APPLICATION OF FOIL ROLLING MODELS TO THIN STEEL STRIP AND TEMPER ROLLING

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SUMMARY

An algorithm for the analysis of rollgap phenomena in thin strip and foil rolling where there is a large ratio of roll flattening to strip thickness, is described. This draws on the work of Fleck et al. 1992 1) who developed a foil rolling model which assumed homogeneous compression and calculated the deformed roll shape by the application of influence functions. Their results obtained for rolling aluminium foil gave reasonable agreement with measurements. The new model includes strip temperature and strain rate analysis, and is applicable to a range of aluminium foil and thin steel strip applications, including tinplate, tinplate temper rolling and double reduction rolling. For each case, typical predicted pressure distributions and deformed roll shapes are presented. A comparison of the results obtained with measured data and conventional circular-arc rolling theories is presented. The effect of using different friction coefficient laws is also demonstrated as part of an investigation into speed effects associated with strain rate and hydrodynamic lubrication.

A major feature of the algorithm is the short execution time which is one to two orders of magnitude faster than that of the Fleck model.

INTRODUCTION

The most accepted theory for the rolling of flat metal products was developed by Orowan in 1943 3). This theory forms the basis of most models used in the analysis of rolling. It is well known that the rolls deform due to the pressure required to plastically deform the metal during the rolling process. Orowan described rolling experiments which showed the roll may assume a flat or even an inverted profile during rolling. However, at this time, it was not realistic to solve the roll deformation problem numerically. Instead a simple model developed by Hitchcock in 1935 4) was used which assumed that the roll maintained its circular profile in the roll/strip contact region, but with a radius greater than its nominal value. This assumption is still employed in the majority of current rolling models.

As rolling technology improved, the rolling of thinner material became more important. Rolling theory suggests that rolling thinner material leads to a more rapid roll pressure increase in the rollgap. Consequently, the magnitude of the work roll deformation relative to the strip thickness increases causing the interaction between the roll pressure development and the roll surface to be more significant. During the period from 1950 - 1990 numerous papers were written on the problem of limiting reduction indicating that a limit exists for thin material beyond which further reduction is possible regardless of the applied roll force. This reasoning was based on the fact that the pressure rise in the strip being rolled exceeded that required to deform the roll and infinite flattening would occur. Despite this, the rolling of thin material, in regions beyond those predicted, was achieved in foil rolling mills.

Meanwhile numerous researchers had attempted to develop a more realistic roll model by the use of influence functions to describe the roll deformation in the radial and tangential directions (see for example Jortner et al. 1960 6), Grimble et al. 1978 9) ). These models gave some prediction improvement but they failed to converge for the thinner materials as the predicted roll profile developed indentations causing algorithm instability. Further developments in this field were made by Chefneux et al. 1977 8) using the Jortner theory for the rolls and the general elasto-plastic equations for the strip. The application of this work to the final tandem mill stands for tin plate mills showed roll indentation as the major departure from the classical models.

Early studies of steel temper rolling by Carlton et al. 1978 7) highlighted the highly sensitive nature of rollgap models using this approach. Simplified circular arc models for on-line automation applications which replicated the behaviour of the non-circular arc Orowan theory were developed. Further improvements were described by Carlton et al. 1990 5) for application to steel temper rolling of annealed steel product down to 0.4 mm in thickness. However, the shortcomings of this approach were immediately apparent when attempts were made to analyse tin plate temper rolling, foil rolling and double reduction rolling data.

A major development in the cold rolling of thin strip and foil was achieved by Fleck and Johnson 1987 9) who assumed that, in extreme cases, there is a region of roll flattening where the roll surface is flat and parallel and the material was essentially unchanged by the rolls. In this region the pressure is no longer determined by the friction at the roll/strip interface but by the pressure required to maintain the roll flat. This model used a mattress model for the roll deformation and in many cases this assumption produced poor results. It also predicted that a limiting reduction does not exist. However, once the roll commences to flatten, the roll load increases rapidly with further reduction as observed in practical situations.

Fleck et al. 1992 7) developed a more realistic model to describe the rolling of foil, by the use of an influence function approach to determine the roll deformation. This model has provided useful
reference results however it is relatively slow and not therefore suitable for automation applications. A variation on the Fleck model has been developed to meet this need and has been found to give superior results to circular arc models in tinplate temper rolling applications. It also appears to have the potential for foil rolling automation.

We will describe this non-circular arc rollgap model, introducing the basic assumptions made in its development. Then, after initially describing the two-stand double reduction/temper mill of Cockerill Sambre, we compare the predictions of the non-circular arc rollgap model and those of the circular arc model with data obtained from this mill. We then provide a more general comparison of the non-circular arc and the circular arc models, highlighting their scope and limitations. The major results and future investigations are discussed in the conclusion.

DESCRIPTION OF THE NON-CIRCULAR ARC ROLLGAP MODEL

This model is essentially equivalent to that described by Fleck et al. 1992, with some additional improvements. A brief review of their analysis is warranted.

The strong interaction between the roll profile and roll pressure (which are directly coupled through the influence function) requires an iterative scheme of solution. The procedure therefore becomes one of commencing with an initial roll profile and determining the resulting roll pressure. The pressure distribution in the regions of plastic reduction is calculated by integrating the differential equation similar to that of Orowan with a two step Runge Kutta procedure. The differential equation for the horizontal force, \( t_x(0) \), in the case of slipping is given by

\[
\frac{dt_x}{dx}(0) = -2(0) \gamma(0) (\tan \phi) \mu(1)
\]

where \( \mu \) is the coefficient of friction, \( \gamma \) is the angular position of the point of interest, \( \phi \) is the angle between the roll surface and the horizontal and \( s(0) \) is the roll pressure which is given by:

\[
s(0) = \frac{t_x(0) + kw}{h(1+\frac{1}{\phi \tan \phi})}
\]

where \( k \) represents the yield stress and \( w \) the inhomogeneity function of Orowan.

Once the pressure profile is obtained, an updated roll profile is determined by the roll profile/pressure influence function. The roll radial deformation \( AR(0) \) is given as

\[
AR(0) = \int_{0}^{\Phi} u(0-\Phi') s(\Phi') d\Phi'
\]

where \( s(0) \) is the roll pressure and the influence function and \( u(0) \) is

\[
u(0) = \left[ (1-v-2v^2) \frac{5}{2} \cos x \text{sgn} \left( \sin x \right) \right]^{\gamma-\psi} + (1-v^2) \sin x \log \left( \tan^2 \frac{x}{2} \right)_{\gamma-\psi}
\]

The iteration of the roll pressure and roll profile distributions continues until a stable solution is found.

We now describe the complications which occur when the roll profile flattens or becomes parallel. The procedure outlined above is identical to that used by previous researchers (see for example Grimble et al. 1978). It results in instability when the roll profile flattens as the frictional force, still assumed to be related to the roll pressure by the Coulomb friction law, causes the roll pressure to increase rapidly. The resulting pressure increase causes the indentation of the roll, implying an increase in strip thickness. In this region the material must be assumed to unload elastically, resulting in a rapid fall in roll pressure. This then causes the roll profile to drop, and this interaction causes the solution to oscillate.

A solution to this instability problem was proposed by Fleck and Johnson 1987 who predicted that in the rollgap there may exist a region where the roll profile remains flat and parallel. In this region, which they describe as the region of constrained plastic flow, no further reduction takes place and the shear or frictional stress at the roll/strip interface remains at a value below that predicted by the Coulomb friction law. They deduce this by reasoning that the two alternative scenarios, that of the roll profile increasing and that of the roll profile decreasing, violate the physical aspects of the problem and the yield criterion respectively. They also show that the value of the frictional force in this region is that value which is required to maintain the roll profile in its flattened form.

The resulting rollgap region now becomes similar to that shown in Fig. 1. We note the following regions, the regions of elastic compression and recovery which are identical to those found in other models, and two regions of plastic reduction which are separated by a region of constrained plastic flow, where the material is transported without change in thickness.

The result of this assumption on the modelling is apparent. It is no longer possible to predict the roll pressure by the plasticity equations in the constrained plastic region. The roll pressure solution in this region must be obtained by inverting the roll profile/roll pressure relationship defined by the roll deformation influence function. Hence in discrete form, if the roll deformation, \( AR_i \), is given by

\[
AR_i = \sum_{j=1}^{n} u_i s_j
\]

then for the flattened region,

\[
\sum_{j=1}^{n} u_{i-1} s_j = (AR_i - AR_{i-1}) \sum_{j=1}^{n} u_{i-1} s_j - \sum_{j=1}^{n} u_{i-1} s_j^2
\]
where $j_1$ represents the first node in the flattened region and $j_2$ is the final node in the flattened region. As the coefficients $A_{ij}$ for $i = j_1, \ldots, j_2$ may be determined from the assumption that the roll is flat, it is possible to invert the above equation and solve for $s_i$ for $i = j_1, \ldots, j_2$. Having determined the roll pressure profile, the value of the roll frictional force may be obtained directly from the horizontal equilibrium equation.

Both models assume that Coulomb friction exists between the roll surface and the strip. This assumption is convenient but difficult to verify without accurate measurement. In practice, it is necessary to postulate a value for the coefficient of friction and this is usually obtained by matching the measured roll force. The prediction accuracy for other available measurements, such as rolling torque and forward slip, is an indirect measure of the effectiveness of these model assumptions.

The current model contains a generic yield stress model which represents the effects of temperature, strain rate and work hardening. The significance of these effects are highlighted in the double reduction and wet temper rolling examples which will be used to demonstrate the model performance. A simplified temperature model has been included. This model calculates the approximate temperature rise in the strip and the roll, and allows the effect of the temperature on the material yield stress to be evaluated. Currently, the temperature model assumes the strip is thin and that temperature variations through the thickness of the strip are not significant.

The main results produced by the non-circular arc rollgap model include the roll load, the rolling torque and the slip. The roll load is obtained by integrating the roll pressure distribution over the arc of contact. The slip is obtained by calculating the mass flow at the neutral point assuming the strip velocity is homogeneous; that is, independent of the vertical direction. The roll torque calculation is based upon the energy balance approach rather than the moment arm approach as this appears to provide more consistent results due to better numerical conditioning. Theoretically, these approaches should be identical, even for a non-circular arc. The principle energies are identified as the energy of reduction, $E_r$, friction, $E_f$, and tension, $E_t$, from which one may deduce the specific roll torque as

$$T = \sum_{i=1}^{n} \frac{dE_i}{dx}$$

where

$$E_r \equiv \frac{1}{2} \int_{H}^{h} (\dot{v}^2 + \dot{h}^2) \, dx$$

and $f$ is the forward slip, $k$ is the material yield stress, $h$ is the strip thickness, $\dot{v}$ is the velocity difference between the strip and the roll, $\mu$ is the coefficient of friction, $s(x)$ is the roll pressure, $\tau$ is the strip exit tension and $\tau'$ is the strip entry tension.

An indication of the applicability of the non-circular arc model to foil rolling is provided in Fig. 2 which displays results obtained for the rolling of aluminium foil of various thicknesses at 50 per cent reduction. These results are similar to those produced by the Fleck (1992) model.
EVALUATION OF MODEL IN TINPLATE ROLLING AT COCKERILL SAMBRE

In early 1991, IAS became involved in performance studies aimed at improving thickness and elongation control on the two-stand, double reduction (DR)/temper mill of Cockerill Sambre, Tilleur, Belgium. This mill operates with stand one either wet or dry and stand two always operating dry. In general little reduction occurs on stand two. The mill rolls annealed tinplate material with entry thicknesses ranging down to 0.18mm, and reductions typically between 20 and 30 per cent for DR rolling. As a temper mill, elongations of approximately six per cent are achieved in wet temper mode and less than one per cent in dry temper mode. As part of the investigation, data was recorded from the mill, and attempts were made to predict rolling parameters using a conventional circular-arc rolling model. Two problems became apparent: firstly, a converged solution for the rollgap pressure was not obtainable in the lower range of thicknesses, and, secondly, the high levels of slip recorded on the mill were not predicted in any cases. Only by using the simplified, semi-empirical rollgap model of Carlton et al. 5, with appropriate tuning, were sensible roll force solutions obtainable for all cases, and still the high slip values were not predicted accurately.

A major automation upgrade of this mill was completed in 1993. This project involved a technology package including advanced control systems with mill setup models and model based control systems to compensate for product dimension and material property changes. The kernel of the mill setup calculation was the semi-empirical rollgap model previously mentioned. By appropriate tuning, reasonable performance was achieved on double reduction and wet temper modes, although it relied on a good adaption scheme, particularly for the slip calculation. For dry temper rolling, where total elongation is less than 1 per cent, the model relies heavily on adaption, and has difficulty in providing accurate predictions where large changes in nominal strip thickness occur. Results from the circular arc rollgap model indicated that the mill was operating in the regime of high roll flattening, so it was considered appropriate to try the new, non-circular arc model. Friction coefficients in rolling are known to vary significantly with roll surface wear, rolling speed, and lubricant condition, so it is difficult to test a model in absolute terms. The procedure followed was to adjust the friction coefficient until the force predicted by the model matched the measured force, and then to compare the measured torques and slips. The estimated friction coefficients are described in Tables 1 and 2.

To allow the torque predicted by the model to be compared with the measured electrical torque it was necessary to adopt a torque model to allow for motor/drive efficiency and rolling mill stand losses. The model chosen was:

$$G = \frac{TW + GL}{\eta}$$

where $G$ is the predicted motor torque, $\eta$ is the motor and drive train efficiency, $T$ is the specific roll torque calculated by the model, $W$ is the strip width and $G_L$ is the stand torque loss. The stand torque loss $G_L$ was assumed to have the form:

$$G_L = a_1 + a_2PW$$

where $a_1, a_2$ are mill dependent constants, $P$ is the specific roll force. As mill torque loss measurements were not available for this mill it was necessary to estimate these values from the available data. Assuming a drive efficiency $\eta$ of 0.9 and measured data from numerous dry temper, wet temper and double reduction cases, the stand $G_L$ losses (expressed in kN.m) were estimated as:

$$G_L = 3.34 + 0.532PW, \text{ (kN.m)}$$

DOUBLE REDUCTION CASES

The rollgap pressure distribution and thickness profiles generated by the new program confirmed that significant roll flattening was occurring in the DR rolling. Figs. 3 and 4 show pressure distributions and rollgap profiles for 30 per cent reduction and mill entry thicknesses of 0.20 and 0.39 mm. In each case, even with the higher thickness, there is a distinct flattening of the roll near the middle of the contact zone, accompanied by the characteristic "bulge" in the pressure distribution.

Four cases were treated in detail, covering a range of thicknesses with the results shown in Figs. 5 and 6. In each case, the procedure was to match the predicted and measured force by adjusting the friction coefficient. In cases 1 and 2, representing entry thicknesses of 0.203 and 0.229 respectively, it
was difficult to exactly match the force with the circular arc model, since the model was very sensitive to the value of the coefficient of friction. There was no such difficulty with the non-circular arc model.

predictions follow the trend evident in the measurements. The assumption of a circular arc of contact which is used in the conventional model is the main reason why this model is incapable of predicting high slips. In most cases, except for those involving extreme tension unbalance, the neutral point falls on the exit side of the mid-point in the rollgap. Under these conditions, the quadratic approximation to the circular arc ensures that the thickness at the neutral point will be less than $h+0.25(H-h)$, and that consequently, the slip will be less than 25 per cent of the total elongation (i.e. $\frac{100(h-\frac{3}{4}h)}{h}$). In the non-circular arc model, the neutral point becomes a neutral zone where the roll flattens, at a thickness $h_n$, and the slip is given by:

$$f = \frac{h_n}{h-1}$$

In most cases, the observed thickness at the flattened zone was roughly half way between the entry and exit thicknesses, giving a slip equal to approximately 50 per cent of the elongation. For a reduction of 30 per cent, this corresponds to a slip of about 21 per cent.

To evaluate the model further, Case 5 of Table 1 in Appendix A was analysed using the non-circular arc model. This example was chosen because the rolls were newly ground, and it was known that the effective coefficient of friction would be higher, producing larger roll force torque and slip measurements. For this example, matching the specific roll force of 16.3 kN/mm gave a predicted specific torque of 18.6 kN.m/mm and a predicted slip of 23.2 per cent. This corresponds well with the measured values of 18.5 kN.m/mm and 27.4 per
cent for the torque and slip respectively.

**WET TEMPER CASES**

A total of four wet temper rolling cases were analysed as summarised in Table 2. Again, the rollgap pressure distributions and the strip profile distributions for two of these cases show that significant areas of roll flattening occur (see Figs. 7 and 8).

Comparative results for the circular and non-circular arc models are presented in Figs. 9 and 10. These cases were chosen to reflect the range of rolling situations found in the wet temper mode. The coefficient of friction was adjusted to match the specific roll force predicted by the model with the measured value. This was only possible for the circular arc model in the final case where the material is thickest and the specific roll force is not as great as in case 3.

The specific torque errors for both the circular and non-circular arc model are consistently within ±1 kN/m. It is not reasonable to discuss the relative error in torque predictions in these circumstances as the rolling torque is much less than the torque losses.

Again, the significant difference between the models appears in their prediction of slip. As previously discussed, the slip predicted by the circular arc model would be limited to a maximum of 1-1.5 per cent, well below the measured values. The graph shows that the slip predicted by the non-circular arc model agrees well with the measured values. Furthermore, it is interesting to note that these values are large when compared with the temper rolling elongations (which range from 3.5-5.5 per cent).

**COMPARISON OF CIRCULAR ARC AND NON CIRCULAR ARC ROLLING MODEL**

It is interesting to compare the predictions of the circular and non-circular arc rolling theories for a range of rolling conditions. In this section we investigate nominal rolling conditions described by Table 3, which correspond to typical double-
reduction and wet temper cases described above. To investigate the effect of diminishing strip thickness on the circular and non-circular arc rolling theory predictions we held the reduction constant and varied the strip entry thickness. The predictions for specific roll force and forward slip are displayed in Figs. 11 and 12. The following observations are immediately apparent. For the thinner cases, the roll force predicted by the circular-arc model increases dramatically as the strip entry thickness decreases and results in the failure to obtain a solution at the lower thicknesses. As the strip thickness increases, the roll forces predicted by both the circular and non-circular arc models converge to similar values as is expected because the deformed roll profile approaches its circular, undeformed shape.

Fig 11: Comparison of specific roll force predicted by circular arc and non-circular arc models for rolling conditions described in Table 3.

Fig 12: Comparison of slip predicted by circular arc and non-circular arc models for rolling conditions described in Table 3.

In general, the non-circular arc rollgap model produced lower specific roll forces and roll torques for the same set of rolling parameters. An intuitive explanation for this phenomenon may be obtained by observing that the effective radius of the non-circular model in the region of plastic deformation is smaller than the nominal roll radius, whereas the deformed roll radius in the circular arc model is larger than the nominal roll radius. The smaller roll radius allows easier reduction.

The differences in slip observed in earlier comparisons of the circular and non-circular arc models are repeated in these results. The non-circular arc rolling theory predicts substantial increases in the slip as the strip thickness falls below 0.25 mm. From the graphs we may conclude that as the entry thickness falls below this value, the roll flattening is becoming significant, increasing the slip. It is also interesting to observe that at this point the circular arc model predictions for roll force tend to become unreliable. This suggests a simple but effective method of monitoring the onset and degree of roll flattening which is occurring in practical situations. Extending this idea further, for a given product, the accurate measurement of forward slip can give an indication of the average friction coefficient between the strip and the roll. This could be employed to monitor the status of roll wear or lubricant effectiveness.

INVESTIGATION OF SPEED EFFECTS

In double reduction and wet temper rolling, the roll force exhibits a strong speed dependence. Two contributing factors which can be investigated with the current model are the change in yield stress with strain rate and the change in friction coefficient with speed. In Fig. 13 the effect of strain rate on specific roll force is illustrated by the solid line for a typical wet temper rolling situation previously described. The strain rate effect is seen to be greatest at lower rolling speeds due to its assumed logarithmic dependence. At higher rolling speeds, the effect of strain rate upon average yield stress decreases. A higher sensitivity of the specific roll force to the average yield stress however maintains the strain rate effect beyond that expected.

Fig 13. Variation in specific roll force with speed.

In practice, at low speeds the specific roll force decreases with increasing rolling speed before climbing due to the strain rate effect. This is directly related to the change in lubrication mechanism within the rollgap causing a decrease in the coefficient of friction with increasing rolling speed. To illustrate this, an exponential friction/speed relationship similar to that observed experimentally has been assumed and the previous analysis repeated with this modified friction law. These results are illustrated by the broken line of Fig. 13. The resulting curve describing the variation in specific roll force with rolling speed agrees well with that observed in practice.

CONCLUSION AND FUTURE WORK

The new, non-circular arc rollgap model described overcomes the basic limitations of the circular-arc model, in that it can produce solutions to cases involving thin strip and high specific roll forces, where significant roll flattening occurs. Furthermore,
it is capable of realistic slip calculations.

The application of the non-circular arc model to dry temper rolling with small reductions remains a future area of investigation. Problems to be addressed include the refinement of the treatment of the elastic components of the strip deformation which become significant at small elongations. The result of this work will be compared with the elasto-plastic approach described in the communication of Chefneux et al. Another important area of investigation is the friction characteristics which exists at the roll-material interface, and types of friction other than Coulombic are being evaluated.

In its current form, the non-circular arc program is only just suitable for on-line use, each calculation requiring a few seconds on a fast computer. The model will be employed in an on-line foil mill automation application in 1994 in Europe. Further developmental work should see the derivation of a simpler model for general purpose automation applications. The model provides a particularly useful tool for fundamental investigations of thin strip and foil rolling processes, notably friction mechanisms and the interpretation of measured data.

REFERENCES


APPENDIX: DATA USED IN EXAMPLES

Table 1: Schedule data for DR cases

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Table 2: Schedule data for wet temper cases.

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Table 3: Schedule data for comparison of circular and non-circular rolling theory

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</tr>
</tbody>
</table>

| K    | 560-560 | 560-580  | 560-580 | 560-580   |

Table 4: Schedule data for comparison of circular and non-circular rolling theory