The study of tapered laminated composite structures: a review

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Abstract

Following laminated composite plates and beams, tapered laminated structures, which are formed by dropping off some of the plies at discrete positions over the laminate, have received much attention from researchers because of their structural tailoring capabilities, damage tolerance, and their potential for creating significant weight savings in engineering applications. A review of recent developments in the analysis of tapered laminated composite structures with an emphasis on interlaminar stress analysis, delamination analysis and parametric study is presented. A discussion of various approaches to modelling and analysis of interlaminar response of tapered composites using finite elements and non-finite elements is given. Displacement-based finite elements and hybrid finite elements that are commonly used are also reviewed. A review of various studies on delamination failure mechanisms as a result of drop-off plies in the tapered composites is given next, which mainly encompasses a stress-strength approach and a fracture-mechanics approach. Lastly, a variety of methods that are being used in the parametric studies regarding the structural integrity is presented. Overall remarks drawn from the reviewed works are given in the final section of the paper. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

This paper presents a review of the recent advances in the study of tapered laminated composite structures. Following laminated plates and beams, tapered composites formed by terminating or dropping off some of the plies in some primary structures have received much attention from researchers since the mid-1980s. Their elastic tailoring properties and potential for creating more significant weight savings than commonly-used laminated components allow an increasing use of the tapered composites in commercial and military aircraft applications.

A typical example is the helicopter yoke, shown in Fig. 1, where a progressive variation in the thickness of the yoke is required to provide high stiffness at the hub and relative flexibility at the mid-length of the yoke to accommodate flapping. The first commercial composite rotor-blade yoke assembly made from glass-fibre/epoxy composite was fabricated at Bell Helicopter Textron. Constructed completely from S-2 glass, the dual-yoke assemblies on the Bell 430 helicopter endure several times more flight hours than traditional titanium or steel yokes, and also provide improved safety. Much more tolerance to damage than conventional materials and the elimination of corrosion are also displayed by these composite components. Other applications include composite aircraft-wing skins, helicopter flexbeams, flywheels, etc.

A significant amount of research work has been done on the delamination analysis of tapered composites. A review of these developments is given in this paper. The review is restricted to studies published in the English language, mostly during the past two decades.

2. Overall review

Tapered composite structures, formed by terminating some of the plies, create geometry and material discontinuities that act as sources for delamination initiation and propagation. From earlier research work concerning this type of structure, two major categories of work on tapered composites can be identified. The first is to understand failure mechanisms induced by drop-off plies in tapered construction. This work encompasses the determination of the state of inter-
laminar stresses in the vicinity of ply drop-offs, the calculation of strain-energy release rate associated with delamination within the tapered region, and the direct modelling of delamination progress by using finite elements. The initiation and propagation of delaminations could thus be predicted. A large number of investigators have been engaged in conducting research on this subject. The list includes the works of Kemp and Johnson [1], Curry et al. [2], Hoa et al. [3], Fish and Lee [4], Salpekar et al. [5], Murri et al. [6,8,16], Armanios and Parnas [7], Vizzini and Lee [9], Wisnom et al. [10–12], Harrison and Johnson [13], Vizzini [14,32], Rhim and Vizzini [17], Adams et al. [47], Wu and Webber [19], Wu [20], Miravete [21], Thomsen et al. [41–43], Mortensen and Thomsen [18], Mukherjee and Varughese [22], Poon et al. [35], Hofman and Ochoa [31], Ochoa and Chan [45], Davila and Johnson [49] and Trethewey et al. [46]. The second category has been to seek more rational or optimal designs of damage-resistant tapered composite structures by investigating the parameters that have substantial influences on the structural integrity. Parametric studies of tapered composites were conducted by Daoust and Hoa [23], Llanos and Vizzini [24], Thomas and Webber [25], Cuí et al. [26], Vizzini [27], Botting et al. [28], Manne and Tsai [29], Cairns et al. [30], Fish and Vizzini [39,40], Pogue and Vizzini [44] and ESDU Data Item 91003 [48].

This review concentrates on the work mentioned above, specifically (1) stress analysis, (2) delamination analysis, and (3) parametric study.

There are several basic types of tapers that are often used and analyzed, and they can be identified as shown in Fig. 2. External-ply drop-off tapers, defined as those in which the dropped plies are on a surface of the laminate, were examined by Hoa et al. [3], Daoust and Hoa [23], Wu and Webber [19], Wu [20], Thomsen [41,42], and by Miravete [21]. Daoust and Hoa [23] developed an extensive finite-element program using displacement formulation to study the effect of some parameters on the strength of the laminate. Both internal and external ply drop-offs were examined and compared in their work. They concluded from their study that under tension, bending and torsion, internal ply drop-offs are roughly twice as strong as external drop-offs. Wu and Webber [19] applied a quasi-three-dimensional iso-parametric finite-element model for the linear elastic static analysis of tapered laminated plate of infinite width subjected to a uniform in-plane load. Numerical results were given for a single-step plate with various arrangements for the ply fibre directions. Very high peak stresses were predicted in the corner region of the step, but these were reduced when a resin fillet was included in the theoretical model for the step region. Following this analysis was a continuation work by Wu [20], in which non-linear material behavior was considered to account for the redistribution of stresses in the resin that would occur in the presence of the peak stresses. Compared with the linear results, the non-linear ones show that the peak stresses are reduced by about half as a result of non-linear deformation of the resin and the non-linear model gave more realistic prediction of interlaminar stress distribution and failure mode at the ply drop-offs. Thomsen et al. [41,42] used a simple mechanical model to investigate the local bending effects of ply drop-offs in CFRP/honeycomb sandwich panels. The interaction between the core material and the face laminates was modelled using a two-parameter elastic foundation model. It was concluded from the examples given that the elastic response is strongly influenced by the presence of a supporting core material and that out-of-plane stiffness of the honeycomb core, the bending stiffness of the base-line face laminate and the bending stiffness of the dropped sub-laminates provide significant bending effects induced by ply drop-offs. Experimental investigation based on the use of electronic speckle-pattern interferometry (ESPI) was conducted to validate the simple model and it was shown that the theoretically predicted and measured out-of-plane deflection profiles correlated well with respect to the local bending response induced by the ply drop-offs. The model was extended to apply to delamination failure analysis [43] and stress analysis for internal drop-off tapered laminates [18]. Miravete [21] presented a study of mechanical behavior of variable-thickness composite beams subjected to transverse load. A theoretical model based on a plane-strain finite-element theory was carried out to analyze the stress distribution near the areas of change of thickness, which is strongly dependent on thickness ratio. For low values of the angle of variation of thickness, the strength is outstanding and the variable-thickness effect does not alter the mechanical behavior of the plate. For high angle of thickness variation, the strength is lower because of the variable-thickness effect and failure occurs at the location where thickness varies. The delamination mechanism is a result of high interlaminar shear stress generated by the variable-thickness effect.

Midplane-ply drop-off tapers, defined as those in which the plies are terminated at the midplane, was examined by Hofman and Ochoa [31] with a shear deformable composite element. The shear deformable element was
modified to accommodate variable-element thickness with mid-plane layer drop-off. With the example problem, it is shown that the tapered element formulation in the QHD40 element, which was developed by Ochoa and Chan [45] for analysis and design of complex shape composite components, adequately models tapered and layered plates. The finite-element modelling, however, was simplified by negating the presence of small resin pockets caused by forming the terminated layers.

Two types of internally dropped ply tapers can be identified in terms of loading direction. Longitudinal-ply drop-off tapers are defined as those in which the internal discontinuities of the laminate are parallel to the applied load. In general, this type of taper was used to change the stress state in the free edge in order to suppress the delamination caused by stress-free edge effects. Vizzini [32] used a quasi-three-dimensional finite-element approach for strength prediction. In his analytical work, which correlated well with experimental evidence, Vizzini found that modelling a discontinuity with an associated resin pocket provides direct evaluation of the stresses in the region where failure occurs. Pogue and Vizzini [44] extended the structural tailoring techniques to the suppression of the delamination at the stress-free edge by dropping plies just around the edge of the laminates. This is one of four edge-alteration techniques applied to prevent the delamination induced by the stress-free edge of composite laminates. The introduction of the taper around the edge can change the state of stress at the free edge while introducing an internal discontinuity. It may help to decrease the interlaminar stresses which arise as a result of the stress-free edge; however, the newly introduced internal edge may be prone to internal damage that is difficult to detect non-destructively. The fact that benefit and detriment of tapering exist simultaneously shows that much care must be taken in choosing an appropriate tailoring technique. Ochoa and Chan [45] also examined the longitudinal taper under tensile, bending and torsion loads using finite elements in analyzing laminates with 90° plies dropped symmetrically just inside of the free edge. They found that under tensile loading, interlaminar stresses at the free edge were reduced significantly, while under bending and torsion loading only a small amount of increase of interlaminar stress values was found.

Transverse-ply drop-off tapers, defined as those in which the dropped plies are terminated from the interior of the laminate and the variation of the thickness goes along the primary loading direction, are concentrated upon in the remainder of this review because of their prevalent application in engineering. Most of the previous works with regard to delamination analysis of transverse-ply drop-off tapers were towards ideally two-, quasi-three- or full three-dimensional representation of the geometry of the taper. In general, they may consist of either 0, 90, ±45, ±15° plies or a certain combination of them with a tapering angle of less than 15°, usually with 5.7° for a 10-to-1 taper ratio. Most of them are symmetric, while a few papers investigated asymmetric laminates that are typically used in applications where a flat surface is important, such as wing skins. Both numerically intensive finite-element models and simplified analyses have been attempted. Experimentally, tapered specimens have been manufactured from graphite/epoxy and glass/epoxy composites and tested under quasi-static loading conditions and fatigue. The schematic of tapers with internally dropped plies is shown in Fig. 3.

3. Interlaminar stresses

The central part of an investigation of delamination analysis of tapered laminates lies in how to describe accurately the interlaminar stress state in the critical region of components. The finite-element method is the most prevalent and powerful tool in dealing with geo-
metrically complex problems such as tapered composites, as applied by a large number of authors. However, some authors either aimed at developing a simple physical model to demonstrate stress-transfer mechanisms at the drop-off location or desired to develop a complicated model to find out the true stress distribution at the critical region. All of these methods are named here as non-FEM approaches.

3.1. Non-FEM approaches

A simple model for interlaminar stresses at the interface between the continuous plies and drop-off plies was developed by Armanios and Parnas [7] on the basis of equilibrium conditions on the continuous sublamine (belt) and of local stiffness variations at the ply drop locations. In this model, the resin pockets were assumed as primarily shear-stress carriers and ply-drop locations as extensional and concentrated shear springs. The estimation of the interlaminar stresses was determined by application of a minimum complementary-energy principle. Although the interlaminar shear stress from their model was in qualitative agreement with a finite-element solution, it failed to capture the tensile nature of the interlaminar normal stress at the ply drops.

Vizzini [14] employed the so-called shear-lag model to analyze interlaminar stresses in the region around drop-offs. In his model, resin layers were assumed to act as media carrying shear stresses only, and fiber layers as media carrying extensional stresses. The stress state at the ply drop region was modeled as a three-zone problem, in each of which force equilibrium was implemented. The resulting governing differential equations, subjected to satisfaction of constraints on displacement at the boundary by finite-element results from the global analysis, the inter-zone constraints on displacements and forces, and constraints resulting from degenerate cases involving zero thickness, were solved with assumed polynomials for the displacements in the fiber layers. In comparison with the finite-element results, he found that the shear-lag model can capture a majority of the load-transfer mechanisms about internally dropped plies under extensional stresses.

A simple mechanical model, which was originally developed by Thomsen et al. [41–43] to investigate bending effects of the sandwich laminate with external drop-offs, was extended to obtain the stress distribution in laminates/sandwich laminates with internal ply drop-offs. The structural modelling for the sub-laminates was based on Kirchoff assumptions, and the classical thin-laminate theory (CLT) was used to describe the constitutive behavior of the sub-laminates. The resin layers were modelled as continuously distributed linear tension/compression and shear springs. In the case where the drop occurs in the face laminate of a sandwich panel, the interaction between the laminates and the core material is modelled using a two-parameter elastic foundation model, which accounts for the shear interaction between the laminate and the core. One of them is used to determine the compressive/tensile strain in the core material, and the other to determine the shear strain in the core material. The system equations subject to the prescribed boundary conditions were solved through the multi-segment method of integration. It was shown that the interlaminar stresses calculated by the proposed simplified approach correlated very well with the finite-element analysis. Meanwhile, the adaptability and limitation of the model were also presented through the discussion on structural parameters of sandwich laminates such as elastic wavelength, thickness of resin layers, cell-size of the honeycomb, etc.

Based on the Hellinger–Reissner variational functional, Harrison and Johnson [13] developed a stress-based method of approximation for the prediction of interlaminar stresses in the vicinity around ply drops. The approach chosen was to follow Pagano’s laminate structural theory which modeled the laminate by a series of layers with the stress field assumed within each layer. The stresses were assumed to be explicit functions of the thickness coordinate with stress variables as coefficients. These stress variables were functions of the longitudinal coordinate only. Substituting the assumed stress field
into the Hellinger–Reissner variational principle and invoking the stationarity condition with respect to all admissible stresses and displacements led to a system of differential-algebraic equations (DAEs) that could be solved by the finite-difference method. The solution for interlaminar stresses in the modeled tapered laminates that were assumed to be under generalized plane deformation was found to be in good agreement with the finite-element solution.

In general, simplified mechanical models, as shown in the above work, for the interlaminar stress analysis of tapered laminates provides more physical insight than that provided by FEM, and reasonable results in comparison to that calculated with FEM are reachable based on physically appropriate assumptions.

3.2. Displacement-based finite-element approaches

As for other composite structures, the majority of approaches to predict interlaminar stress and delamination in tapered composites are based on finite-element methods. Displacement-based finite elements and assumed stress hybrid elements are most commonly applied in these areas and deserve more discussion in this review.

In displacement-based finite-element models, also called compatible models by Pian and Tong [33], the displacements are assumed and are required to be continuous over the whole domain. They are the most commonly used finite-element models because of their inherent ease of development for most applications and efficiency of computation. Their ease of development is a result of the relatively loose restriction of continuity in the assumed displacement field which, on the other hand, leads to a loss of accuracy in predicting stresses because equilibrium of the stresses within the elements is satisfied only in an integral sense.

A full three-dimensional displacement-based finite-element approach was employed in some of the studies, including the works by Adams et al. [47], Hoa et al. [3], and Daooust and Hoa [23]. In Ref. [47], a non-linear material response and thermal residual stresses of porous laminates with ply drop-offs were investigated. Free-edge effects, however, were not included as a result of the generalized plane-strain assumption imposed. Finite-element meshing adopted in this model was very coarse in that at the ply drop region no longitudinal mesh refinement was made. In Ref. [3], the three-dimensional mesh at the ply drop region was refined by a submodelling technique. This approach involved successive reduction and refinement of the mesh in the region of interest while retaining the results of the previous iteration as boundary conditions for the refined mesh. The purpose of this method was to have a refined mesh in the region of large stress gradients while keeping the number of degrees of freedom of the solution required for each pass of the finite-element solver within the capability of the computer that was available. In Ref. [23], an extension of this work, three-dimensional finite elements were employed again with the development of a more efficient computer program for parameter analysis.

Some of the studies employed quasi-3D (Q3D) displacement-based finite element approaches by reducing the domain into a two-dimensional boundary problem based on the assumptions of either generalized plane deformation [37], or generalized plane strain. In both the theories, all the cross-sections would remain plane, and the stresses, geometric and material properties, and strains would be independent of the co-ordinate normal to the plane of analysis. The difference between these two theories lies in the fact that the former allows bending about the coordinates comprised of the analyzed plane and twisting about the remaining coordinate, of which the variables are independent. This theory is applicable to non-symmetric laminates under extension loading condition prescribed so as to accommodate deflections and rotations caused by the eccentricity of the load path. The latter is ideal for the analysis of long symmetric structures under a tensile loading condition. Typical applications of Q3D approaches based on the above theories for transverse-ply drop-off tapers include the work of Kemp and Johnson [1] and Curry et al. [2]. In each of these models, the displacements normal to the plane of the model were still included and therefore these models have five or six non-zero components of strains. In Ref. [1], four elements through the thickness in the plies in the vicinity of the drop-off were set up to perform the analysis as required to satisfy reasonably the continuity of interlaminar stresses and strains. The analyzed results show that the interlaminar stresses reach the maximum at the ply drop location. Curry et al. [2] conducted a global/local approach to tapered composite analysis. The global analysis was performed using the general-purpose computer program STAGS, while the local analysis for determining the three-dimensional state of stress in the vicinity of the dropped plies was based on the generalized plane deformation. Their study shows that interlaminar normal stress in the interface or resin layer reaches a maximum at the end of the dropped plies, and at the same location where the interlaminar shear stress is close to its maximum value. Variughese and Mukherjee [15] also made a global/local approach for the analysis of tapered composites. Considering that the drop-off need not pass through a nodal line in a global analysis, they developed drop-off elements that can be independent of the location of the drop-offs. The elements were used in a global analysis to reduce the size of the global structural matrix and showed more flexibility in the meshing division. An accurate stress distribution around the ply drop-offs was determined by local ana-
ysis with refined finite elements over the critical region and the input from the global analysis as the boundary conditions. Good correlation was found by comparing the results obtained using this approach with published results based on three-dimensional modelling.

Some of the authors, however, were more interested in performing plane-stress and plane-strain finite-element analysis by reasonably reducing the domain of the problem into two dimensions in order to avoid the computationally intensive nature of a three-dimensional finite-element model. Salpekar et al. [5] and Murri et al. [6,8,16] are among those authors who performed interlaminar analysis and, furthermore, determined the strain-energy release rate associated with the delamination growth.

3.3. Assumed stress hybrid finite-element approaches

The majority of finite elements used for stress analysis of tapered composites is based on displacement formulation, particularly employed in commercial software packages. This is because of the simple approach to the element formulation provided by the displacement model. However, there are some disadvantages inherent in the displacement approach in an analysis of a laminated composite which have limited its application in accurately describing the response of the critical area in tapered laminated composites. The main disadvantages of displacement elements include the fact that they cannot satisfy continuity conditions on displacements and transverse stresses at interlaminar surfaces as a result of the discontinuity in material properties, and the fact that the convergence of the displacement element model for problems with large gradients of stresses, as in the case of the drop-off location in tapered composites, is very slow [36]. Moreover, the modelling leads to an excessive requirement of computer resources for finer element meshing that is needed to determine more accurate structural and local responses of the composite. In general, analysis of tapered composites based on displacement approaches can only provide qualitative and trend information on responses of the structure under certain loading.

On the other hand, assumed stress hybrid elements, motivated by an attempt to overcome the disadvantages of displacement elements, was developed in 1964 by Pian [34] and have since been extensively applied in the analysis of regular laminated composites. As in the equilibrium model, this hybrid element model uses assumed equilibrating stress fields within the elements which enhance stress accuracy and also uses assumed boundary displacements in terms of nodal values such that they satisfy inter-element continuity.

Fish and Lee [4] first introduced hybrid elements in the analysis of tapered composites. In their work, 3D assumed hybrid elements were used to develop a methodology for the prediction of delamination onset in tapered composite laminates containing multiple ply-drop steps. The model contained 433 eight-node brick elements and six-node pentahedral elements for a total of 2916 global degrees of freedom. The eight-node hexahedral elements were based on an assumed stress hybrid formulation and could provide more accurate stresses than the linear displacement element. The six-node pentahedral elements were based on the assumed displacement formulation. The influences of the sub-laminates above and below the ply-drop steps were investigated. Both experimental testing and finite-element modelling of the tapered region were conducted. The failure of the tapered laminates is a result of the interlaminar shear stress and occurs at the last ply-drop step.

This approach has been systematically employed by the research group at University of Maryland. In addition to the above, the topics that they studied with this method also include delamination failure-mechanism analysis [9], delamination prevention techniques [24], effects of realistic taper geometries on the stress state at critical regions [27], shear-lag analysis about an internally-dropped ply [14], delamination of ply-drop configurations and tailoring concepts [28,39,40]. Vizzini [32] and Fish and Lee [4] also used the \textit{Q3D assumed stress hybrid} method to perform strength analysis of laminated composites with internal discontinuities parallel to the applied load, and to examine the free-edge effects in a dropped-ply specimen, respectively.

Another elaborate model that consists of shell elements, solid elements and transition elements was developed by Davila and Johnson [49] to capture post-buckling response in the internally dropped laminates. The shell elements employed to model the majority of the laminate is a nine-node assumed natural strain (ANS) element with 5 degrees of freedom per node. The solid element used to model the ply drop-off region is a 20-node serendipity brick element with 3 displacement degrees of freedom per node. A transition element that has 15-node element with 51 degrees of freedom per node and permits the connection of shell and solid element was constructed by degenerating the 20-node solid element. The influences of the geometric non-linearity on the stress concentration and the delamination initiation were examined through the analysis by this advanced element.

4. Delamination analysis

Delamination analysis of tapered composites involves determination of interlaminar stresses using finite-element methods as described in the first part of this review, prediction of delamination onset location, and simulation of delamination crack propagation.

In order to predict delamination onset and growth and, hence, the performance of the various laminates
studied, some kind of failure analysis was applied. Two general approaches exist for this purpose. They are the strength-of-materials approach (stress-strength approach) and the strain-energy-release-rate approach (fracture-mechanics approach). In the strength-of-materials approach, the local stress or strain state is compared to the material strength allowables. In the strain-energy-release-rate approach, which is based on fracture mechanics, the laminate is assumed to fail when the available strain energy of a delamination crack in a ply interface exceeds the critical strain-energy-release rate for the material.

4.1. Strength-of-materials approach

In application of this approach to perform delamination analysis, usually more than one failure criterion was used to predict the weakest location over the whole structure. Frequently, different criteria were used for prediction of in-plane and out-of-plane failure of the plies as well as for out-of-plane failure between plies.

Kemp and Johnson [1] used the maximum stress criterion to predict the failure in the rich resin surrounding the dropped plies, while applying the Tsai–Wu criterion for intralaminar failure prediction. With these criteria under consideration, they found that majority of the first failure events in either tension or compression is a resin failure in a few cases for which failure occurred at the re-entrant corner.

Both interlaminar and intralaminar failure criteria were used by Curry et al. [2] for their analysis. The interlaminar criterion they used, which is based on a matrix failure mode developed by Hashin [50], was evaluated at all interfaces between plies with different fiber orientations in the local model, while the intralaminar criterion was a modification of Tsai–Wu, in which only the strength parameters that correspond to the failure mode were included. The failure analysis with the above criteria and finite analysis results indicated that the first major failure event for the laminate studied was a delamination at the interface between the dropped ply and continuous ply that appeared to initiate at the end of the dropped ply. These failure analyses, however, underestimated the experimental failure load by more than 30%.

Fish and Lee [4] used a modified Tsai–Wu criterion to predict the out-of-plane failure of the composite laminates in their study. They introduced the average stress concept for the situations where the stress state is dominated by a single stress and applied it to the out-of-plane stress distributions obtained from the numerical analysis; thus, the maximum stress failure was considered. They found that the maximum stress criterion, using an interlaminar stress averaging distance of one ply thickness, provided consistent and accurate delamination onset predictions for the laminates investigated, which was also supported by the experimental observations.

On the basis of the assumption that the primary failure in the tapered composites is to be delamination and to occur in the interply resin layer, Vizzini [27,32] employed the von Mises stress criterion, an isotropic failure criterion, to be a measure of the overall stress state for a given configuration. In Ref. [27], the maximum von Mises stress in the realistic laminate with an ill-formed pocket modelled as to be four-sided rather than triangular as is usually assumed, or with unsymmetric ply drops, occurred around the last ply drop-offs. He found that the results agreed well with his finite-element analysis and, further, the von Mises stress for the laminate with a fracture void increased by more than 50%, which indicated that the presence of a void greatly affected the stress state around the ply drop and that the interlaminar stress criterion that excludes voids will overpredict the onset of damage. In Ref. [32], the von Mises criterion was used to determine the strength of the resin pocket at the discontinuity. Falling within the scatter of the experimental data for delamination initiation, this resin pocket model predicted very well the initiation of damage for the laminates dominated by the internal edge failure.

Harrison and Johnson [13] used a delamination fraction concept, which was proposed by Brewer and Lagace [38], as a measure to investigate the effect of eccentricity and stiffness discontinuity on the tendency of laminates with dropped plies to delaminate. In combination with their mixed variational approach, they found that the highest delamination fraction value is contributed by both the interlaminar normal and shear stresses at the ply drop region and show that it is the stiffness discontinuity rather than eccentricity of the laminate that has a larger influence on the interlaminar stresses and eventual delamination.

Thomsen and Mortensen [18] applied a point stress criterion for prediction of delamination failure in composite laminates with external ply drop-offs. The stress at a certain characteristic length away from the drop step was evaluated with a simplified analytical model. Delamination failure is considered to occur if the calculated stress exceeds the strength of the interply resin layers. An empirical formula for the characteristic length calculation and the respective characteristic length stress criterion were suggested. It was concluded that the proposed approach works well as a result of the good match between experimental and analytical results.

It is seen that in a strength-of-materials approach for delamination analysis, two kinds of criteria, i.e. the point-stress approach, as used in Ref. [43], and the stress-averaging approach, can be identified in terms of the techniques used to obtain principal stresses, both of which were introduced as a result of the singular nature.
of interlaminar stress distribution at ply drop-off positions, where the interlaminar stress peaks occur and by far exceed the interface material allowables at the load levels where delamination failure can be detected experimentally. Thus, it is impossible to provide physically meaningful prediction delamination initiation in laminated tapered composites by direct application of the calculated peak stresses together with some point-stress criteria. To overcome the difficulties induced by the inherent singularity, some other techniques like 'stress averaging' or 'effective/characteristic length' were applied and proved to be effective in dealing with delamination initiation analysis of tapered laminates.

4.2. Strain-energy-release-rate approach

Strain-energy-release rate is a concept from fracture mechanics. It may be interpreted as the amount of work required to close a delamination by an incremental length. Much of the work has been done to calculate the modes of strain-energy-release rate using the finite-element method for delamination in tapered composites.

In Ref. [48], the general design guidelines and analysis capability for the prediction of delamination of tapered composites are presented. Two cases are studied, i.e. zero transverse strain for a wide plate or zero transverse load for a longitudinal strip. It is indicated that a value for the critical strain-energy-release rate associated with delamination between layers is required for delamination initiation and growth analysis, and notes are included on how that may be measured experimentally. Guidance is given on good design practice in tapering a laminate to reduce the likelihood of delamination. Results from the analysis are compared with limited experimental data from the literature, and agreement is seen to be reasonable.

Salpekar et al. [5] conducted delamination analysis of tapered composites with this approach. The virtual crack-closure technique (VCCT), combined with 2D finite-element model, was used in their analysis to obtain the strain-energy-release rate components, in mode I \((G_I)\), and in mode II \((G_{II})\), based on the local forces at, and ahead of, the delamination tip and the relative displacements behind the delamination tip. Two models were shown in this work, one for the interlaminar stress distribution along the interface BCDE (Fig. 3) and the other for the strain-energy-release-rate variation for various size delaminations assumed along the interface BCDE. With the first model, they found that the interlaminar normal stress shows peaks near the ply drop-offs and the largest one occurred at the transition point D. The sudden changes in the stress distributions at the drop-off location indicated that stress singularities are more likely present. In the second model, the strain-energy-release rate was calculated for a delamination assumed to initiate at the juncture of the intersection of the taper and the thin laminate, point D. The delamination along the thin section of the laminate consisted predominantly of the mode I component, while the delamination along the thick section initially consisted of the mode I component and was replaced by the mode II component afterwards. This linear fracture-mechanics approach presented a new vision to predict failure of tapered composites, but it lacked an experimental investigation to validate the conclusions drawn.

Trethewey et al. [46] also employed linear elastic fracture mechanics to determine the mode I and mode II components of the strain-energy-release rate. Their analytical model was based on shear deformation plate theory with a through-width delamination embedded at the interface between continuous and discontinuous sublaminates. The influence of geometry and material properties on the structural performance of the tapered laminate was determined with the parametric study. It was shown that among the geometric parameters, the number of discontinuous layers at a single axial position had the strongest influence, while the crack size of the delamination had a less pronounced effect.

Murri et al. [6] later extended their analysis to fatigue delamination onset prediction in unidirectional tapered laminates, using nearly the same techniques as in Ref. [5] except for the experiments included to verify the analysis results. It was shown in the experiments that, for some of the laminates studied, initial stable delamination that often started with a resin crack at the drop-off and final unstable delaminations that initiated at the junction of the thin and thick sections of the laminate were observed. Finite element calculations for the strain-energy-release rate associated with the initial resin delamination showed good agreement with this phenomenon.

Ref. [8] by Murri et al. was a follow up of the analysis in Ref. [5], in which delamination of tapered multi-angle laminates under tension fatigue loading was examined numerically and experimentally. In addition to the delamination existing at the interface between the belt and core as in Refs. [5,6] the matrix ply cracking resulted from the presence of \(±45°\) plies in the laminate of analysis was also modelled by observed failure mode. Only one type of the laminates examined tended to fail as modelled using the finite-element analysis since the delaminations in other types of laminates were dominated by matrix ply cracks and were always at locations other than interface BCDE.

In Ref. [16], Murri et al. examined the effect of combined tension/bending loading on glass/epoxy laminates with a non-linear taper and internal ply-drops. The delamination growth originating from the initial tension crack at the drop-off was simulated in the 2D finite-element model by releasing pairs of multi-point constraints at the critical interfaces, and the strain-energy-release rates were thus calculated using VCCT for a delamination...
starting at the ply drop-off location and growing toward the thick or thin section. They found that the initial delamination grows first toward the thick section where the delamination is predominately $G_{II}$ (shear mode), and grows all the way, as the fatigue loading was continued, to the junction of the tapered and thick section where the delamination is predominately $G_{I}$ (opening mode). The results obtained from their model also indicated that the mode ratios are very sensitive to the discrete angle changes in the model.

Wisnom et al. [10] presented the results of the tests carried out on the rapidly tapered specimens with dropped $\pm 45^\circ$ and $0^\circ$ plies in order to determine static and fatigue strengths. The failure modes for the three types of specimens studied are fiber breaks initiated near the first dropped ply or delamination occurring at the dropped $0^\circ$ plies which are more susceptible to delamination than $\pm 45^\circ$ plies. Wisnom et al. continued this analysis in the paper [12] by comparison of tapered laminate delamination with delamination in internal cut plies under fatigue loading. They found that the delamination in the cut-ply specimens propagated in both directions from the cut, whereas for the dropped-ply specimens it propagated only to the thick end with a slower delamination rate than for the cut-ply specimens as a result of the effect of through-thickness compressive stresses in the region where delamination initiated. Normalization of strain-energy-release rates calculated from a simple equation was made by dividing the cyclic strain-energy-release-rate range $\Delta G$ by the fracture energy $G_c$ deduced from a static tension delamination test, and obtained the similarity between the delaminations in the specimens under fatigue and static loadings.

Wisnom et al. [11] chose three asymmetrical composites with $0/\pm 45^\circ$ lay-ups loaded in tension to carry out an experimental investigation on the effects of the tapered geometry and the stiffness of the discontinuous plies for asymmetrically tapered sections based on the conclusion that the strain-energy-release rate associated with the discontinuous plies was the critical factor controlling delamination into the thick section, with the effect of the tapered geometry being of secondary importance. In comparison with the previous results for the thick section delamination failure mechanisms in the symmetric tapers studied in Ref. [12], they concluded that the asymmetry does not appear to have a significant effect on thick-section delamination. Existence of thin-section delamination induced by specimens tapered geometry, which is less severe than thick-section delamination, however, showed a different delamination behavior from the previously tested symmetric specimens and, therefore, further investigation was encouraged to explain it. The taper angle and the degree of consolidation in the region around the dropped plies are likely the reasons suggested by the authors for this behavior. A summary of the results for transverse-ply-drop tapers is listed in Table 1. It is indicated in this table that the specimens often used for analysis and testing were made from glass/epoxy or graphite/epoxy, and were configured in multidirectional and symmetric form. The majority of the work was based on experimental programs wherein the test coupons were subjected to static and/or fatigue loading. Various finite-element modelling and non-finite-element approaches were used in the analysis of tapered laminates. Strength-of-materials criteria and fracture criteria were almost equally applied by the authors. The maximum interlaminar shear stress was found by most works to appear at the ply drop step, while the maximum interlaminar normal stress was found to appear at the ply drop step by about half of the authors, and at the taper root by the other half of the authors. The final delamination will grow into the thick and thin sections simultaneously, but the location of delamination initiation was found to appear at the ply drop step in some works, and at the taper root in others.

There are many factors contributing to the stress state, but among them the most important one is the configuration of a laminate. Thus, for different configurations of a laminated composite, it is impossible to make a categorical statement as to the location where stress peaks appear. However, the contradiction in this regard with an identical configuration under the same loading conditions and constraints can only be attributed to the methodology used in modelling the taper. It seems that the model where a three-dimensional assumed hybrid element was implemented with the inclusion of interply resin layers, approximates the true stress state better than others. A zero-thickness resin layer causes singularity at the ply drop region, and variation of interply layer thickness can alter the stress state at the ply drop region. So, a model without relatively accurate thin interply resin layers is inadequate if refined results are desired.

5. Parametric study and design considerations

In order to design damage resistant tapered structures, many parameters such as taper geometry, locations of ply drops, and configurations of ply-drops through the thickness that would affect delamination at dropped plies, have been studied [23–28,39,40]. Moreover, optimization design considerations for the external tapered composite structures have been investigated [29,30].

Daoust and Hoa [23] developed an extensive finite-element program for the study of tapered laminates. The parameters that influence the strength of the laminate were also examined through evaluating the efficiency of the tapered laminates. The efficiency is defined as a ratio of the maximum applied load with the drop-
off to the one without drop-off, and it can be calculated using the finite-element method. The analysis results show that internal drop-offs are roughly 2 times stronger than external ones; that the layer drop-off does not affect torsion resistance; that extending the length of the drop-off hole while keeping the same drop-off height reduces interlaminar stress level.

Llanos and Vizzini [24] evaluated two commonly used tailoring techniques in free-edge delamination prevention in flat laminates, i.e. addition of a softer inner layer (structural adhesive) and ply angle alteration, for the prevention of delamination in tapered structures. Another technique applied and approved to be efficient by the comparison of the analysis results and experimental observations was to add resin layers in their model. It was demonstrated that the addition of adhesive film reduced the interlaminar stresses in some of the components analyzed, but provided no significant change in the others. Both alterations of the small internal edge of the last ply drop and the complete ply drop can be introduced to produce a substantial reduction in the magnitudes and gradients of the interlaminar stresses at the last ply drop region.

Thomas and Webber [25] used linear elastic fracture mechanics combined with simple strength of material theory to predict the tensile delamination load of a tapered laminated plate subject to a certain geometric variation. It was shown that the delamination load is very sensitive to the thickness of the dropped sublaminate and that varying the lay-up angle of a dropped sublaminate from 0° to 90° with respect to the direction of loading could increase the delamination load as the angle tended toward 90°.

Cui et al. [26] investigated the effect of the distance between neighboring drop steps in a staircase arrangement, with the objective of finding out the point at which the interaction between neighbouring steps becomes significant. The critical stepping distance cal-
culated with a simple formula was used to assess the extent to which the step spacing affects the delamination stress. The step spacing could have a significant effect on delamination, particularly within small range of step spacing. Based on their newly developed variable fracture energy concept they also concluded that the fracture energy is not a material constant.

Vizzini [27] studied the effects of realistic taper geometries on the stress state at and near the ply drops using finite-element analysis. Ill-formed pockets, non-symmetric ply drops, and fracture resin pockets (voids) were considered in his model. It was concluded that all realistic geometries tend to increase the interlaminar stress state and the effect that these geometries have on the damage onset point and the failure mode is of importance. Therefore, any quantitative results from analysis models that do not take into account realistic geometries may be questionable and even misleading.

Botting et al. [28] examined through finite-element analysis the stress state in tapered laminates with different ply-drop configurations with the inclusion of a stress-free-edge effect. One standard staircase ply-drop configuration and the three alternate ply-drop configurations, as shown in Fig. 4, were investigated. In all cases studied, the results from finite-element analysis showed that altering the ply-drop configuration could decrease the stress state at the ply drop. This conclusion was experimentally validated by the improvement in the damage onset stress of the laminate investigated. Fish and Vizzini [39,40] continued this work by analyzing unidirectional glass/epoxy tapered laminates with four different ply-drop configurations and failure modes, as shown in Fig. 5. Two of them were chosen for further study under cyclic loads. Their analysis indicated that tapered laminates can be tailored for stiffness and strength by altering the internal ply-drop configuration. The overlapped-dispersed configuration could achieve stable delamination initiation and growth and provide the best overall structural performance with an intermediate delamination strength, the highest bending stiffness retention, and good damage tolerance characteristics.

Manne and Tsai [29] investigated how sublaminates made of multiple plies at various orientations, combining one or more materials are repeated or dropped in different zones of the structure, yielding the external ply-drop taper while ensuring physical continuity of the fibers in all composite layers. The orientation and thickness of each ply group in this reference sub-laminate as well as its number of repetitions in the zones across the structures were optimized with the objective of minimum weight, subject to the constraints of strength, stiffness and manufacturing complexity. This new design methodology was an attempt to combine the considerations of low weight and easy manufacturing requirements. The introduction of a reference sub-laminate, called base sub-laminate or design unit, could achieve these double objectives. The sublamine repeated or dropped a given number of times forms a laminate. The optimum uniform-thickness design and the best quasi-istropic design are, respectively, 16 and 110% heavier than the optimum ply drops one.

Cairns et al. [30] explored various factors, such as thickness, ply stacking sequences, ply drop geometries and manufacturing considerations for design of composite blades with ply drops. Fatigue loading was also considered with respect to delamination initiation and growth. Delamination-prevention techniques, such as the inclusion of random mat fabric between the ply drop and the continuous layer, ‘feathering’ (alternating tows are pulled out to provide a less defined delamination site), ‘Z-spiking’ (removing the scrim from the fabric and driving the fiber tows into the lower layers), and addition of an adhesive region were used to enhance the structural integrity. Two epoxy adhesives were applied

![Fig. 4. Schematic drawings of alternate ply-drop sequences in Ref. [28].](image1)

![Fig. 5. Schematic drawings of alternate ply-drop configurations and failure modes in Refs. [39,40]. Notation for drop-off sequence and configuration: (i) \|, (ii) \ and (iii) \ stand for a ply or sub-laminate that is (i) dropped, (ii) folded and dropped, and (iii) continuous, respectively. Piles or sublaminates are countered from the top to the bottom of the laminate, excluding the belt and core plies. The drop-off sequence starts from the left side of each notion.] (image2)
to repair the delamination in an exterior ply drop sample. The general conclusions drawn from their work about delamination and preventing delamination can therefore be summarized as follows. An optimum configuration of dropping plies is to have an internal ply drop, with a combination of either ‘Z-spiking’, an adhesive, or ‘feathering’ used in the construction. The aforementioned techniques are shown in Fig. 6.

6. Concluding remarks

To describe accurately the delamination mechanisms in tapered laminate composites and to correlate them well with experiments have been challenging tasks for researchers for more than a decade. The difficulties in modelling tapered composite structures for stress analysis, and delamination initiation and growth lies in their geometric and material discontinuities as well as the free-edge effect. All of these would give rise to complicated stress distribution around the ply drop-off region; typically, interlaminar stresses that would cause delamination failure of the whole structure.

Both FEM and non-FEM approaches were implemented to determine the interlaminar stress profile around ply drop-offs, initial delamination onset location along with a strength-of-materials-based criterion, and to stimulate delamination mechanisms by releasing failed elements. They were also used to calculate the strain-energy-release rates associated with delamination within tapered laminates for the fracture mode I (opening) and mode II (shearing). Hybrid elements were overwhelmingly applied in 3D problems with the consideration of including the free-edge effect, while easily formulated displacement elements were preferred by most authors for solving the 2D problem.

Several distinguishable methodologies for analysis of delamination initiation and propagation can be identified according to finite modelling, the failure criteria chosen, and special techniques authors applied.

Vizzini and his associates used 3D or Q3D assumed stress hybrid elements to determine accurately the interlaminar stresses together with the application of a stress averaging concept and applied strength-of-materials criterion for failure analysis. The stress free-edge effect was included using a coarse mesh in some of their analyses. Resin layers were introduced in their models so as to reflect the accurate stress state at and near the drop steps in tapered laminates. It was found in their analysis that, in most of the laminates studied, both maximum interlaminar stresses (normal and shear) and delamination initiation located at the ply drop step (point C in Fig. 3), where stress singularity was likely generated. A delamination initiation location is significantly affected by local taper angles, the amount of offset of the ply drops and presence of fracture pockets in realistic tapers [27]. They concluded that delamination growth simulated by the progressive damage finite-element model could agree with experimental observation and hence correctly predict stable and unstable growth only with assumed initial damage [9]. With respect to parametric study, structural tailoring techniques, such as edge alterations, changing ply-drop configurations and addition of film adhesive between the critical interfaces, were investigated to find out the effects of these changes on the structural integrity.

Murri, O'Brien and Salpekar [5,6,8,16] used a 2D displacement-based finite-element and strain-energy-release rate approach to analyze tapered laminates loaded in tension and/or fatigue. In their studies, the maximum interlaminar normal stress was found to locate at the taper root (point D in Fig. 3), while the maximum shear stresses at the drop-off step (point C). The location where a delamination initiated was either assumed [5,6,8] to be at the taper root or directly borrowed [16] from the experimental observation. Delamination growth could be further simulated by the finite-element model.
However, the efficiency and accuracy of the simulation wholly depends on the delamination initiation site chosen. Wisnom et al. conducted their experimental and analytical works through behavior comparisons between tapered laminates with drop-off plies and laminates with cut plies/untapered laminates with the same discontinuous plies, and between tapered laminates with drop-off plies and the same geometrical laminates with low-stiffness fill-in discontinuous plies. On this basis, they concluded that, for both symmetrically and unsymmetrically tapered laminates, the strain-energy-release rate associated with discontinuous plies was the critical parameter controlling delamination into the thick section, while the taper geometry effect was the primary factor for delamination into the thin section. Confirmed by finite-element analysis, a simple calculation model for the strain-energy-release rate was applied in their analysis of delamination initiation.

Another research area of interest, which is pursued by Thomsen et al., is on the study of the interlaminar response in sandwich panels with internal or external tapered face sheets. They resort to a simplified model, instead of a FEM model, as a tool to perform the analysis. Experimental investigations were conducted to validate analytical results.

All other works focused mainly on stress analysis around critical regions over the tapered laminate.

It can be seen in this review that the finite-element method is an almost indispensable tool in the analysis of tapered composites, either for determining interlaminar stresses and strain-energy-release rate or for verifying simple formulations. Both hybrid and displacement elements were widely employed, with the former providing more accurate stresses than the latter. Finite-element analysis of taper composites was implemented by researchers by directly applying elements developed for the analysis of continuous media to the laminated tapers, without taking into account the multilayer nature of the laminated structures. In addition, neither of these two types of element could completely satisfy displacement and traction continuities simultaneously along bi-material interfaces in tapered composite structures. Moreover, the overwhelming dependence of finite elements on modelling the tapers would require huge computational resources for accurate analysis. Because finite-element modelling is carried out subject to personal experience and the methodology chosen, a variation of results would be expected, even for the same problem. Non-FEM approaches that are formed based on physical concepts, and that can provide equivalent accuracy to the FEM approach, deserve special attention. Fracture mechanics and damage mechanics may provide an effective way for delamination analysis of laminated tapers according to their structural characteristics.

Experiments are essential for tapered laminate analysis. Correct prediction of delamination initiation and propagation by finite-element modelling and the strain-energy-release approach depends on the initial crack site observed in experiments. An initial crack site assumed without more theoretical justifications may lead to wrong conclusions about delamination failure mechanisms unless an experimental program is conducted to validate them.

In contrast to the strain-energy-release-rate approach, the strength-of-materials approach is more efficient in seeking for the point of delamination initiation, as was done in some of the works. However, the strength-of-materials approach, which is based on the mechanics of materials, cannot characterize delamination propagation as efficiently as the strain-energy-release-rate approach because the delamination propagation undergoes a progressive growth phenomenon. Therefore, both these approaches have their own merits and limitations in failure prediction.

In addition to the strain-energy-release rate, the crucial parameters that have significant effects on delamination in tapered laminates also include the geometry of the taper, especially the angle of the taper. Therefore, ideally modelled tapered sections without consideration of variations formed as a result of manufacturing tolerance are inadequate if a quantitative analysis is desired.

Increasing the structural integrity of the tapered section can be realized by application of a few rules that have been drawn from the parametric study. For example, the addition of film adhesive around the dropped plies to strengthen the region about the drop can modify the load transfer, thus decreasing the interlaminar stress state. This method is successful in certain configurations. Reconfiguration of the ply drop may also increase the structural integrity and alter the failure mechanism. The taper with dropped plies interleaved with continuous plies displays better behavior than the other taper with grouped dropped plies. A minor alternation or modification to the constitution and geometry of the taper may lead to a completely different mechanical behavior for the structure studied.

Based on the reviewed works, the following aspects with regard to analysis and design of internal tapered laminated composite structures need to be investigated further from the point of view either of filling up blank areas of interest or of improving the methodologies available.

Firstly, it is noted that material non-linear analysis of internal tapered laminates has never been conducted by previous researchers. However, a realistic response in internal tapered laminates significantly depends on constitutive laws that describe material properties. Both composite laminae and resins that constitute a laminated structure exhibit non-linear stress/strain properties, especially for shear stress/strain relations of lamina and for all properties in resins. Material non-linear properties must be incorporated for a quantitative analysis of tapered laminates.
Secondly, further investigations on the improvement of finite-element modelling incorporating interlaminar characteristics of laminated composites is required so as to increase computational efficiency shown in displacement-based FEM or assumed stress FEM. Development of new finite elements is required in this regard.

Thirdly, investigation into the influences of some structural parameters, such as resin toughness on interlaminar fracture toughness of tapered laminates, is needed in order to gain insight into employing toughened composites laminated tapers in engineering applications.

Fourthly, reactions of multiple delaminations in the tapered laminate have not been considered so far. Failure mechanisms induced at geometric and material discontinuities can be thoroughly understood only after gaining this insight.

Lastly, optimization of tapered laminated composite structures such as helicopter-yoke arms and rotor blades should be targeted. This work will involve devising a method of defining variations of structural properties as functions of the construction and materials of optimized laminates. The optimization techniques should also incorporate manufacturing constraints.

References


