ELASTIC-PLASTIC ANALYSIS OF THE CYLINDRICAL CONTACT BETWEEN LAYERED BODIES

Bachelet, L., Sainsot, P.
Institut National des Sciences Appliquées
Lyon, France

ABSTRACT
Metallic gaskets are frequently used in applications operating under extreme conditions (very high or low temperatures). The elastic-plastic contact conditions between a layered toroidal seal and its seat are presented in this paper using a bi-dimensional finite element method. This problem is similar to the contact between an elastic cylinder and an elastic substrate coated by a soft elastic-plastic layer. Three different types of behavior could be defined according to the thickness of the layer: (A) very thin, (B) very thick and (C) intermediate layer. First we propose a new equivalent elastic modulus, taking into account the properties of the layer and substrate. This new formula is a function of the ratio of the thickness of the layer to the contact width. Then, by using a dimensional approach, two criteria related to the layer thickness (very thin or very thick) are defined. Finally, for intermediate layers, the specific shape of the pressure field is explained.

1. INTRODUCTION
In this paper, the contact between an elastic cylinder and an elastic-plastic layer bonded on an elastic substrate is studied using a finite element analysis. In order to predict and understand the different types of behavior of such contact, numerical simulations have been performed, an analytical expression for the equivalent elastic modulus in a layered medium and some dimensionless forms are proposed.

2. MODEL AND OBSERVATIONS
The initial problem consists of a contact study between a layered toroidal seal and its seat. As the tor dimensions are very large compared to the contact, this problem is reduced to the indentation of an elastic-plastic layer (silver) bonded on an elastic substrate (steel) by an elastic cylinder (steel). So, a 2-D finite element model, using the ABAQUS code, was built. Depending of the operating conditions (load, thickness and plastic properties of the layer...), the contact characteristics vary from elastic behavior (case a - low load or thin layer) to fully-plastic behavior (case b - large load and thick layer). For intermediate thicknesses (case c) of the layer the pressure field presents a particular shape, with an increase of the pressure at the center of the contact.

3. THEORETICAL BACKGROUND
3.1 Elastic domain
For a line contact, Hertz theory defines the maximum pressure \( P_h \) and the contact half-width \( b \). They are a function of the mechanical properties of the cylinder and the medium. For a coated medium (thickness \( t \)) the elastic properties are a mix of the layer properties and the substrate properties. In this study we have considered that the equivalent elastic modulus of the medium \( E_m \) is the weighted average of the elastic properties of the layer and of the substrate:

\[
\frac{1}{E_m} = \alpha \frac{1-u_l^2}{E_1} + (1-\alpha) \frac{1-u_s^2}{E_s},
\]

the subscripts \( l \) and \( s \) refer to the layer and to the substrate. Elastic simulations have been conducted which give the relation \( b/b_0 \) and \( t/b_0 \) (the subscripts \( o \) refers to the uncoated case). Using the exponential shape of this curve, a simple approximation for \( \alpha \) has been obtained:

\[
\alpha = 1 - \exp\left(-0.9 \frac{1}{b_0}\right).
\]

3.2 Plastic domain
The Von Mises criterion \( \tau_{\text{max}} = \sqrt{3}Y \) is used to determine if the contact behavior becomes plastic. For a line contact, \( \tau_{\text{max}} = 0.32P_h \) and is located at a depth \( d = 0.7b \). The Von Mises criterion becomes \( P_h = 1.8Y \). However, for a layered medium, this condition of plasticity is not sufficient. As only the layer is elastic-plastic, the maximum shear stress must be located in the layer. Then, the medium behavior becomes plastic if \( t > 0.7b \).
So, a criterion for the thin layers could be proposed.
First, the Hertz contact half-width \( b_h \) is given by
\[
\frac{b_h}{t} = 7.2 \frac{R \cdot \sqrt{Y}}{t \cdot E'}.
\]
So, the medium behavior is plastic if
\[
\frac{b_h}{t} < \frac{1}{0.7},
\]
and the layer is thin (i.e. \( \tau_{\text{max}} \) is located in the substrate) if
\[
\frac{t}{R} < \frac{5 \cdot \sqrt{Y}}{E'}.
\]

For a sufficiently thick layer (case b), the maximum contact pressure in the plastic domain is limited by the material hardness (Tabor 1951), which, for a perfectly plastic layer, corresponds to three times the yield stress \( P_{\text{max}} = 3 \cdot Y \).

Moreover, as the pressure field shape is flat, the contact half-width, in the plastic domain, takes a simple form according to the load applied
\[
b_{\text{pl}} = \frac{W}{2 \cdot P_{\text{max}}} = \frac{W}{6 \cdot Y}.
\]

This theory (called "hardness theory") is valid only for the elastic-plastic homogeneous media or very thick layers.

For intermediate layers (case c), the maximum pressure increases after a given load \( W_{\text{lim}} \) (Figure 1).

![Fig. 1 – Maximum contact pressure versus load](image)

In the next part, the results of the finite element simulation are used to understand and predict this specific behavior.

### 4. RESULTS, EXPLANATION AND DISCUSSION

#### 4.1 Normalized forms

In order to describe the different behavior of a layered medium, some dimensionless expressions are presented. At the beginning of the loading, the medium behavior is elastic. A good representation is obtained using the Hertz equations
\[
P_h = \frac{1}{4} \cdot E' \cdot b_h.
\]
The elastic domain is valid until
\[
\frac{E' \cdot b_h}{Y \cdot R} < 7.2.
\]
For \( P_{\text{max}}/Y > 1.8 \), the behavior becomes plastic and for thick layers, the maximum contact pressure is limited by the layer hardness
\[
\frac{P}{Y} = 3.
\]
Increasing the load, the case c appears for \( b_h/t = 1.4 \). So a new criterion for the thick layers can be
\[
\frac{t}{b_{\text{pl}}} > \frac{1.4}{1.4}.
\]

#### 4.2 Explanation

The different types of behavior from the elastic domain to the "hardness domain" have already been identified in previous studies, but the end of the "hardness domain" has never been enhanced. To explain this new phenomenon, the distribution of the equivalent plastic deformations \( \varepsilon_p \) is used. The maxima for \( \varepsilon_p \) are located at the interface between the layer and the substrate and at the contact center. Indeed, during the loading, the plastic flow of the layer moves towards the outside of the contact zone up to be obstructed by the elastic substrate. So the plastic deformations lock at the interface between the layer and the substrate and the hydrostatic pressure increases at the contact center. Then the contact pressure increases too and leaves the "hardness domain".

### 5. CONCLUSION

In this paper, the elastic-plastic contact between a cylinder and a layered medium has been studied. Three different types of behavior have been observed, they are governed by the thickness of the layer:

- Very thin layers for which the medium follows the elastic behavior of the substrate. Here, the layer has a small influence on the contact and the deformations are elastic and located in the substrate.

- Very thick layers for which the behavior of the medium is imposed by the properties of the layer. The deformations are elastic then plastic and are located in the layer. The presence of the substrate does not affect the contact behavior.

- Intermediate cases for which the layer is neither very thin nor very thick. Here, the medium behavior is influenced by the layer and by the substrate. During the loading, the elastic deformations move from the layer to the substrate. So the equivalent elastic modulus of the medium evolves and its expression is a weighted average of the material properties of the two components. At the beginning of the plastic domain, the medium follows the perfectly plastic behavior of the layer and the contact pressure is limited to the layer hardness. Finally, after a given load is reached, the substrate limits the plastic flow of the layer and the contact pressure increases at the center of the contact. So the medium leaves the "hardness domain".

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### REFERENCES


