Fatigue damage mechanics of composite materials. II: A damage growth model

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1 INTRODUCTION

Observations of fatigue damage in carbon fibre/epoxy laminates described in our first paper (Part I)\(^1\) indicate that the dominant failure mechanism is matrix cracking in various forms, including splitting within 0° plies, transverse ply cracking within 90° plies and delamination between 0° and the off-axis plies. Any fibre fracture that occurs in a 0° ply in cyclic loading is a consequence of matrix cracking in the 90° ply.\(^1\) However, there is a fibre element; a high fibre-packing density implies that the matrix is highly constrained. Fatigue damage, therefore, consists of several interacting, planar cracks whose formation and growth depend on the properties and fatigue behaviour of a constrained matrix.

In Part I\(^1\) it was shown that fatigue damage at notches in composite materials could be characterised by the split length, \(l\), at the notch tip (Fig. 1). The starting point of our model is the fatigue crack growth law ('Paris' law) valid for many isotropic materials, including epoxy resin:\(^2\)

\[
\frac{da}{dN} = \lambda_1 (\Delta K)^m
\]

where \(da/dN\) is the crack growth rate, \(\Delta K\) is the stress intensity factor range and \(\lambda_1\) and \(m\) are empirical constants.

This can be recast in terms of the split growth rate, \(dl/dN\), and energy release rate, \(G\) (using \(K^2 = Y G\), where \(Y\) is an appropriate modulus):

\[
\frac{dl}{dN} = \lambda_3 (\Delta G)^{m/2}
\]

Others have followed a similar approach. For instance, O'Brien\(^3\) used a power law of this form to describe the growth of edge delaminations in a laminated material, and Gustafson and Hojo\(^4\) developed a similar expression in which the mode I and mode II contributions to \(\Delta G\) were treated separately. This is inadequate, however, when splits and delaminations grow in combination at the notch tip, as they do in carbon fibre/epoxy laminates. As the split length, \(l\), increases, the associated delamination area grows with a dependence on \(l^2\), implying an increasing resistance to further crack advance (Fig. 1). It is more appropriate, therefore, to normalise \(\Delta G\) by the current toughness, \(G_c\) (as measured in monotonic loading), giving

\[
\frac{dl}{dN} = \lambda_3 \left[ \frac{\Delta G}{G_c} \right]^{m/2}
\]

where \(\lambda_3\) is a constant. In essence, the rate of damage growth is related to the ratio of the
driving force, $\Delta G$, to the current damage growth resistance, $G_c$. Poursartip suggested a similar formulation suitable for delamination growth in which $R$ curve behaviour (increasing toughness with crack length) occurs.

In the second paper of four, a model based on eqn (3) is formulated and used to predict fatigue damage growth at a notch. First, we present an introduction to the concept of damage in laminates as applied to damage growth in monotonic loading.

2 DAMAGE GROWTH IN MONOTONIC LOADING

Previous papers have addressed the application of fracture mechanics to the growth of matrix cracks in monotonic loading. Damage which forms at a notch in a $(90/0)_s$ laminate is considered to grow by the simultaneous formation of splits in $0^\circ$ plies, transverse ply cracks and triangular-shaped delamination zones at $(90/0)$ interfaces. For the time being, we ignore the existence of transverse ply cracks. The growth of the damage zone is self-similar, i.e. the shape of the delamination front remains constant as does the angle it makes with the split; it simply grows in scale (Fig. 1(a)).

An energy balance can be formulated for one quadrant of the specimen (width $W/2$, length $L/2$, thickness $2t$) which consists of one $0^\circ$ ply and one $90^\circ$ ply (Fig. 1(b)). For an increment of split growth, $\delta l$, the energy absorbed in forming new crack surfaces, $\delta E_{ab}$, is given by:

$$\delta E_{ab} = G_i \delta l + G_d (l \tan \alpha) \delta l$$

where $G_i$ is the energy absorbed per unit area of split, $G_d$ is the energy absorbed per unit area of delamination, $t$ is the thickness of a $0^\circ$ ply and $\alpha$ is the delamination angle at the split tip.

Some of the global energy of the system (strain energy + potential energy due to the applied load) is dissipated when the split extends with a corresponding increase in specimen compliance, $\delta C$,

$$\delta E_i = -\frac{1}{2} P^2 \delta C$$

$P$ is the applied load on one quadrant of the specimen (Fig. 1), given by

$$P = \sigma_0 (W/2)(2t)$$

where $\sigma_0$ is the remote applied tensile stress on the specimen.

For the damage zone to grow, $\delta E_i \geq \delta E_{ab}$:

$$\frac{1}{2} P^2 \delta C \geq G_i \delta l + G_d l \delta l \tan \alpha$$

For the limiting case ($E_i = E_{ab}$):

$$\frac{P^2}{2 \delta l} = G_i + G_d \frac{t \tan \alpha}{t}$$

$G_i$ is the effective toughness of the laminate and $G$ is the strain energy release rate. This expression can be evaluated if $\partial C/\partial l$ is known. It is difficult to calculate it analytically; instead, a finite element model was constructed to determine it numerically. A brief description of the method, together with results, are given in Appendix 1. For a range of split lengths between $l/a = 0$ and $l/a = 6$ ($2a$ is notch length), $\partial C/\partial l$ was found to be constant, dependent only on the angle $\alpha$ and the fibre lay-up geometry.

Rearranging eqn (8), the load for split initiation is given by

$$P_i = \left[ \frac{2G_i \delta l}{\partial C}_{l=0} \right]^{1/2}$$

For subsequent split growth under monotonic loading:

$$l = \frac{P^2 (\partial C)}{2G_3 \tan \alpha - G_d \frac{t}{\tan \alpha}}$$
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Dividing both sides of eqn (10) by notch length, \( a \), and substituting for \( P \) and \( \partial C / \partial l \) (scaled from the finite element result, \( \partial C / \partial l \mid _{FE} \) (eqn A1.1)), this expression can be normalised as follows:

\[
\frac{l}{a} = \frac{(\sigma_a)^2 l (W/2)_{FE} (2a)_{FE} \partial C}{G_d \tan \alpha (2a/W)} \mid _{FE} - \frac{G_d}{G_o} \left[ \frac{t}{a \tan \alpha} \right]
\]

(11)

In practice, \( G_o \ll G_d \), hence, the second term of eqn (11) is small, allowing the simplification:

\[
\frac{l}{a} = \frac{(\sigma_a)^2 l (W/2)_{FE} (2a)_{FE} \partial C}{G_d \tan \alpha (2a/W)} \mid _{FE} - \frac{G_d}{G_o} \left[ \frac{t}{a \tan \alpha} \right]
\]

(12)

Equation (12) indicates that for a given \( 2a/W \), the ratio \( l/a \) is dependent only on applied stress, \( \sigma_a \). The normalised split length with applied stress curves for all (90/0), specimens should be coincidental, regardless of width. Figure 2 shows split growth data for three (90/0), specimens (\( W/2a = 3 \)) under monotonically increasing load. Equation (12) produces a good fit to the data using selected values of \( G_d = 400 \text{ J m}^{-2} \) and \( G_o = 158 \text{ J m}^{-2} \). (These values are identical to those chosen by Kortschot and Beaumont for a similar material system.) A value of \( \alpha = 3.5^\circ \) was determined experimentally for the (90/0), laminate.

Now we apply the model to fatigue damage.

3 FATIGUE DAMAGE MODEL APPLIED TO (90/0), LAMINATES

We start with eqn (3). Inserting the definition

\[
\Delta G = \frac{(\Delta P)^2}{2t} \frac{\partial C}{\partial l}
\]

(13)

and the result derived from eqn (8)

\[
G_c = G_s + G_d \frac{t \tan \alpha}{t}
\]

(14)

gives

\[
\frac{dl}{dN} = \left[ \frac{1}{2} (\Delta P)^2 \left( \frac{\partial C}{\partial l} \right) \right]^{m/2}
\]

(15)

For an initial split length, \( l_0 \), the split length after \( N \) cycles of constant load amplitude is given by the integrated form of eqn (15):

\[
l = \frac{1}{G_o \tan \alpha} \left[ \left( \lambda (\Delta G) \right)^{m/2} \frac{m+2}{2} \right]
\]

(16)

where \( \Delta G \) is given by eqn (13). The split length in the first cycle, \( l_0 \), can be determined from eqn (12). It only remains to identify values for \( \lambda \) and \( m \).

Figure 3 shows split growth data for a (90/0), laminate cycled at three different peak stresses. Assuming a delamination angle, \( \alpha \), of 3.5\(^\circ\), \( \lambda = 8 \times 10^{-4} \) and \( m = 14 \), a good fit is obtained to the data. The value of \( m = 14 \) is in agreement with the fatigue results of Gustafsson and Hojo.

The delamination angle, \( \alpha \), is related to split length (eqn (16)) essentially by the parameter \( \phi \):

\[
\phi = \left[ \frac{\partial C}{\partial l} \right]^{m/(m+2)} \tan \alpha
\]

For \( \alpha = 3.5^\circ \), \( \partial C / \partial l = 2 \times 10^{-7} \text{ N}^{-1} \), \( \tan \alpha = 0.0612 \), \( \phi = 2.13 \times 10^{-5} \); for \( \alpha = 7^\circ \), \( \partial C / \partial l = 3 \times 10^{-7} \text{ N}^{-1} \), \( \tan \alpha = 0.1223 \), \( \phi = 1.51 \times 10^{-5} \).

The two values of \( \phi \) are within 30% of each
other; therefore, an inaccuracy of 100% in determining $\alpha$ leads to only a small error in predicting the extent of damage.

The effect of load amplitude on split growth in a (90/0), laminate is shown in Fig. 4. Data of split length (for $N > 10^6$ cycles) are shown for a peak stress of 324 MPa and $R$ ratios between 0.1 and 0.7. The model provides a reasonable description of the observed decrease in split length with increasing $R$ ratio. Furthermore, there is some evidence of an additional mean stress effect that is not predicted by the model.

For laminates with different values of $W/2a$, $\partial C/\partial l$ can be obtained either by modifying the finite element mesh, or by using elasticity theory to extrapolate from the values of $\partial C/\partial l$ for $W/2a = 3$. Values of $\partial C/\partial l$ for $W/2a = 9$ are included in Table A1.1. The model predicts small variation in damage growth for $W/2a > 3$, which is supported by experiment.\textsuperscript{1}

4 FATIGUE DAMAGE GROWTH IN (90/0), LAMINATES

For laminates containing 0° and 90° plies of varying thickness, the nature of damage is essentially the same. The delamination angle, $\alpha$, is between 3°.5° and 7°, tending towards the upper limit with increasing 0° ply thickness. This generic similarity allows the same finite element model to be used for all laminates investigated. The meshes used for the (90/0), specimens can be altered by simply adjusting for relative ply thickness. It is assumed that the damage pattern is similar in all plies and that the delamination shape is the same at all interfaces.

The basic fatigue damage growth law (eqn (3)) has to be modified to cope with the different laminate geometry which controls damage growth. In the following section these modified growth laws are derived. For convenience, the growth laws are kept in differential form but used in integrated form.

4.1 Damage growth in (90/0), laminates

For this family, $\Delta G$ increases linearly with ply thickness; $i \times t$. As for the (90/0), case, there are only two interfaces at which delaminations can grow. If delamination growth is the dominant energy-absorbing process (and it is, since $G_d > G_s$, and delaminations have a greater surface area than splits), then the ratio $\Delta G/G_s$ increases as $i$ increases. The fatigue law has the form:

$$\frac{dl}{dN} = \lambda \left[ \frac{1/2(\Delta \sigma)^2(W/2)(2it)(W/2)F_E(2a) FE}{G_s it + G_d \tan \alpha} \right]^{m/2}$$

Figure 5 illustrates the increasing rate of damage growth with increasing ply thickness. There is good agreement between experimental data and the predicted split growth curves.

4.2 Damage growth in (90/0), laminates

Laminates with 90° and 0° plies of unequal thickness can be modelled in a similar way. The relative ply thicknesses of the layers in the finite element meshes are adjusted, and the resulting values of $\partial C/\partial l_{FE}$ are scaled appropriately.
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The fatigue law is, therefore, modified to

\[ \frac{dl}{dN} = G_i nt + G_d (2n - 1) t \tan \alpha \]

As the number of plies increases, the model predicts that the rate of damage propagation decreases slightly. In this instance, the model tends to underestimate damage growth (Fig. 7). This may be the result of unequal damage growth through the thickness of the laminate. When damage is measured by X-ray, the greatest extent of splitting is recorded, not the average split length which the model predicts. In summary, the model includes the important parameters and processes which govern fatigue damage behaviour. The ability to model fatigue damage growth in a family of laminates without altering the parameters that appear in the equations is a significant advance on previous methods.

5 Damage Growth in (90/0)m Laminates

For these laminates, it is assumed that damage evolution is controlled by splitting in the 0° plies and by delamination at the interface between the 0° ply and the innermost -45° plies. The shape of the delamination was identified in Part 1. The finite element representation is described in Appendix 2.
5.1 Damage growth in monotonic loading

Consider 1/8 of the specimen \((W/2, L/2, 90°+45°/-45°/0)\) as shown in Fig. 8. The energy absorbed by splitting in the 0° plies and by delamination at the −45/0 interface is given by

\[
\delta E_{ab} = G_t \delta l + G_a \left( \frac{l + k}{2} \right) \delta l
\]

where \(k\) is the dimension defining the intersection of the delamination with the centre line of the notch (Fig. 8).

It was observed that the delamination is bounded by off-axis ply cracks tangential to the notch tip. Thus,

\[
k = (\sqrt{2} - 1) \rho
\]

where \(\rho\) is the notch tip radius. The energy released due to the increase in compliance \(\delta C\) remains (eqn (5)):

\[
\delta E_r = \frac{1}{2} P^2 \delta C
\]

For split growth in monotonic loading, the energy balance \(\delta E_r = \delta E_{ab}\) is satisfied. Thus, the equation equivalent to eqn (7) for this laminate is

\[
\frac{1}{2} P^2 \delta C > G_t \delta l + G_a \left( \frac{l + k}{2} \right) \delta l
\]

For the limiting case:

\[
\frac{P^2 \delta C}{2l \delta l} = \frac{G_t}{G_a} \left( \frac{l + k}{t} \right)
\]

from which

\[
l = \frac{P^2 \partial C}{G_t \partial l} \frac{2G_a}{G_t} - k
\]

The finite element representation of this damage mode provides values for \(\partial C/\partial l\) which are independent of split length (see Appendix 2).

Figure 9 shows data for split growth in monotonic loading. The damage appears to ‘pop in’ at a stress of about 120 MPa, which cannot be predicted by the model. We believe that growth mechanisms different from those proposed may operate at small split lengths. If \(l < k\), eqn (25) is not appropriate. By ignoring the damage initiation load, and fitting eqn (25) to the subsequent split growth, good agreement between theory and data is obtained with values of \(k = 0.41\) mm (\(\rho = 1.0\) mm), \(G_t = 300\) J m\(^{-2}\), \(G_a = 158\) J m\(^{-2}\) and \(\partial C/\partial l = 1.52 \times 10^{-7}\) N\(^{-1}\). The value of \(G_t\) is that used for a cross-ply laminate, indicating perhaps a change in mode of loading of the delamination compared to the (90/0) laminate.

5.2 Damage growth in fatigue

The form of the fatigue law remains (eqn (3)):

\[
\frac{d \ell}{dN} = \lambda \left[ \frac{\Delta G}{G_c} \right]^{m/2}
\]

Combining eqns (3), (13) and (24):

\[
\frac{d \ell}{dN} = \lambda \left[ \frac{1/2(\Delta P)^2}{G_t + G_a } \frac{(\partial C)}{\partial l} \frac{(l + k)}{2} \right]^{m/2}
\]

Since \(\lambda\) and \(m\) are constants for a given matrix/interface, they should retain the values used in the cross-ply model. Likewise, since \(G_a\), \(G_t\) and \(k\) have been established from the monotonic tensile tests it should be possible to apply the model without further calibration. The predicted split growths and experimental data are plotted for two peak stresses (\(\sigma_{max} = 100\) and 200 MPa) (Fig. 10). A good prediction is achieved.
By adjusting the ply thickness in the finite element model (Appendix 1), the damage model can be used to predict fatigue damage growth in (90/+45/-45/0), laminates. Predicted curve and data are shown in Fig. 11. A good agreement is achieved without further adjustment of the model.

The extension of the model from one family of laminates to another of different lay-up is a major achievement. Although a slight change in the value of $G_0$ was found necessary from monotonic tensile tests, neither of the constants of the fatigue model required alteration. The success of the model combining monotonic and cyclic damage growth has been clearly demonstrated.

6 CONCLUDING REMARKS

In this paper, a fatigue law for notched laminates has been formulated. The law is an extension of a model for damage growth in monotonic loading, from which some of the parameters of the law can be derived. While the law is successful in predicting the extent of damage growth by utilising prior knowledge of the fatigue damage pattern, it does not attempt to predict the path of damage (a three-dimensional finite element approach would be necessary to do this). This represents a limitation, although there is some evidence that knowledge of the damage modes observed in one laminate can be extended to predict damage that develops in laminates of a similar lay-up. A word of caution: the effects of residual stresses and transverse ply cracks on the damaging processes have been omitted from the present work.

The development of a damage-based fatigue law fulfils the first part of the modelling procedure proposed in our first paper (Part I). In the remaining two papers (Parts III and IV), we relate the damage state to changes in mechanical properties.

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REFERENCES

APPENDIX 1: FINITE ELEMENT MODEL OF DAMAGED NOTCHED CROSSPLY LAMINATES

The finite element model is a modification of the one used by Kortschot and Beaumont\textsuperscript{9} which consists of two superimposed identical layers of two-dimensional plane stress elements. The layers have the in-plane properties of the 90° and 0° plies, where the elastic properties of a single ply are

\[ E_{11} = 135 \text{ GPa} \quad E_{22} = 9.6 \text{ GPa} \quad G_{12} = 5.8 \text{ GPa} \]
\[ \nu_{12} = 0.31 \quad \nu_{21} = 0.022 \]

where \( E_{11}, E_{22} \) are the moduli measured parallel and perpendicular to the fibre directions, \( G_{12} \) is the in-plane shear modulus, and \( \nu_{12} \) and \( \nu_{21} \) are the principle Poisson ratios.

A split in a 0° layer is modelled by using nodal pairs. The two layers share the same nodes everywhere except in the delaminated region, where separate nodes are generated in the 90° ply. Only one quadrant of the specimen, consisting of one 0° and one 90° ply, is modelled, and appropriate symmetry conditions are applied. Because this investigation concerns centre-notched specimens some modifications were required to Kortschot and Beaumont’s model for edge-notched specimens. In particular, the symmetry conditions were rearranged. The mesh uses a square-ended notch, which is a reasonable approximation to round-tipped notches blunted by split extension. A typical mesh (\( l/a = 2 \)), is shown in Fig. A1.1. For all meshes the overall length was 15.167 \( a \). Meshes with different values of \( W/2a \) were also constructed.

The mesh was loaded by applying a displacement of 0.05\( a \) to the top edge of the specimen. In Fig. A1.2, the compliance of the meshes is plotted against split length for delamination angles of 3.5°, 7° and 14°. In all cases, the compliance, \( C \), is directly proportional to the split length, \( l \), so that \( \partial C / \partial l \) is independent of split length. The relationship between the value of \( \partial C / \partial l \) obtained from the FE model (\( \partial C / \partial l \text{FE} \)) and those for the actual (90/0), specimen is given by the scaling equation:

\[
\frac{\partial C}{\partial l} = \left( \frac{W}{2} \right)_{\text{FE}} \left( \frac{2l}{2r} \right)_{\text{FE}} \frac{\partial C}{\partial l} \text{FE}
\]

\[(A1.1)\]

Fig. A1.1. Detail of the two-ply finite element mesh. The entire mesh consists of two layers of the same form.

Fig. A1.2. Normalised compliance \( C/C_0 \) versus \( l/a \) for various delamination angles.
The same FE model can be used to derive data for all laminates of the form \((90/0)_m\) in which there is an equal total thickness of \(90^\circ\) and \(0^\circ\) plies. The scaling equation becomes

\[
\frac{\partial C}{\partial l} = \frac{(W/2)_{\text{FE}}(\text{int})_{\text{FE}}}{(W/2)(2\text{int})} \frac{\partial C}{\partial l}_{\text{FE}} \tag{A1.2}
\]

The method can be extended to laminates with unequal thicknesses of \(0^\circ\) and \(90^\circ\) plies by altering the relative ply thicknesses of the layers of the finite element mesh. Table A1.1 shows values of \(\frac{\partial C}{\partial l}\) for a range of laminates and different values of \(W/2a\).

### APPENDIX 2: FINITE ELEMENT REPRESENTATION OF \((90/\pm 45/\pm 45/0)_s\)

Figure A2.1 shows the finite element mesh used for this damage pattern. The method employed was the same as for the cross-ply laminates. Two identical layers were used: one represents the \(0^\circ\) ply with the appropriate properties; the other represents the \((90/\pm 45/\pm 45)\) plies with equivalent laminate properties derived from the unidirectional-ply properties using laminated plate theory. The equivalent \((90/\pm 45/\pm 45)\) ply was three times as thick as the \(0^\circ\) ply. The two layers were separated in the delaminated region (dotted area in Fig. A2.1); elsewhere, the elements of the two layers shared the same nodes. Figure A2.2 shows a graph of compliance versus split length for the series of meshes used in this study. The compliance varies linearly with increasing split length, implying \(\frac{\partial C}{\partial l}\) is independent of split length. The method of scaling was the same as for the cross-ply specimens using eqn (11). A \((90/\pm 45/\pm 45/0)_s\) laminate was modelled by using the same mesh, but halving the \(0^\circ\) ply thickness. Values of \(\frac{\partial C}{\partial l}\) for these notched laminates are shown in Table A2.1.

### Table A1.1. Compliance results for multi-directional laminates

<table>
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<th>(W/2a)</th>
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<td>((90/0)_s)</td>
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<td>(1.45 \times 10^{-13})</td>
</tr>
<tr>
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</tr>
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Note: The values of \(\frac{\partial C}{\partial l}\) shown above refer to FE meshes with \(a = t = 1\) m, and should be scaled accordingly using eqns (A1.1) or (A1.2).

### Table A2.1. Compliance results for multi-directional laminates

<table>
<thead>
<tr>
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<th>(\frac{\partial C}{\partial l})_{FE} ((N^{-1}))</th>
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<td>(7.91 \times 10^{-14})</td>
</tr>
</tbody>
</table>

Note: The values of \(\frac{\partial C}{\partial l}\) shown above refer to FE meshes with \(a = t = 1\) m, and should be scaled accordingly using eqns (A1.1) or (A1.2).