profile, $B_0 = 2.9, W = 31$, \( r = 0.39, U = 4.9, b_0 = 3.35 \text{ mm}, R = 50 \text{ mm}, W' = 37 \text{ kN m}^{-1} \) (c) Strip profile, $B_0 = 3.1, W = 19, r = 0.39$, \( U = 5.2, b_0 = 1.6 \text{ mm}, R = 24 \text{ mm}, W' = 11 \text{ kN m}^{-1} \).

differences in the roll behaviour was investigated by performing some calculations in which the undeformed roll shape was taken to be either circular or parabolic. Figs 6(a)(ii) and 7(a)(ii) show substantial differences in the predicted strip reduction for these two roll shapes (recall that use of the parabolic profile leads to a prediction of zero reduction for Fig. 7), despite the actual differences in profile being slight. Comparable changes in the separation $y$ of the undeformed rolls at the inlet to the bite [Eqns (8) and (9)], achieved by changes in roll radius with a constant parabolic roll shape,
Fig. 7. Thin strip results. (a) (i) Theoretical pressure profile and (ii) Strip profile. $B_0 = 1.2, \bar{W} = 32, r = 0.30$, $U = 4.7, b_0 = 1.25$ mm, $R = 50$ mm, $W' = 32$ kN m$^{-1}$. (b) Strip profile, $B_0 = 1.4, W = 27, r = 0.14, U = 3.9$, $b_0 = 1.05$ mm, $R = 50$ mm, $W' = 17$ kN m$^{-1}$.

make a relatively insignificant change to the theoretical master curve predictions of Fig. 4, changing the values of $B_0$ and $\bar{W}$ by around 10%. Although it is not suggested that the calculations using a circular shape are significantly more accurate than those with a parabolic shape, this calculation serves to illustrate the sensitivity to changes in the details at the edges of the contact. As $B_0$ decreases, the behaviour of the contact is increasingly dominated by these details at the edge of the contact (cf. Figs 6(a)(i) and 7(a)(i), in a way similar to that observed for elastohydrodynamic lubrication [17].

5.2. Time-dependent flow

Creep deformation of the strip may have occurred during the 5–10 s while the rolls were being separated. An elastic-plastic model of the workpiece will not predict the increase in thickness through the bite which is observed in the unloaded strip. However creep flow during unloading would be expected to lead to indentation-type deformation of the relatively unconstrained regions near the edges of the contact. This is consistent with the thickening observed in the measured profiles. Similar behaviour is observed by Orowan in his experiments with cold-rolled brass strips.

5.3. Changes in frictional traction and yield stress

Arrows included in Fig. 4 show the shift in the position of experimental points that would occur due to a reduction in estimates of the friction coefficient $\mu$ or yield stress $Y$ of 25%. The sensitivity to
these factors suggest that slight differences in \( \mu \) or \( Y \) could account for scatter in Fig. 4, but not the large differences between measured and theoretical loads at the smallest values of \( \Phi_0 \).

The assumed rigid–perfectly plastic behaviour of the strip is a relatively crude approximation to the actual flow behaviour, which includes strain hardening and time-dependent flow. In addition there is significant uncertainty about the actual value of the yield stress. Since the results of the elastic flattening calculation (Section 5.2) show that small differences in roll shape at the inlet and exit can have a significant effect for small values of \( \Phi_0 \), slight changes in flow behaviour through the bite could also lead to the significant discrepancy observed between theory and experiments at small \( \Phi_0 \).

In the theoretical calculations, the frictional traction \( \tau \) is taken equal to \( \mu \nu \) in the regions of plastic reduction. For the conditions of most of the experiments, this implies that \( \tau \) exceeds the shear yield strength \( Y/2 \) of the material. To overcome this deficiency, friction is frequently modelled using the expression

\[
\tau = mY/2,
\]

where \( m \) is termed the friction factor. The computer program of Fleck et al. was modified to use this friction factor model. Careful numerical calculations using this model suggested that the exit reduction region vanished for \( \Phi_0 \) less than about 5. The load predicted using the friction factor model with \( m = 0.7 \) is compared in Fig. 4 with Coulomb friction calculations for \( \mu = 0.35 \) and \( U = 5.5 \). The effect on the strip profile is illustrated for a typical intermediate strip thickness case in Fig. 6(a). Figs 4 and 6 show that use of the friction factor model can reconcile the relatively small difference between theory and experiments results at intermediate values of \( \Phi_0 \) around 3. However, at the smallest values of \( \Phi_0 \), the friction factor model does not predict the large knock-down factors on load observed experimentally, suggesting that differences in the frictional tractions in the bite are not the cause of the large differences between theory and experiments.

5.4. Elastic behaviour of the strip

Although it is likely that elastic behaviour of the strip has some influence on the mechanics in the bite, particularly for small reductions, it is assumed in the theory used in this paper that the strip is rigid. Inclusion of elastic deformation in the strip is likely to have the maximum impact at the exit of the contact, where there is a large elastic region of unloading and at the pressure spike, where small changes in roll shape are associated with large changes in roll pressure.

5.5. Inhomogeneous deformation

In the theoretical analysis it is assumed that deformation is homogeneous. Since the theoretical extent of the pressure spike is considerably less than the thickness of the strip in many of the calculations (cf. Figs 6 and 7), the assumption of homogeneous deformation is poor at this point. This may partly explain why the change in strip thickness in the exit region is gentler than predicted by theory. Effects of inhomogeneous deformation away from the pressure spike are considered to be a relatively minor.

6. CONCLUSIONS

Experimental rolling of plasticine strips with elastomer rolls is described. Conditions cover the range of behaviour from thick strip, in which roll elastic deformations are small, to thin strip, in which elastic deformation of the rolls greatly affects the behaviour. Results support the measurements of Orowan of a central region with no reduction and provide experimental confirmation of the thin strip rolling model proposed by Fleck et al. [4]. Strip profiles clearly show a short region of reduction in the inlet to the bite and a central region which is relatively flat, in accord with the theory. The profiles do not however show a short region of reduction at the exit as predicted. For intermediate strip thicknesses the measured loads are in reasonable agreement with theory. For the thinnest strips, although the form of the dependence of load on reduction and inlet strip thickness is as predicted by theory, the measured loads are almost an order of magnitude lower than predicted. Although in industrial rolling it will be appropriate to model the rolls as elastic half-spaces, this is not an accurate model for the conditions of these experiments. It is suggested that either this half-space approximation or errors in using a rigid perfectly plastic constitutive model for the plasticine cause the main differences between theory and experiments at the thinnest gauges.
Time-dependent creep of the plasticine during unloading and inhomogeneous deformation near the pressure spike may also affect the behaviour at the exit. Use of a friction factor model is shown to lead to a relatively minor knock-down in theoretical load, and so cannot explain the differences between theory and experiments found for the thinnest strips.

In the experiments presented here, a constant coefficient of friction of 0.35 has been used in the theoretical modelling, consistent with the French chalk lubricant used. In industrial metal rolling it is generally necessary to have a lower friction coefficient to reduce rolling loads and to limit scuffing or metal pick-up on the rolls. This is achieved by running the mill under mixed lubrication conditions, where there is some hydrodynamic film action generated by the lubricant. Since it is not possible to estimate the friction accurately in these circumstances [10], measurements using industrial plant cannot differentiate between errors in the friction model and errors in the model of roll deformation. By confirming the main points of the roll deformation model of Fleck et al. in a situation where friction is relatively well defined, the industrial problem with interaction between roll deformation and frictional factors can now be approached with more confidence. However, it is clear that the load under conditions of high roll deformation is sensitive to the details at the inlet and exit, so that it will be necessary to check mill data carefully for consistency, for example using measures of forward slip as well as load and torque.

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