This paper addresses geometric variability of woven textiles used in composites manufacturing. Variability in tow directions and unit cell size is quantified by applying an image analysis procedure to two representative materials; a pre-impregnated carbon/epoxy satin weave textile and a commingled glass/polypropylene fabric, and the spatial autocorrelation of stochastic variables is characterized. It is found that variability in tow orientations is significant in both the pre-impregnated material and the fabric, whereas variability in the unit cell size is significant only in the commingled fabric. Variability in the weft directions is more significant than in the warp direction. Highly anisotropic spatial autocorrelation of tow orientations is observed in both materials with the major direction of autocorrelation normal to the corresponding set of tows. The correlation structures identified are decomposed using Cholesky factorization and Monte Carlo simulation of a stochastic textile is performed. This enables stochastic simulations of forming to be carried out based on a simplified finite element model of woven material draping. The results of these simulations show that variability in the woven material geometry induces significant variations in the formed geometry.

Nomenclature

Ao = amplitude parameter of the autocorrelation model
B = intercept parameter of the autocorrelation model
C = Covariance matrix
d = decay parameter of the autocorrelation model
g = textile field
h = textile repetition pattern
L = Cholesky root of the covariance matrix
l0 = nominal unit cell size
lwarp = unit cell size in the warp direction
lweft = unit cell size in the weft direction
W = Vector of independent normally distributed variables
u = textile elementary unit
Z = Vector of correlated normally distributed variables
θwarp = warp direction
θweft = weft direction
µ = average
ρ = correlation
σ = standard deviation
τx = convolution lag in the x direction
τy = convolution lag in the y direction
ϕ = phase parameter of the autocorrelation model

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I. Introduction

Manufacturing, storage and handling of textiles introduce variations in fiber architecture. These variations influence all stages of the manufacturing of a composite component and the properties of the final product. Local variations in textile architecture may affect the outcome of the draping/forming stage and introduce variations in hydraulic and thermal properties that can be of significance in subsequent manufacturing steps such as impregnation/consolidation and curing/cooling. Variability in the textile material propagates to the final product causing local disturbances in the fiber architecture which in turn affect the local mechanical properties. The extent and spatial distribution of these variations determines their significance with respect to the expected properties used in component design.

Process modeling of continuous fiber reinforced composite materials is carried out under the assumption that properties and geometrical features of the material are fully deterministic. In this framework, the tows of a woven textile are always assumed to be orthogonal, and tow widths and spaces constant and equal to a nominal value. This assumption is carried forward to all process models and, ultimately, to the final composite component performance evaluation.

Recently Long et al. [1, 2] examined the variability of non-crimp fabrics and its effects on draping and the filling stage of liquid composite molding by assuming that geometrical variables of the textile in the case of draping simulations are spatially uncorrelated and normally distributed. These investigations indicated that variability affects significantly the outcome of both processes.

It is expected that variations are not distributed uniformly over a region of textile material, as fiber continuation and friction at tow crossovers in the case of woven textiles, force disturbances to spread to neighboring locations. Consequently, a realistic stochastic simulation of these materials requires information on the spatial autocorrelation structure of stochastic variables in addition to their global variance.

The present work addresses the full characterization of stochastic properties of dry and pre-impregnated woven textiles. The stochastic variables considered include the fiber tow angles and directions and the size of the unit cell of the woven fabric in the two tow directions. The spatial structure of correlation is investigated and a stochastic simulation procedure that allows incorporation of autocorrelation and cross-correlation is implemented. This simulation provides the input required for subsequent stochastic simulation of forming.

II. Image Analysis

The image analysis methodology used here was developed to enable estimation of tow directions and unit cell lengths in both tow directions to be made, assuming approximate a priori knowledge of the size of the unit cell of the textile. Fourier transform of a linear periodic image yields a two-dimensional spectrum with directional structure with maxima in the direction normal to the orientation of the image. This fact has been used in image analysis of woven textiles to identify the direction of tows by adding the power of pixels along radial directions [3, 4]. Although this procedure is computationally very efficient, its accuracy is low when a local determination based on the information of a limited number of unit cells of the periodic structure is required. In order to improve accuracy, the results of the Fourier transform can be enhanced using a correlation search.

The autocorrelation of an image, i.e. the correlation of a mask to areas of equal size centered at every point of the image, can uncover repeated patterns and yield accurate local quantitative information. The periodic structure of a woven textile $g(x, y)$ can be described as a convolution of an elementary unit $u(x, y)$ (unit cell) by a pattern of repetition $h(x, y)$. The structure of the woven material is represented as a two-dimensional array of image pixel values and the convolution is expressed as follows:

$$g(x, y) = u(x, y) \otimes h(x, y)$$

$$= \sum_{\tau_x=0}^{\infty} \sum_{\tau_y=0}^{\infty} u(\tau_x, \tau_y) h(x-\tau_x, y-\tau_y)$$

(1)

Correlation search uncovers the locations where the pattern of repetition in Eq. (1) maximizes and it can be used locally to determine the positions of four unit cells adjacent to the central region of each image in the direction of the tows. This procedure is computationally expensive. However, prior knowledge of the nominal size of the unit cell can be used, combined with approximate tow directions obtained from the Fourier transform, to limit the range of the search and to allow the execution of this procedure in a fraction of the time required for a full search.

Image analysis is performed in five steps as follows:
- Fast Fourier transform (FFT) of the image brightness field to derive a two-dimensional power spectrum.
- Calculation of radial energy of the spectrum as a function of direction.
- Estimation of the two directions of maximum radial energy.
- Calculation of approximate unit cell positions using the maxima of radial energy and a nominal cell size.
- Estimation of unit cell positions using correlation search in the vicinity of the approximate cell positions.
- Calculation of tow directions, cell angle and cell lengths from the cell positions.

The image analysis methodology was applied to images of two woven textiles, a five harness satin weave 6k carbon/epoxy pre-preg (Hexcel) and a twill 2/2 woven commingled E-glass polypropylene fabric (Vetrotex-Twintex). Figs. 1 and 2 show images of the two textiles with the corresponding FFT spectra and autocorrelation distributions. The results of the image analysis procedure, illustrated as the four lines starting from the centre of each image and ending at the estimated location of the adjacent unit cells, are superimposed on the fabric images. The nominal size of the unit cells is 10 mm for the carbon/epoxy pre-preg and 20 mm for the commingled E-glass polypropylene fabric. It should be noted that the FFT spectra are rotated by 90°, in order to illustrate the coincidence of the maximum radial energy directions with the direction of tows.

The spectra obtained by the Fourier transform are sharp in the case of the carbon/epoxy prepreg. The frequency spectrum of the images of commingled glass/polypropylene textile is broad due to the degree of fiber misalignment in the tows of the fabric. The autocorrelation estimation results in very sharp peaks in the cases of both materials. This allows accurate estimation of the unit cell positions.

Digital images of the two materials were acquired using an Olympus digital camera. The material was set on the table of an Omicron coordinate measurement system, while the camera was mounted on its measuring head in order to record the exact position of each image. The images were analyzed using an implementation of the analysis procedure outlined here on VB.NET. The code loads images sequentially, while the user is able to accept or reject the results of the analysis. The sample size was 410 images for the carbon/epoxy pre-preg and 420 images for the glass/polypropylene fabric. The average distance of images was approximately 10 mm for the carbon/epoxy pre-preg and 20 mm for the glass/polypropylene fabric.
III. Statistical Properties of the Unit Cell

The unit cell angle varies in a range of about 5° for the carbon/epoxy textile and 20° for the glass/polypropylene fabric. The distribution of the angle over the sampling region is random, but existence of long scale trends can be observed. The trend is expected to have an effect on the results of subsequent analysis of the statistical properties of the measured variable.

The basic statistical properties of the unit cell parameters of the carbon/epoxy textile are summarized in Table 1, and the correlation matrix is given in Table 2. The corresponding distributions are illustrated in Fig. 3. Here \( \theta_{\text{warp}} \) and \( \theta_{\text{weft}} \) denote the warp and weft directions. Angles are measured in the clockwise direction relative to the nominal warp direction of the material. \( l_{\text{warp}} \) and \( l_{\text{weft}} \) are the unit cell lengths in the two tow directions, whereas \( l_0 \) is the nominal unit cell length. Unit cell lengths are presented as percentage ratios with respect to nominal lengths. The statistical properties reported include the average \( \mu \) and the standard deviation \( \sigma \) of the sample. Also, the single measurement standard deviation \( \sigma_{\text{repeat}} \) calculated from the analysis of 20 images of the same location of the textile, and the standard deviation due to uncertainty \( \sigma_{\text{uncertainty}} \) calculated as the error propagated from the uncertainty arising from the finite resolution of the images.

It can be observed that there exists significant variability in the cell angle and the tow directions. The variability of the orientation of warp tows appears significantly lower than the variability of the weft direction. The variability of unit cell lengths is negligible as the sample standard deviation is of similar magnitude to the repeatability tests for this material. Consequently, unit cell lengths can be considered deterministic.

The correlation matrix (Table 2) indicates that the two tow directions are uncorrelated which implies that variability is generated independently in the two directions. Also, the two unit cell lengths are uncorrelated, and tow directions are uncorrelated to lengths. This result is mainly due to the insignificance of the variability of unit cell lengths for the carbon/epoxy pre-preg. Examination of the distributions of cell angle, tow directions and cell lengths shown in Fig. 3 suggests that all variables are normally distributed. The value of warp direction ranges from -1° to 1° and of weft direction from 85° to 92°. The range of values for the normalized cell length is about 5% in the warp and weft directions, with these deviations being of similar magnitude to the repeatability of the image acquisition and analysis procedure.

Table 3 summarizes the statistical properties of the glass/polypropylene fabric. All standard deviations are significantly higher than the repeatability and the uncertainty standard deviation. In general, this dry fabric has significantly higher variability than the woven pre-preg tested and, as a consequence, all variables examined can be considered as stochastic. Orientation variability in the weft direction is higher than in the warp direction, whereas unit cell length variabilities in the two directions are of similar magnitude.
The correlation matrix of the glass/polypropylene material is given in Table 4. Correlation coefficients are low indicating that the two tow directions are independent and variability in the tow orientation is uncorrelated to variability in the unit cell length.

Table 4. Correlation matrix of the unit cell parameters of commingled E-glass polypropylene fabric.

<table>
<thead>
<tr>
<th></th>
<th>( \theta_{\text{warp}} (^\circ) )</th>
<th>( \theta_{\text{weft}} (^\circ) )</th>
<th>( l_{\text{warp}}/l_0 (%) )</th>
<th>( l_{\text{weft}}/l_0 (%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{\text{warp}} (^\circ) )</td>
<td>1.00</td>
<td>0.03</td>
<td>0.16</td>
<td>0.01</td>
</tr>
<tr>
<td>( \theta_{\text{weft}} (^\circ) )</td>
<td>-0.03</td>
<td>1.00</td>
<td>0.08</td>
<td>-0.11</td>
</tr>
<tr>
<td>( l_{\text{warp}}/l_0 (%) )</td>
<td>0.16</td>
<td>0.08</td>
<td>1.00</td>
<td>-0.03</td>
</tr>
<tr>
<td>( l_{\text{weft}}/l_0 (%) )</td>
<td>0.01</td>
<td>-0.11</td>
<td>-0.03</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Figure 3. Distributions of the unit cell variables of carbon/epoxy pre-preg; a) Warp direction; b) Weft direction; c) Unit cell length in the warp direction; d) Unit cell length in the weft direction.

Figure 4. Distributions of the unit cell variables of commingled E-glass polypropylene fabric; a) Warp direction; b) Weft direction; c) Unit cell length in the warp direction; d) Unit cell length in the weft direction.
The distributions of the cell lengths and the tow directions for the glass/polypropylene fabric are illustrated in Fig. 4. All variables follow a normal distribution. The value of the direction of warp tows ranges from -5° to 5° and of the direction of warp tows from 80° to 97°. The range of values for the normalized cell length is about 17% in the warp direction and 20% in the weft direction.

IV. Autocorrelation Structure

The analysis of statistical properties indicated that the cross correlation between different stochastic variables of the textile is negligible allowing for independent simulation of the underlying factors. However, the autocorrelation structure of the variables of interest should be investigated in order to quantify the spatial dependence of variability. Spatial autocorrelation is calculated as a function of distance and orientation by computing the correlation between two samples of all pairs of points in the original sample that are located at a specific distance and orientation and within a discretization range.

The directional spatial autocorrelation of the stochastic variables of the carbon/epoxy satin weave pre-preg is illustrated in Fig. 5. It can be observed that strong autocorrelation exists in the orientation of weft tows, whereas the autocorrelation in orientation of warp tows is negligible. Autocorrelation results for the unit cell lengths in the two directions are not presented here as lengths do not show significant variability in this textile. The autocorrelation of weft orientation is persistently high at the 0° direction which coincides with the direction of warp tows. Autocorrelation in the 90° direction crosses the zero level at about 60 mm, a distance that corresponds to 6 unit cells. Autocorrelation drops off within 10 to 20 unit cells at orientations between the warp and the weft direction. Negative weft orientation autocorrelation is observed at high distance for directions of fast decaying autocorrelation as a result of long range trends in the stochastic variable.

The results of spatial autocorrelation analysis for the glass/polypropylene fabric are illustrated in Fig. 6. The autocorrelation is highly anisotropic, in a similar fashion to that observed for the carbon/epoxy material. According to the results presented in Table 3 both tow orientations and unit cell lengths are stochastic in this material. However, the autocorrelation results presented in Fig. 6 show that unit cell lengths are not spatially autocorrelated. In contrast to the carbon/epoxy pre-preg, both tow orientations show significant autocorrelation. The autocorrelation in the orientation of warp tows is persistent in the 90° direction, which coincides with the weft direction. The opposite effect occurs in weft tows where the high correlation direction coincides with the warp direction. Autocorrelation parallel to the direction of the corresponding tows drops off at about 100 mm or 5 unit cells for warp tows and at 120 mm or 6 unit cells for weft tows. At directions between the warp and the weft, autocorrelation drops off within 6 to 12 unit cells for warp tows and within 7 to 15 unit cells for weft tows. Autocorrelation tends to be stronger for weft tows. It should be noted that the weft direction is also the variable with the most significant variability in both materials.
The autocorrelation surfaces obtained for the stochastic variables of the two materials in this study are modeled using the expression:

\[
\rho(r, \theta) = (1 - \phi_0 \cos(\theta - \phi)) e^{-r/\phi_1} + \phi_0 \cos(\theta - \phi) + B
\]  

This expression is an anisotropic extension of the exponential correlation function, which is used widely to simulate Markov processes. The model parameters for the weft orientation of the carbon/epoxy satin weave prepreg and for the warp and weft orientations of the glass/polypropylene commingled fabric are given in Table 5. The quality of the fit obtained using Eq. (2) is illustrated in Figs. 5 and 6. The model simulates very closely the autocorrelation of the glass/polypropylene fabric. In the case of the carbon/epoxy material the fit follows the anisotropic structure and decay rates successfully, but underestimates the remote plateau of autocorrelation.

Table 5. Parameters of the autocorrelation surface.

<table>
<thead>
<tr>
<th>Material</th>
<th>( d ) (mm)</th>
<th>( \phi_0 )</th>
<th>( \phi ) (°)</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon/epoxy weft direction</td>
<td>65.0</td>
<td>0.83</td>
<td>1.67</td>
<td>-0.60</td>
</tr>
<tr>
<td>Glass/PP warp direction</td>
<td>128.8</td>
<td>-1.17</td>
<td>5.93</td>
<td>0.42</td>
</tr>
<tr>
<td>Glass/PP weft direction</td>
<td>172.5</td>
<td>1.67</td>
<td>5.11</td>
<td>-0.96</td>
</tr>
</tbody>
</table>
V. Decomposition of Autocorrelation and Stochastic Simulation

Stochastic simulation of the geometry of the textile requires the generation of realizations of a vector \( \mathbf{Z} \) of stochastic variables with a given correlation structure expressed by a covariance matrix \( \mathbf{C} \). \( \mathbf{Z} \) comprises values of warp and weft tow directions as well as unit cell sizes at different locations. Analysis of the experimental data has shown that all variables involved in the textile unit cell geometry are normally distributed, whereas second order stationarity is assumed.

\[
\mathbf{Z} = \begin{bmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_k \end{bmatrix}
\]

\[
\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1k} \\ C_{21} & C_{22} & \cdots & C_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ C_{k1} & C_{k2} & \cdots & C_{kk} \end{bmatrix}
\]  

(3)

(4)

The covariance matrix is symmetric and positive definite and it has a Cholesky root \( \mathbf{L} \) such that \( \mathbf{C} \) can be factored as follows:

\[
\mathbf{C} = \mathbf{LL}^T
\]  

(5)

Realizations of \( \mathbf{Z} \) can be generated by the transformation

\[
\mathbf{Z} = \mathbf{LW}
\]  

(6)

where \( \mathbf{W} \) is a vector of \( k \) independent normally distributed variables.

The procedure outlined in Eqs. (3)-(6) was implemented in a computer code that uses the autocorrelation model expressed by Eq. (2) and the measured variances (Tables 1 and 3) to calculate the covariance matrix, a standard Cholesky decomposition routine\(^6\) and a random number generator\(^7\).

Realizations of a 240 mm × 240 mm square piece of carbon/epoxy satin weave pre-preg were generated on a 12 × 12 grid which coincides with the finite element mesh of subsequent forming analyses. The warp and weft orientations were considered to be stochastic. The variance of the two variables was set equal to the values reported in Table 1, whereas the weft orientation was considered to be spatially autocorrelated with correlation following Eq. (2) with the parameters given in Table 5. Three of these realizations are shown in Fig. 7.

![Figure 7. Three realizations of the stochastic textile simulation for the carbon/epoxy pre-preg.](image)

VI. Monte Carlo Simulation of Forming

Random text process textile realizations produced using the simulation procedure outlined in the previous section provide the input for Monte Carlo simulation of the forming process. The process model used to simulate forming over a hemisphere is a simplified finite element model which represents the woven fabric as an assembly of bar elements which are connected at the corners of the unit cell of the model (Fig. 8.a) via pin joints\(^8\). The sides of the unit cell represent the tows of the textile and the diagonal bars govern the non-linear shear behavior of the assembly. This arrangement allows the trellising action observed in practice to be reproduced when the effective
stiffness of the tow bars is significantly higher than the stiffness of the diagonal bars. Tow elements are considered elastic in tension. Incorporation of wrinkling due to tow buckling is based on an activation/deactivation strategy applied to tow elements. The non-linear stress-strain response of the shear element (Fig. 8b) is simulated by a strain dependent plastic material model. The action of the blank holder on the woven material is incorporated in the model by including a group of bar elements connected to the nodes in contact with the blank holder. These elements are fixed with respect to all degrees of freedom on their free ends. They replicate friction via their elastic-perfectly plastic behavior.

Fig. 8c illustrates the formed pattern resulting from the model application to the case of the carbon/epoxy satin weave pre-preg. Occurrence of wrinkling due to tow buckling is observed at normal positions near the base of the hemisphere. More details on the application and validation of the forming model can be found in Ref. 9.

Forming optimization studies have indicated that applying a variable blank holder force with higher forces in the bias direction of the woven material results in less wrinkling in the formed component. The validity of these results when random fluctuations of the woven material geometry are taken into account is examined here. Two forming situations are considered; one with uniform peripheral force distribution and one with the peripheral force in the bias direction of the material increased by 50% while the total force is kept constant.

The simplified finite element model was executed for each realization and the total wrinkling at the end of forming was estimated by summing up wrinkling strain over the 312 tow elements of the model. Table 6 summarizes the results of these runs and Fig. 9 illustrates the distribution of total wrinkling strain in the forming of a satin carbon/epoxy pre-preg.

The distributions of total wrinkling shown in Fig. 9 indicate that the resulting strain follows a normal distribution. The distribution in the case of variable force is broader than in the case of constant force as a result of its higher standard deviation. It should be noted that the value of strain in the unperturbed case is in the right tail of

<table>
<thead>
<tr>
<th></th>
<th>Unperturbed Strain</th>
<th>$\mu$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant force profile</td>
<td>-3.86</td>
<td>-4.29</td>
<td>0.57</td>
</tr>
<tr>
<td>Variable force profile</td>
<td>-2.79</td>
<td>-3.83</td>
<td>0.83</td>
</tr>
</tbody>
</table>

The procedure described in the previous section was used to generate 100 realizations of the textile in each case. The simplified finite element model was executed for each realization and the total wrinkling at the end of forming was estimated by summing up wrinkling strain over the 312 tow elements of the model. Table 6 summarizes the results of these runs and Fig. 9 illustrates the distribution of total wrinkling strain in the forming of a satin carbon/epoxy pre-preg. The unperturbed strain decreases by approximately 40% when the variable peripheral force profile is used. The average strain over the 100 simulations decreases only by about 10% with the optimized force profile, while the standard deviation increases by approximately 40%.
the distributions indicating the adverse effect of imperfections introduced by the stochastic character of the woven material geometry. These results indicate that the variability of woven textiles propagates in forming and induces significant variability in the formed component.

VII. Conclusions

In this study, a procedure for analyzing and simulating random fields, describing the orientations of tows and the unit cell size in the tow direction of woven materials, is presented. An image analysis technique combining Fourier transform and a correlation search is used to measure local tow angles, orientations and unit cell lengths of both pre-impregnated and dry textiles. Efficient use of the procedure is conditional upon availability of an approximate prior estimate for the unit cell size, which is easily obtainable in the case of woven textiles.

Results obtained from image analysis show that variability of tow orientations in a carbon/epoxy satin weave pre-preg and a commingled glass/polypropylene fabric and that variability in the weft direction tends to be higher than in the warp direction. In contrast, unit cell lengths show significant variations only in the case of the dry fabric. Cross-correlations between all variables are found to be insignificant.

Examination of the autoregressive character of stochastic variables shows that orientation of weft tows in the carbon/epoxy pre-preg and orientation of weft and warp tows in the glass/polypropylene commingled fabric have strong and highly anisotropic spatial autocorrelation. Persistent correlation of tow orientations occurs at their corresponding normal direction, whereas correlation drops off within 5 to 15 unit cell lengths at directions other than the normal. Negative autocorrelation at long distances occurs due to long range trends in the stochastic variables.

These results indicate that stochastic simulation of textiles should allow both tow orientations and unit cell lengths to be treated as stochastic variables, while anisotropic autoregression should be incorporated in the simulation of tow orientations. A simulation procedure based on Cholesky decomposition is developed to address these requirements. The stochastic simulation provides the geometrical input required by a simplified forming finite element model which is computationally efficient and can be used repetitively in Monte Carlo simulation of woven material draping or forming. Results of the simulation show that variability of the woven material geometry induces significant variations in the formed geometry. In addition, forming conditions which were found to be optimal in a deterministic optimization framework are still preferable but introduce higher variability in the final geometry when the stochastic nature of the textile is considered.

Acknowledgments

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References