ABSTRACT

A nonlinear reduced-order modeling approach based on Proper Orthogonal Decomposition (POD) is utilized to develop an efficient low order model, based on ordinary differential equations, for mechanical gas face seal systems. An example of a coned mechanical gas face seal is given in a flexibly mounted stator configuration is presented. The axial mode is modeled, and simulation studies are conducted using different initial conditions and forcing inputs. The results agree well with a fully meshed finite difference model, while the resulting model order is significantly decreased.

INTRODUCTION

In flexibly mounted stator mechanical gas face seal systems, the dynamic behavior of the floating stator is significantly influenced by the thin gas film’s stiffness and damping. To date, because of the highly nonlinear character of the thin gas film dynamics, the dynamic analysis of gas face seals mainly relies on direct numerical simulation of motion. However, direct numerical simulation is computationally intensive, making it impossible to use in a real-time control system. Although a few linear models have been used successfully for constant operating conditions (e.g., shaft speed, axial clearances) to study gas seal systems, these models do not describe the nonlinear characteristics of the thin gas film. Therefore, an effective model that is applicable to a wide range of operating conditions will make it more convenient to design mechanical gas face seals and their control systems.

MECHANICAL GAS FACE SEAL MODEL

Mechanical gas face seals have three degrees of freedom: axial stator translation ($Z$) from its equilibrium clearance ($C_0$) and two tilts. Forces and moments are exerted on the stator from a flexible support and from hydrodynamic and hydrostatic pressure within the thin gas film. The flexible support is assumed to have axial stiffness, $k_z$, and damping, $d_z$, and angular stiffness, $k_{\gamma}$, and damping, $d_{\gamma}$.

The pressure distribution within the thin gas film is determined by solving the compressible form of the Reynolds equation [1],

$$\nabla \left( \rho h^2 \nabla p - 6 \mu \nabla \rho \delta \phi \right) = 12 \mu \frac{\partial (\rho h)}{\partial t} \tag{1}$$

where $r$ and $\theta$ are the polar coordinates, $\nabla$ is the polar coordinate gradient operator, $p$ is the pressure, $h$ is the film thickness, $\mu$ is the gas viscosity, $\Omega$ is the shaft speed, and $\phi$ is the unit vector in the angular direction. The boundary conditions are $p(r,\theta,t) = p$, $p(r,\theta,t) = p_{in}$, and $p(r,\theta,t) = p(r,2\pi,t)$ where $p$ and $p_{in}$ are the external pressures at the inner and outer radii, respectively.

In (1) the pressure is coupled to the film thickness, $h$, and the squeeze term, $\partial h / \partial t$, through the stator degrees of freedom. Motion of the stator is governed by the following equations of motion [1]

$$m \ddot{Z} = -F_{z,eq} + F_z + F_c - k_z Z - d_z \dot{Z}$$

$$I_{\gamma} \ddot{\gamma} = M_{\gamma} - k_\gamma \gamma - d_\gamma \dot{\gamma} \tag{2}$$

where $F_{z,eq}$ is the axial equilibrium force resulting from the static deflection of the support and external back pressure and $F_c$ is the control force generated from an adjustable back pressure. The gas film force, $F_z$, and moments, $M_z$ and $M_{\gamma}$, are computed by integrating the pressure profile over the stator area.

A fully meshed finite difference model (FDM) is constructed to calculate the dynamic response of the gas pressure. The order of the full model is the number of the nodes in the finite difference mesh, $N_{nod}$, plus the kinetic model order, which is six.

Construction of POD Modes for Thin Gas Films

To construct the POD basis functions for the thin gas film, first the system dynamics are simulated using the fully meshed FDM model. An ensemble of simulations spanning the desired range of operating conditions are executed, and the spatial distribution of the pressure profile $p(r,\theta,t)$ is sampled at a series of $N_t$ different time steps. The collected pressure profiles are stored in the matrix, $\hat{p}$, which is mean
subtracted. The left-singular vectors of \( \hat{P} \) construct the basis functions (or modes), \( \phi_i \), of the pressure.

Next, the pressure profile is expanded as the product of the time varying coefficients, \( \alpha_i(t) \), and the spatial basis functions, \( \varphi_i(r, \theta) \)

\[
\hat{p}(r, \theta,t) = \bar{p}(r, \theta) + \sum_{i=1}^{N} \alpha_i(t) \varphi_i(r, \theta)
\]

(3)

where \( \bar{p}(r, \theta) \) is the mean pressure profile and \( N \) is the number of basis functions selected (\( N \ll N_i \)).

**Reduced Model Using Galerkin Projection**

The original governing partial differential equation of the mechanical gas face seal system is converted to

\[
\frac{\partial p}{\partial t} = \frac{D(p)}{r^2 h} \left( \frac{h^3}{12 \mu} \frac{\partial^2 p}{\partial \theta^2} \right)
\]

\[
+ \frac{1}{r h} \frac{\partial}{\partial r} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial r} \right) - \frac{\Omega}{2h} \frac{\partial}{\partial \theta} \left( p h \frac{\partial h}{\partial \theta} \right) - \frac{p \varphi(h)}{h} \frac{\partial \varphi(h)}{\partial t}
\]

(4)

The residual function is defined as

\[
R = \frac{\partial \hat{p}}{\partial t} (r, \theta, t) - D(\hat{p}(r, \theta, t))
\]

(5)

The Galerkin condition requires the inner product of \( R \) and \( \phi_i \) to be zero

\[
(R(\hat{p}(r, \theta, t)), \varphi_i(r, \theta)) = 0 \quad i = 1, ..., N
\]

(6)

Substituting (4) and (5) into (6), and using the orthogonality condition for the basis functions, the following equation of the temporal coefficient \( \alpha_i(t) \) is

\[
\hat{\alpha}_i = (D(\hat{p}(r, \theta, t)), \varphi_i) \quad i = 1, ..., N
\]

(7)

That is, the temporal term in the pressure profile approximation can be found by solving the \( N \)th order ODE system in (7). The initial condition for \( \alpha_i \) is

\[
\alpha_i(0) = (p(r, \theta, 0) - \bar{p}(r, \theta), \varphi_i) \quad i = 1, ..., N
\]

(8)

**RESULTS**

The nonlinear modeling methodology described above is applied to a coned mechanical gas face seal whose parameters are given in [2]. For simplicity, only the axial mode is considered. The tilt modes can be modeled in a similar manner.

To generate the basis functions, an ensemble of snapshots is collected from a set of full simulations of an initial condition response, a step response, and two sinusoidal responses (with frequencies of 100 Hz and 500 Hz, respectively). In the full model, the gas film is discretized using a 140(\( \theta \))x45(\( r \)) mesh. The full simulation is run within a wide operating range of the nondimensional compressibility number, \( \Lambda \) (\( \Lambda = 6\mu \Omega r_o^2 \rho_p C_o^2 \)) by varying the initial clearance \( C_0 \) (0.5–5 \( \mu \)m) while maintaining a fixed shaft speed of \( \Omega = 2000 \text{ rad/s} \). The corresponding compressibility numbers are \( \Lambda = 31104.0, 1944.0, 634.8 \) and 311.0.

To validate the constructed reduced models, results are compared with simulations from the full FDM model. Figures 1 and 2 show the simulation results corresponding to a step and sinusoidal responses, respectively, for compressibility numbers of \( \Lambda = 7776.0 \) and \( \Lambda = 486.0 \). Both simulations are run with a shaft speed \( \Omega = 1000 \text{ rad/s} \). The figures show that, with \( N = 5 \) basis functions, the reduced model approximates the full model response very well and shows insensitivity to the compressibility number in the desired operating range.

**CONCLUSIONS**

The results demonstrate that the reduced order model compares well with the fully meshed model, while achieving significant computational efficiency by decreasing system order. The results also show that the reduced order model is “global” in the sense that it is applicable to a wide range of compressibility numbers.

**REFERENCES**
