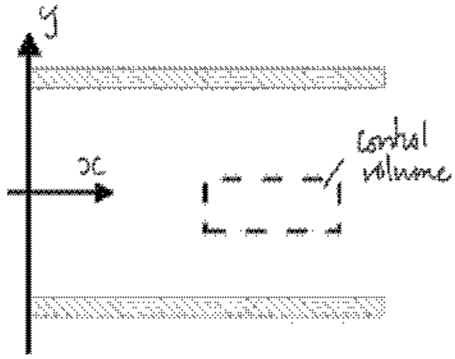
CHAPTER 4

BOUNDARY LAYERS

- Combined Couette and Poiseuille flow
- Boundary layers
- Boundary layer growth
- Bernoulli, streamline curvature and boundary layers
- Pressure gradients in boundary layers
- Boundary layer separation
- Delaying separation

4.1 Combined Couette and Poiseuille flow

What happens when we combine Couette and Poiseuille flow? The force balance and the equation of motion are the same but the boundary conditions are different.

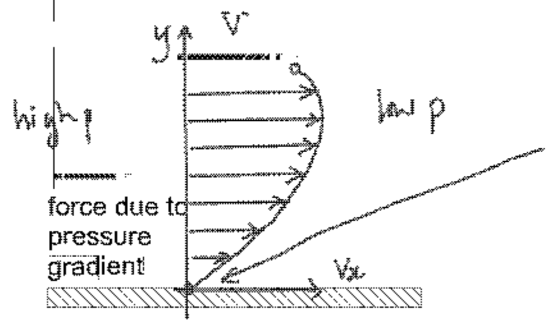


$$\frac{d\tau}{dy} = \frac{dp}{dx}$$

$$\Rightarrow \mu \frac{d^2 v_x}{dy^2} = \frac{dp}{dx}$$

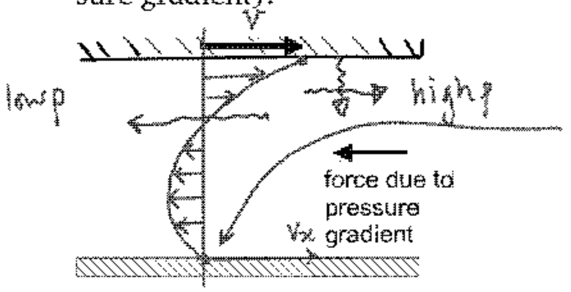
$$\Rightarrow v_x = \left(\frac{1}{2\mu}\frac{dp}{dx}\right) y^2 + By + C$$

$\Rightarrow$ parabolic v-profile



- parabolic
- steep velocity gradient at the bottom plate : $\frac{dv_x}{dy}$ is large

(b) Pressure pushes in the opposite direction to the top plate's motion (*adverse* pressure gradient):



- parabolic
- the flow can reverse next to the bottom plate : $\frac{dv_x}{dy}$ is negative
- There is a competition between the pressure gradient pushing backwards and the momentum diffusion from the top plate pushing forwards

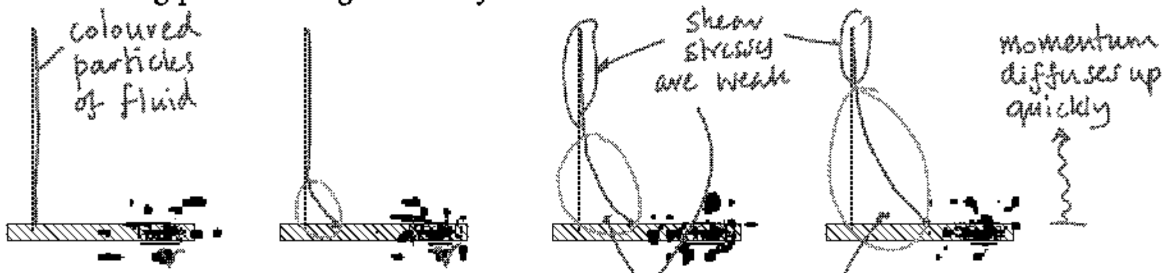
When the top plate moves one way and the pressure gradient pushes the other, the flow can reverse direction. There is a competition between diffusion of the momentum down, which pushes the fluid forwards, and the adverse pressure gradient, which pushes the fluid backwards. This has a very important consequence for boundary layers, which we will consider next.

4.2 BOUNDARY LAYERS

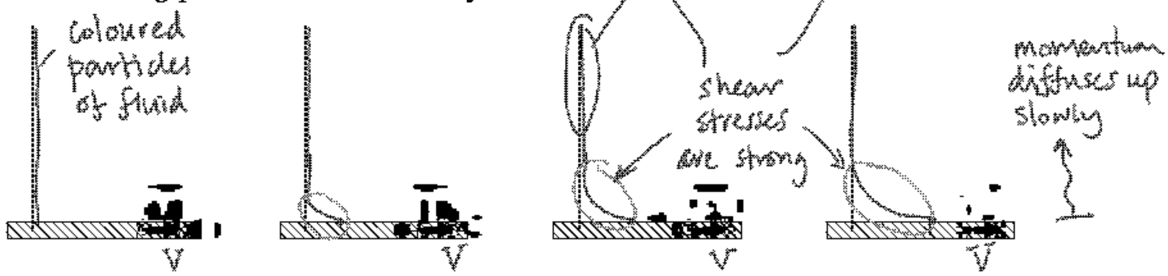
So far we have considered a viscous flow between two flat plates. The fluid next to the plates takes the velocity of the plates (the no slip condition). Momentum then diffuses through the fluid due to molecular motion, giving rise to a shear stress that is proportional to the velocity gradient. The constant of proportionality is the viscosity, μ.

The same thing happens if we have a single plate in an unbounded fluid. The layer of fluid next to the plate takes the plate's velocity. This layer exerts shear stresses on the next layer up, causing it to accelerate or decelerate. This process continues as momentum diffuses up through the fluid. This momentum diffusion is faster in more viscous fluids:

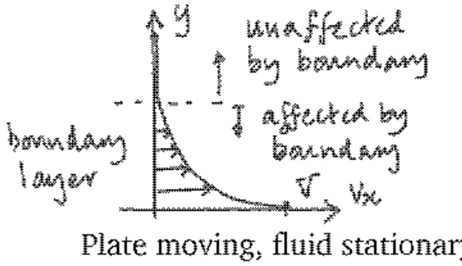
Film of a moving plate in a high viscosity fluid:



Film of a moving plate in a low viscosity fluid:



The region of fluid in which there are appreciable shear stresses is called the *boundary layer*. It is *defined* as the region in which viscous forces dominate over inertial forces (which are mass $\times$ acceleration: $\rho D\mathbf{v}/Dt$). Outside this region, by definition, viscous effects are negligible.



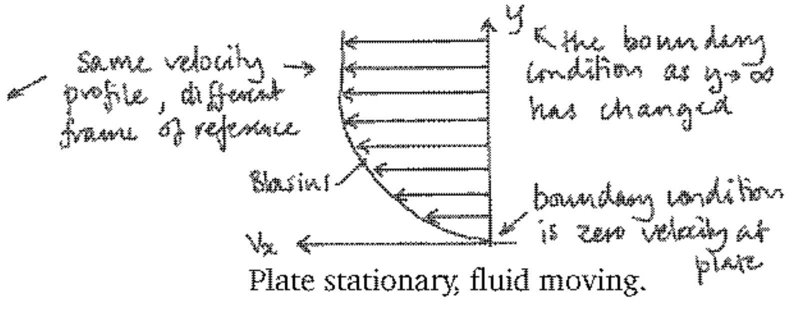
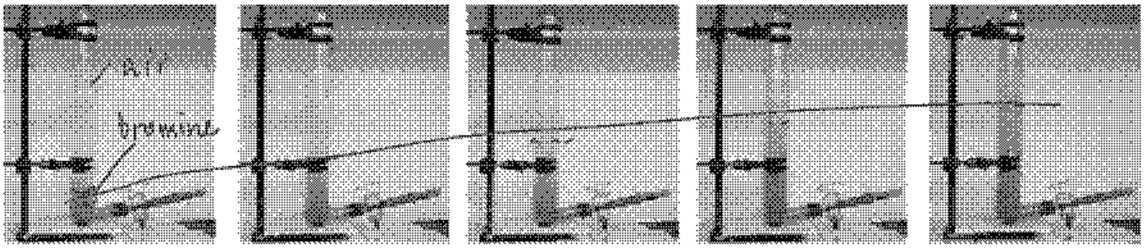


Plate moving, fluid stationary. Plate stationary, fluid moving.

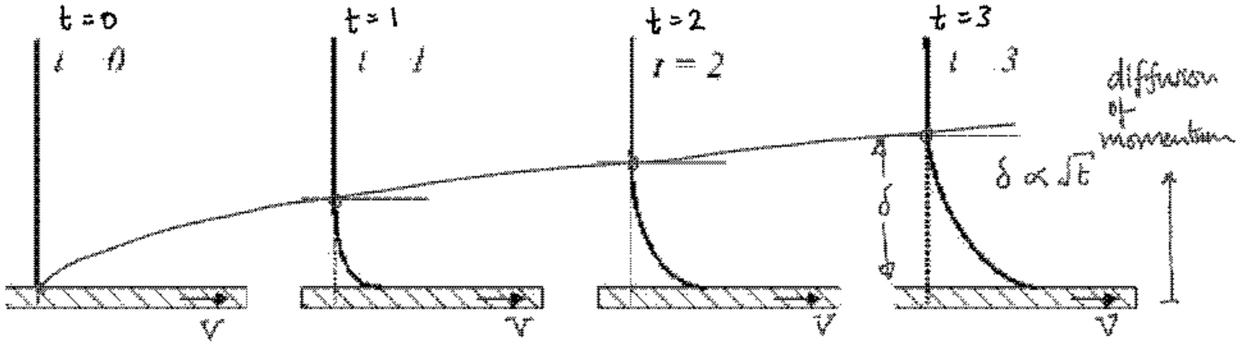
The velocity profile is no longer parabolic or linear (as it was for the flow between flat plates). This is because the top boundary condition has changed.

4.3 Boundary layer growth

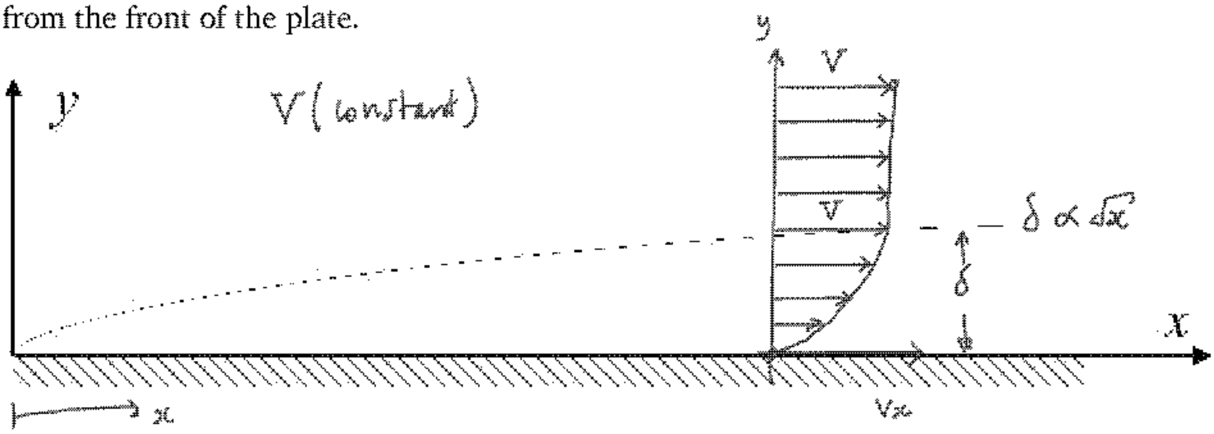
In any 1D diffusion problem, the diffused distance grows in proportion to the square root of time. You may have met this at school with bromine diffusing into air or ink diffusing in water.



Boundary layers are caused by diffusion of momentum. This is a diffusion problem so if the fluid is initially stationary and the bottom plate starts moving at time $t = 0$, the thickness of the boundary layer increases in proportion to $\sqrt{t}$.

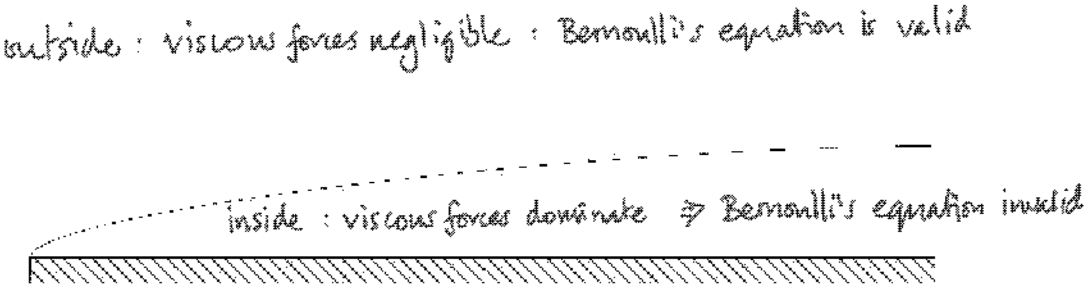


Similarly, if the fluid is moving at a constant velocity over a stationary flat plate, the thickness of the boundary layer increases in proportion to $\sqrt{x}$, where x is the distance from the front of the plate.



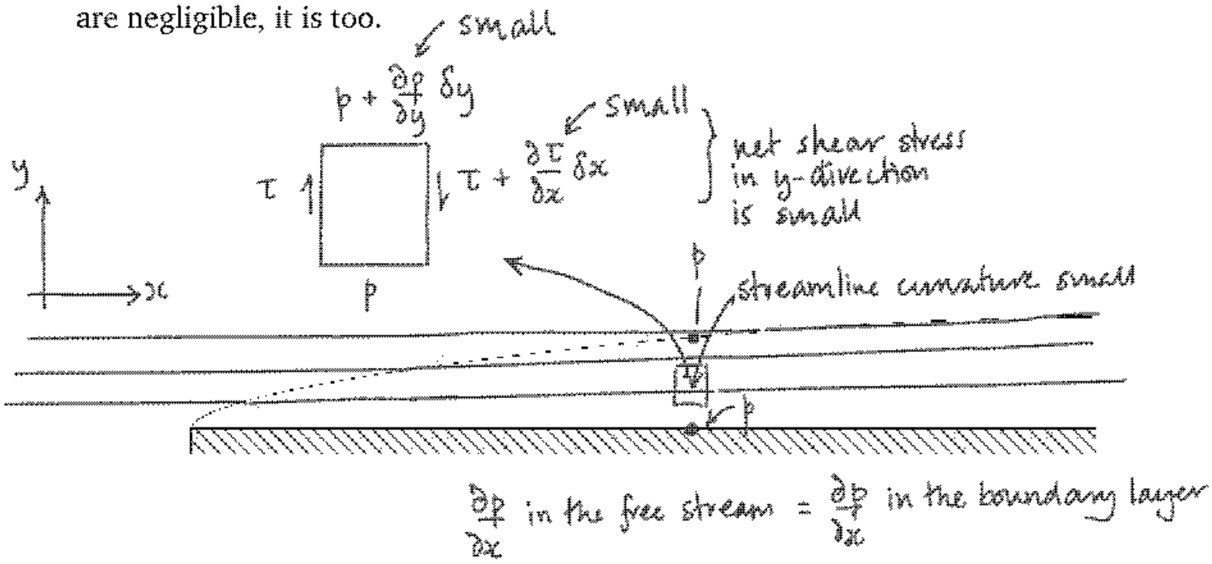
4.4 Bernoulli does not work inside a boundary layer

Bernoulli's equation (chapter 2) was derived by equating pressure forces and inertial forces. In a boundary layer, however, *viscous* forces dominate over inertial forces which means that *Bernoulli does not work inside a boundary layer*. Bernoulli can only be applied outside the boundary layer where, by definition, viscous effects are negligible.



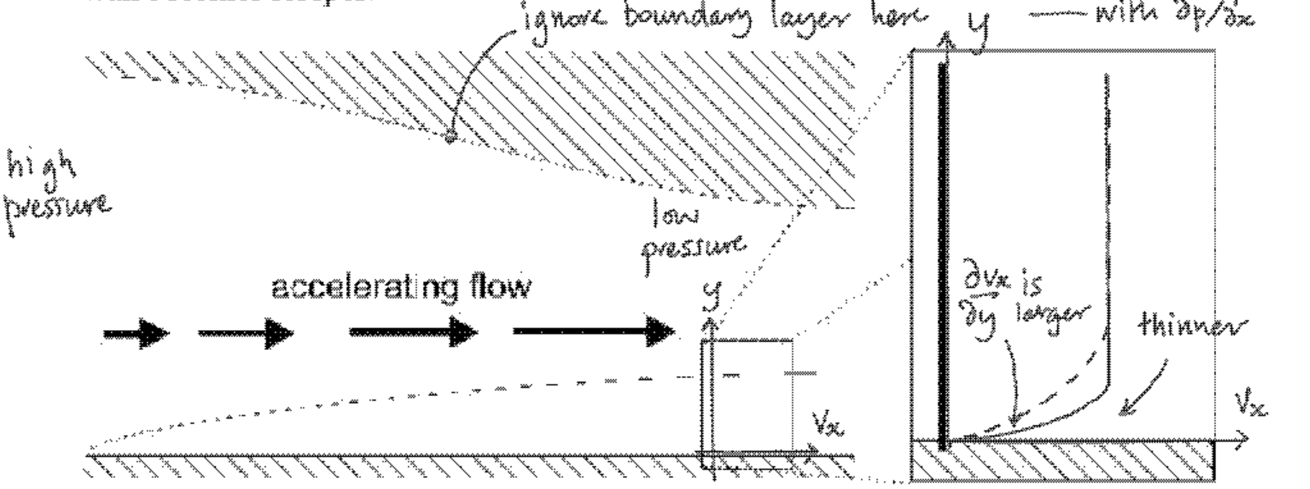
4.5 The pressure gradient across a boundary layer

The pressure at a point in a boundary layer is very nearly the same as that in the free stream just above it. This is for two reasons, which are best illustrated by considering a small element of fluid in the boundary layer. Firstly, the shear stress in the y-direction changes very little in the x-direction, which means that the net shear force in the y-direction is negligible. Secondly, the streamlines within a boundary layer are almost parallel, which means that the vertical acceleration of the fluid is also negligible. The pressure gradient in the y-direction balances with these and, because they are negligible, it is too.

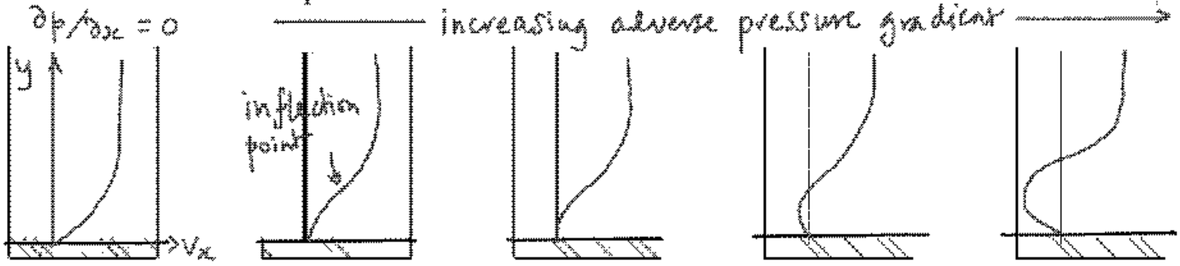


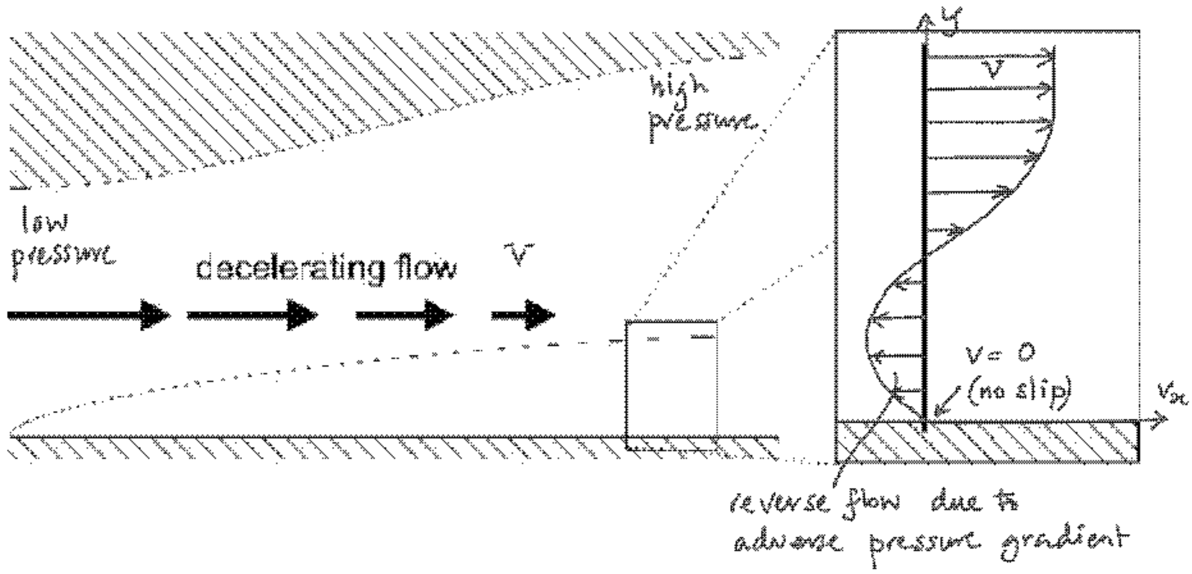
4.6 Boundary layers in pressure gradients

If the free stream is accelerating, then there must be a pressure gradient pushing the flow in the same direction as the free stream. This is similar to combined Couette/Poiseuille flow with a *favourable* pressure gradient. The velocity gradient at the wall becomes steeper:



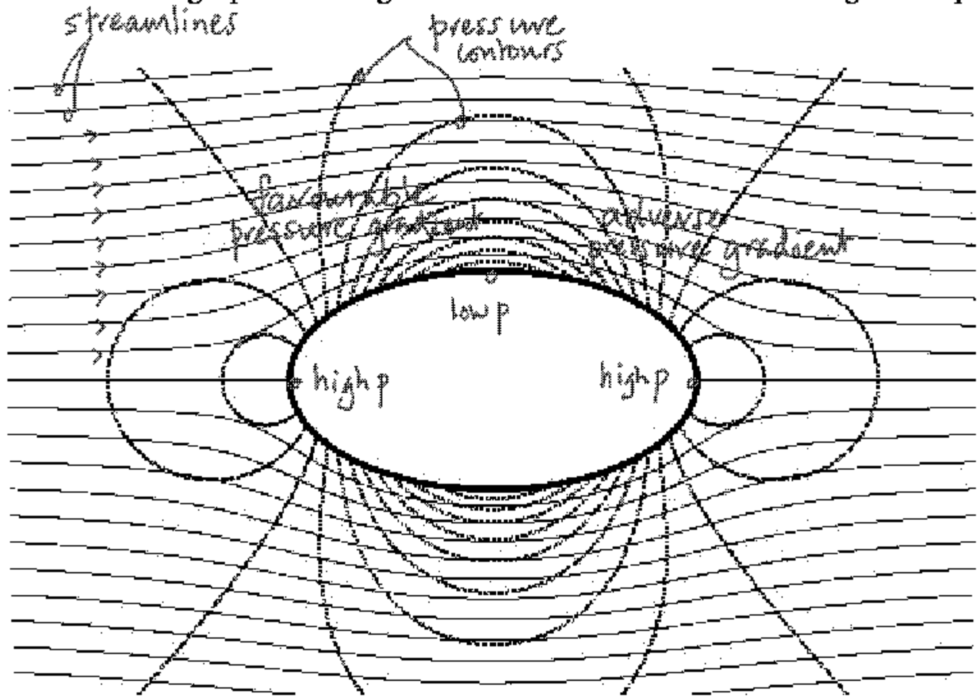
If the free stream is decelerating, then there must be a pressure gradient pushing the flow in the opposite direction to the free stream. This is similar to combined Couette/Poiseuille flow with an *adverse* pressure gradient. The velocity gradient at the wall becomes less steep and the flow can even reverse.



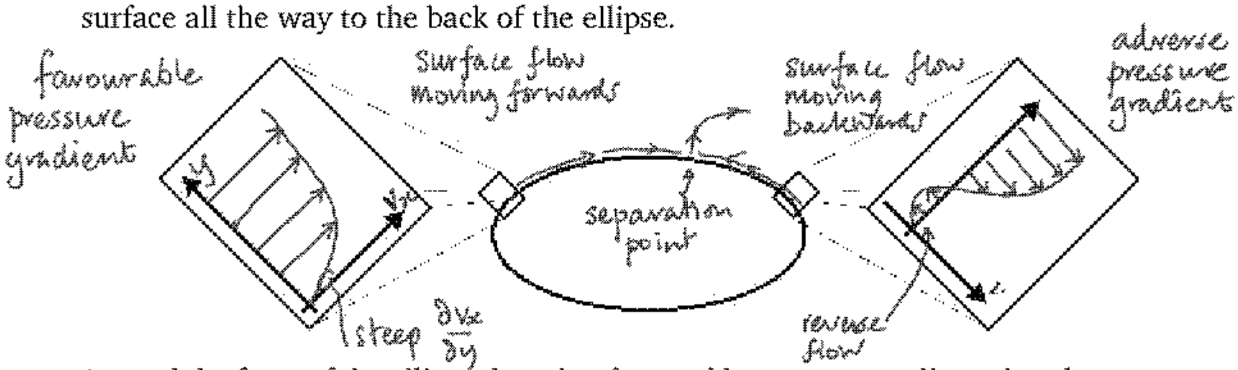


4.7 Boundary layer separation

The streamlines and pressure field of the *inviscid* flow around an ellipse are shown below. There are high pressure regions around the front and rear stagnation points.

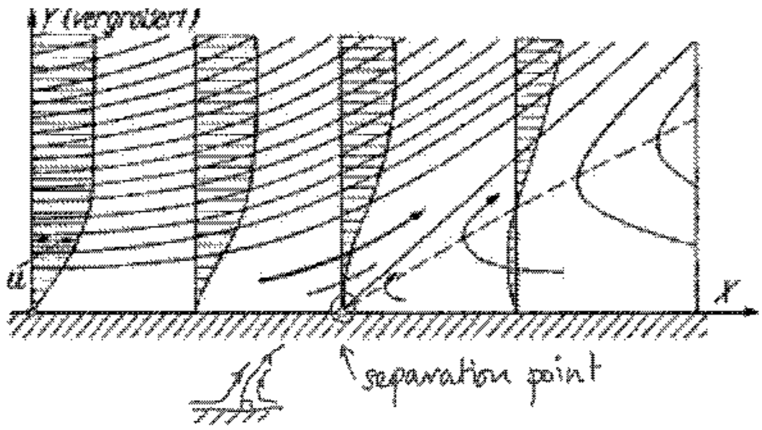


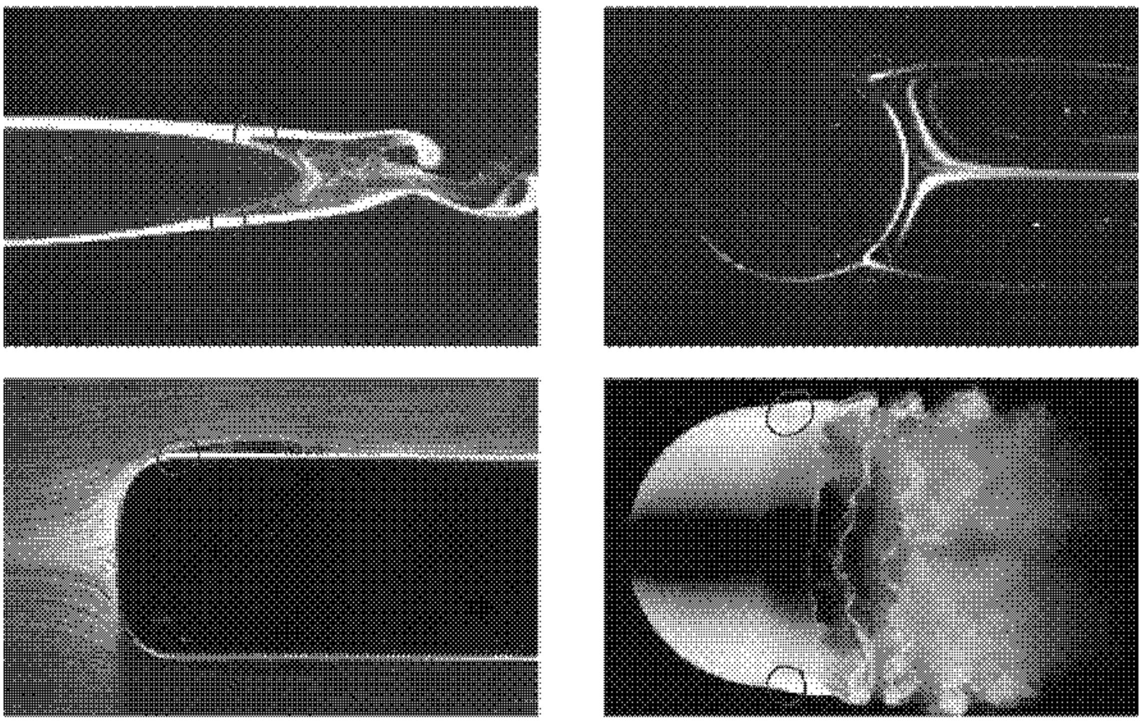
Viscous fluids obey the no slip condition. This means that a thin boundary layer forms around the surface of the ellipse, growing as the fluid moves from the front to the back. We will imagine for the moment that the boundary layer sticks to the surface all the way to the back of the ellipse.



Around the front of the ellipse there is a *favourable* pressure gradient - i.e. the pressure is pushing in the same direction as the bulk fluid motion. The favourable pressure gradient makes the velocity gradient at the wall steeper. Around the back of the ellipse there is an *adverse* pressure gradient - i.e. the pressure is pushing in the opposite direction to the bulk fluid motion. The adverse pressure gradient makes the velocity gradient at the wall less steep and, unless the fluid is extremely viscous, *eventually will cause flow reversal.*

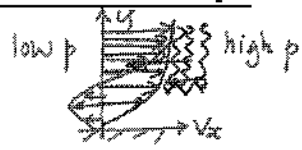
This flow reversal completely changes the flow. The reversing fluid has to go somewhere. It cannot reverse all the way to the front of the ellipse because there is a favourable pressure gradient there and all the fluid is moving forwards. Instead it separates from the body at a mini-stagnation point, which is called the point of separation. Ludwig Prandtl was the first person to realize this. The figure below is copied from his 1905 paper.



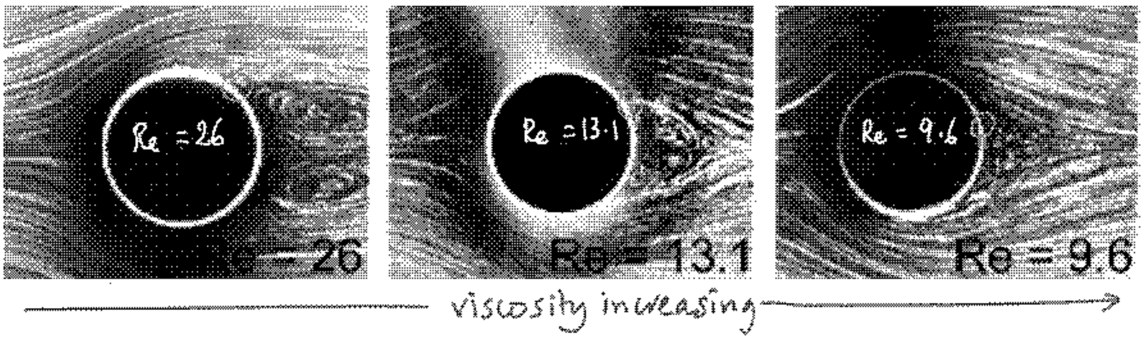


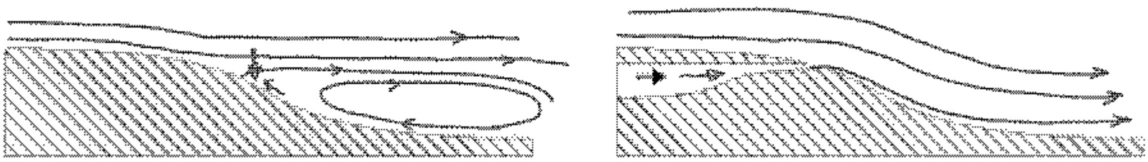
[1]These photographs, and similar ones throughout these notes, are taken from an *Album of Fluid Motion* by Milton Van Dyke

4.8 Delaying boundary layer separation

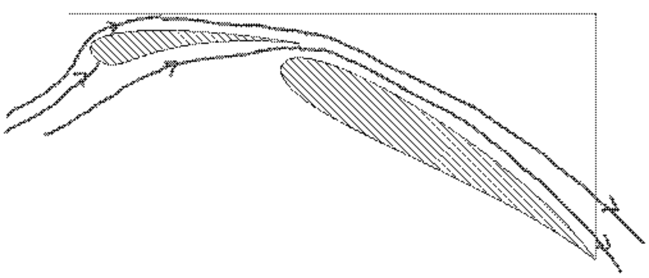


Separation occurs when flow reversal occurs. From our Couette/Poiseuille flow model we know that there is a competition between the adverse pressure gradient, which pushes the flow backwards, and the rate of momentum diffusion through the fluid, which pushes the flow forwards. We can increase the rate of momentum diffusion by increasing the viscosity. This delays separation:



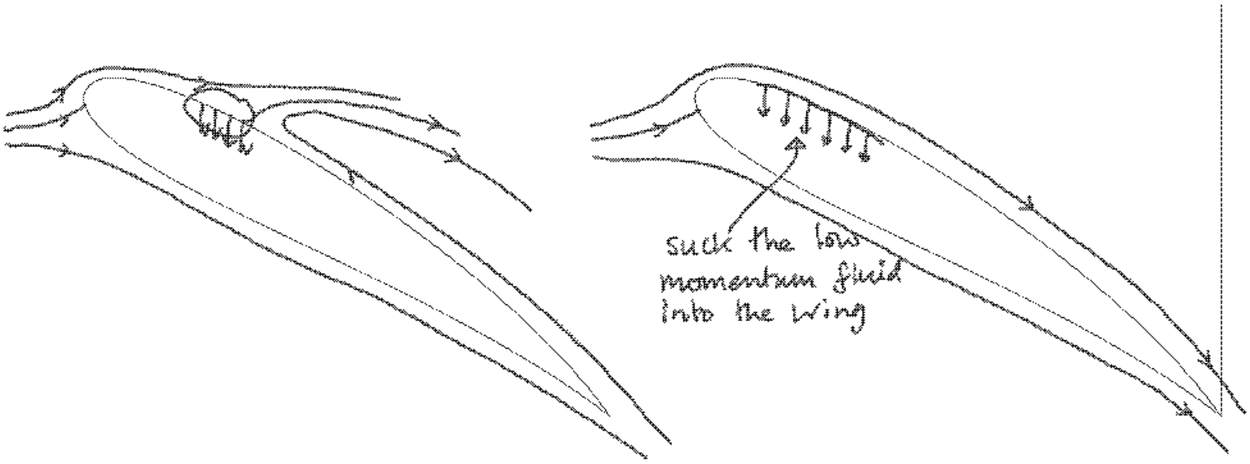


This is part of the principle behind leading edge slats on wings.



the boundary layer remains attached at high angles of attack

Another technique to avoid separation is to suck the low momentum part of the boundary layer into the wing:



This has been achieved on aeroplanes in flight but requires a great deal of power. In chapter 7 we will look at a more common way to increase momentum transfer into the boundary layer and hence avoid separation.