Non-normality in combustion–acoustic interaction in diffusion flames: a critical revision

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Perturbations in a non-normal system can grow transiently even if the system is linearly stable. If this transient growth is sufficiently large, it can trigger self-sustained oscillations from small initial disturbances. This has important practical consequences for combustion–acoustic oscillations, which are a persistent problem in rocket and aircraft engines. Balasubramanian & Sujith (J. Fluid Mech., vol. 594, 2008, pp. 29–57) modelled an infinite-rate chemistry diffusion flame in an acoustic duct and found that the transient growth in this system can amplify the initial energy by a factor, \( G_{\text{max}} \), of the order of \( 10^5 \) to \( 10^7 \). However, recent investigations by L. Magri and M. P. Juniper have brought to light certain errors in that paper. When the errors are corrected, \( G_{\text{max}} \) is found to be of the order of 1 to 10, revealing that non-normality is not as influential as it was thought to be.

Key words: Acoustics, Combustion, Flames

1. Results and discussion

Recent investigations have brought to light certain errors in Balasubramanian & Sujith (2008, labelled B&S in this note). We use the same model, discretization and non-dimensionalization as in B&S. The required corrections to B&S are listed below.

(a) The analytical steady solution, \( Z_{\text{st}} \) (appendix B, p. 54), obtained by separation of variables, is

\[
Z_{\text{st}} = X_i (1 - \alpha) - Y_i \alpha - \frac{2}{\pi} (X_i + Y_i) \sum_{n=1}^{\infty} \frac{\sin(n\pi\alpha)}{n(1 + b_n)} \cos(n\pi y_c)(e^{a_{n1} x_c} + b_n e^{a_{n2} x_c}),
\]

where

\[
a_{n1} = \frac{Pe}{2} - \sqrt{\frac{Pe^2}{4} + n^2 \pi^2}, \quad a_{n2} = \frac{Pe}{2} + \sqrt{\frac{Pe^2}{4} + n^2 \pi^2},
\]

\[
b_n = -\frac{a_{n1}}{a_{n2}} \exp \left( -2L_c \sqrt{\frac{Pe^2}{4} + n^2 \pi^2} \right).
\]

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The growth factor, $G_{\text{max}}$, as a function of (a) the Péclet number, $Pe$, and (b) the fuel slot half-width, $\alpha$. Panel (a) has $\alpha = 0.25$, and (b) has $Pe = 10$.

The non-dimensional coordinates of the combustion domain are $x_c$ and $y_c$. (b) The expressions for $C_m^{(n)}$ and $W_{nk}$ (equation (7), p. 36) are $2/L_c$ times the original terms, due to the Galerkin projection. Furthermore, $W_{nk} = 1/L_c$ when $k = m$. (c) The variable $Y_l$ on the right-hand side of the terms $R_{nm}$ and $J_{nm}$ (equation (13), p. 37) is $Y_i$. (d) The variable $\dot{Q}_{av}$ (equations (18) and (19), p. 39) is to be divided by 2 due to non-dimensionalization over the cross-sectional area. (e) The multiplying factor in front of matrix $M_1$ (appendix B, p. 54) is $1/((T_i + T_{ad})/2)$. (f) The expression for matrix $B_{NN}$ (appendix B, p. 54) is $B_{NN} = -D + A_1 - A_2 + A_3 - A_4 + A_5$, where

$$A_5 = \frac{1}{(T_i + T_{ad})/2} \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 & \sin(\pi x_f) & \sin(2\pi x_f) & \ldots & \sin(K\pi x_f) \end{bmatrix}^T$$

$$\times \begin{bmatrix} J_{00} & \ldots & J_{0M} & 0 & \ldots & 0 \end{bmatrix}$$

and $K$ is the number of Galerkin modes for acoustic discretization. (g) The damping terms in the matrix $S$ (appendix B, p. 55) are $+2\pi \xi_1$, $+4\pi \xi_2$, $\ldots$, $+2K\pi \xi_K$. (h) The numerator of matrix $A_4$ (appendix B, p. 55) is 1 due to non-dimensionalization over the cross-sectional area of the duct.

We perform computations with $50 \times 50$ Galerkin modes in the flame domain and six modes in the acoustic domain. When we increase the number of Galerkin modes to $70 \times 70$ in the flame and 12 in the acoustics, the eigenvalues and singular values change by less than 15%. The fixed parameters are: the fuel mass ratio, $Y_i = 3.2$; the oxidizer mass ratio, $X_i = 3.2/7$; and the average temperature, $T_{av} = 1/0.685$. We set the damping coefficients to $c_1 = 0.013$ and $c_2 = 0.08$ in order to have marginally stable systems. The nonlinear behaviour of this thermo-acoustic system is not considered because it has been fully characterized by Illingworth, Waugh & Juniper (2013).

Figure 1 shows the growth factor, $G_{\text{max}}$, as a function of (a) the Péclet number, $Pe$, and (b) the non-dimensional half-width of the fuel slot, $\alpha$. (Note that we use the same norm as B&S, even though Chu’s norm would be a more appropriate measure of the energy (Chu 1965).) In both cases, $1 < G_{\text{max}} \lesssim 10$. These plots can be compared with figures 9 and 10 in B&S. Figure 2 shows the eigenvalues and the pseudospectra for this thermo-acoustic system. These can be compared with figure 11 in B&S. The pseudospectra around the most unstable eigenvalues are nearly concentric circles whose values decrease rapidly as the distance from the eigenvalue increases. This is a further demonstration that the system is only weakly non-normal,
because a marginally stable but highly non-normal system would have pseudospectra that protrude significantly into the unstable half-plane (Trefethen & Embree 2005). Nevertheless, it is worth noting that Juniper (2011) showed that even a small amount of non-normality can make a system somewhat more susceptible to triggering.

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REFERENCES